Some Notes on Bonds for MATH 630

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Abstract

In these notes some additional discussion of bonds is given (Ch. 4) before proceeding to the subsequent topics such as return on investment and term sructure of interest rates.

1 Introduction

Since we have discussed bonds to some extend already, the present discussion will be limited to primarily examples.

2 Types of Bonds

Initially a short list of some types of bonds is given.

1. Accumulation bond is one in which the redemption price includes the original loan plus accumulated interest. Examples of such bonds are the Series E Savings bonds.
2. Bonds with coupons are periodic payments of interest made by the issuer of the bond prior to redemption. Zero coupon bonds are bonds that pay no periodic interest payments. It just pays a lump sum at redemption date.
3. Registered and Unregistered bonds. A registered bond is a bond issued with the name of the owner printed on the face of the certi

   cate. If the owner decides to sell the bond, the change must be reported to the borrower.

4. Fixed-rate and floated-rate bonds.

5. A mortgage bond is a more secured bond backed by a collateral such as a mortgage on a property. Mortgage bonds are backed by real estate or physical equipment that can be liquidated. A debenture bond is an unsecured bond issued by a civil or governmental corporation or agency and backed only by the credit standing of the issuer.

6. Income or adjustment bonds. They are a type of high risk bonds in which coupons are paid only if the borrower has earned sufficient income to pay them.

7. Junk bonds. A junk bond is a high-risk bond of default in payments. The risk that a bond issuer does not pay the coupon or principal payments is called default risk. Because of this risk, these bonds typically pay higher yields than better quality bonds in order to make them attractive to investors.

8. Convertible bond. A convertible bond is a bond that can be converted into the common stock of the company at the option of the bond owner. The owner of the bond is compensated with the ability to convert the bond to common stock, usually at a substantial discount to the stock's market value.

9. Serial bonds. A set of bonds issued at the same time but having different maturity dates. These are used when a borrower is in need of a large amount of money.

10. Treasury bonds. Issued by the US Treasury. Terms of seven or more years.

11. Treasury bills. Short term debt with maturities of 13, 26, or 52 weeks. T-bills yields are computed as rates of discount. These yields are computed on a simple discount basis. The basis for counting time periods for T-bills is actual/360.

12. Municipal bonds. These are bonds issued by state and local governments to finance large, long-term capital projects (e.g., hospitals, highways, schools).

3 Some Bond Computations

1. A 10 year bond matures for its par value of 5000. The price of the bond is 4320.48 at an 8 percent yield convertible semi-annually. Calculate the coupon rate convertible semi-annually.
   $5000k\frac{1-1.04^{-20}}{.04} = 2038 : 545269$
   $k = 0 : 029999$
   The coupon rate convertible semiannually is 6.00.

2. A 1000 par value bond bearing a 6 percent coupon rate payable semi-annually will be redeemed at 105 percent at the end of 15 years. Find the price to yield an investor 5 percent effective.
   $P = 1135.54.$
   $P = 30a_{30|j} + 1050 + (1.05^{-15} = 1135.54$
   where $j = (1.05^{-15} - 1)$.

3. A 10-year accumulation bond with an initial par value(i.e. face value) of 1000 earns interest of 8 percent compounded semiannually. Find the price to yield an investor 10 percent effective.
   $(1.10)^{-10}(1000(1.04^{20})) = 844.77.$

4 Ch.5 Measuring the rate of return of an investment

Definition 4.1. Internal rate of return (IRR): the interest rate at which the value based on compound interest of all cash flows out is equal to the value of cash flows in. e.g. Compute at starting date or ending day though can use other dates.

Yield to maturity for coupon bonds is an example of IRR.

Example 4.2. L is amount invested at $t = 0$ and $K_1, K_2, ..., K_n$ are the future payments that are all strictly positive. Then $L = K_1\frac{1}{1+i} + ... + K_n\frac{1}{(1+i)^n}$
There is only one solution for $i > -1$. If $L < \Sigma_{j=1}^{n} K_j$ then there is a unique solution $i > 0$.
Consider payments received $A_0, A_1, ..., A_n$ at times $0 = t_0 < t_1 < .... < t_n$ and payments out (disbursements) $B_0, B_1, ..., B_n$ at the same times where $A_j \geq 0$ for all j and $B_j \geq 0$ for all j.

The net amounts $C_k = A_k - B_k$ for $k = 1, 2..., n$ and C_j is the net cash flow at time j.

Definition 4.3. The IRR is any rate of interest such that $\Sigma_{k=0}^{n} C_k \nu^k = 0$

Remark 4.4. Criminal interest rate in Canada: If effective annual rate of interest exceeds 60%.
This criminality does not exist in the US or even in Lawrence.
Project evaluation using net present value (NPV)

Definition 4.5. The net present value method ranks possible investment alternatives by the present value of all net amounts received. An interest rate (cost of capital or interest preference rate) must be chosen and then used in the present value calculations.
Two alternative methods:
1. The dollar-weighted rate of return is the internal rate of return for the fund but it is based on an equation of value using simple interest applied for each transaction point to the end of the year for which the rate is being measured.
2. The time-weighted rate of return for a one year period is determined by *compounding* the returns over successive parts of the year.
An example 5.2.5: The time weighted return for first half year -20% and for the second half year $+25\%$. Thus the time weighted return is $(1 - .20)(1 + .25) - 1 = 0$.
Chap.6 Term Structure of interest rates
Some considerations: credit rating and length of time of loan.

Definition 4.6. The relationship between the time to maturity and the yield rate of fixed income securities (e.g. T-bills, coupon bonds) is called the term structure of interest rates. A graph relating the two is called a yield curve.
The normal term structure: longer term investments have higher associated yields than shorter term investments.

Definition 4.7. A zero coupon bond is a bond that has no coupons and has a single payment at the end of the time of maturity.

Definition 4.8. The term structure of interest rates at the current time is the set of yield rates on zero coupon bonds of all maturities, $(s_o(t), t > 0)$ where $s_o(t)$ is the annual effective interest rate at $t = 0$ for a zero coupon bond maturing at time t.

Definition 4.9. The yield to maturity on a zero coupon bond is the spot rate for a t-year maturity zero coupon bond.
Another type: Treasury STRIPS (Separate Trading of Registered Interest and Principal of Securities)
An investor can hold and trade the interest and principal components of some T-notes and bonds as separate securities. Interest payments and principal payment become separate securities.