

SAMPLE PROBLEMS FOR EXAM I

MATH 630 Actuarial Math, 2/18

For the exam show your work to ensure credit for the solution of each problem. Put a box around the answer to each problem. No use of calculators, books or notes is allowed.

1. A borrower repays a loan with 25 equal annual payments of 500 on 1 July of each year. The annual rate of interest is 10%. Find an expression for the value of the loan at the end of 10 years.
2. A loan of 10,000 is repaid with quarterly equal payments commencing three months after the loan for 5 years at 12% convertible quarterly. Find an expression for the loan balance immediately after the payment at $t=2$ years.
3. John borrows X at time $t=0$. He repays the loan during the next 20 years making an annual payment of P at the end of each year for each of the first ten years and then making semiannual payments of $2P$ for the remaining 10 years. If $P=500$ and the annual interest is $i=.06$ then determine X .
4. Jane and Mary have equal amounts of money to invest. Jane purchases a 10 year annuity with annual payments of 2500 at the beginning of each year. Mary invests her money in a savings account earning 9% effective annual interest for two years. At the end of two years, she purchases a 15 year annuity with annual payments of Z at the end of each year. Both annuities are valued using an effective annual rate of 8%. Find the value of Z .
5. A real estate agent has two offers for a house: (i) 40,000 now ($t = 0$) and 40,000 in two years ($t = 2$) or (ii) 28,750 now ($t = 0$), 23,750 in one year ($t = 1$), and 27,500 in two years ($t = 2$). The agent remarks that one offer is "just as good" as the other offer. Find the two values of the effective annual interest rate which make the agent's remark correct.
6. A perpetuity has annual payments that commence at the end of the first year. The payments are 1 at the end of the first year, $t=1$, then a payment of 2 at $t=2$ and a payment of 3 at $t=3$ and the payments continue

in this manner so that the sequence of payments is $(1, 2, 3, 1, 2, 3, 1, 2, 3, \dots)$. The interest rate is 12%. Find the value of the annuity at $t=0$.

7. Let i be the annual rate of interest. Find an expression for the initial value of an annuity that commences on 1 January and pays 2 at the end of each month for 15 years.