SAMPLE PROBLEMS FOR FINAL EXAM

MATH 630 Actuarial Math, 4/19

For the exam show your work to ensure credit for the solution of each problem. Put a box around the answer to each problem. No use of calculators, books or notes is allowed. No use or viewing of electronic devices is allowed.

1. An investor deposits 10,000 in a bank account, 2.5 years later the investor deposits an additional 2000 and 3.5 years after the initial investment the investor deposits 3000. Find explicitly the value of the bank account five years from the original deposit if
   i) the bank pays simple interest of 4% per year.
   ii) the bank pays 8% interest convertible quarterly.

2. A perpetuity provides biannual payments of 2000 for the first 10 years with the first payment two years after the purchase of the perpetuity and then after the first 10 years the perpetuity makes semiannual payments of 1000. The effective annual rate of interest is .05. Find the purchase value of the perpetuity.

3. Give an expression for the book value of a bond at the end of two years after the second coupon payment for a ten-year bond that has face (par) value $F$ and annual coupons with rate $r$ if the bond was purchased to yield the annual rate $j$.

4. A ten-year bond has face value 1000 and redemption value 1200. The bond pays semiannual coupons and is purchased for 1150 to yield 10% convertible semiannually. The first coupon is $X$ and each subsequent coupon is 4% greater than the preceding coupon. Find an explicit expression for $X$.

5. Determine the price of a 1000 par-value ten year bond with coupons at 11% convertible semiannually and for which the yield rate is 5% per half-year for the first five years and 6% per half-year for the last five years.

6. Give an expression for the book value of a bond at the end of two years after the second coupon payment for a ten-year bond that has
face (par) value $F$ and annual coupons with rate $r$ if the bond was purchased to yield the annual rate $j$.

7. A man deposits 1000 into Fund A at time $t = 0$, which pays interest at the end of each six month period at a fixed nominal annual rate $i_0$ that is convertible semiannually. These interest payments are immediately taken from Fund A and reinvested in Fund B which earns interest at an effective annual rate of .06. At the end of 15 years the man has earned an effective annual yield rate of 7.56%. Find an expression that determines the nominal rate of interest being earned in Fund A?

8. A loan $L$ is paid by $n$ equal payments of amount $K$ with an interest rate $i = .05$ per period. If $L = 1000$ and $K = 100$ then express the outstanding balance immediately after the tenth payment.

9. Deposits of 1000 are placed into a fund at the end of each year for the next 25 years for the subsequent purchase of a perpetuity. Five years after the last deposit, annual payments commence and continue forever. If $i = .09$ then find the amount of each payment.

10. Helen wishes to borrow 7000 to buy a car. The lender offers a loan in which the principal is to be repaid at the end of five years. In the meantime interest at 11% effective is to be paid on the loan and the borrower is to accumulate her principal by means of annual payments into a sinking fund earning 8% effective. Determine the annual interest payment and her annual sinking fund payment.

11. Consider a one period binomial option pricing model where $S_0 = 2$, $u = 3$, $d = .5$, $r = .2$ for a call $(S_1 - 2)^+$ that is exercised at time one. Find the non-arbitrage price of the call option at $t = 0$. Find also the portion $\Delta_0$ of stock to be purchased at $t = 0$. 