Butterworth filters

A Butterworth filter is a type of analog filter that is designed to have a flat frequency response in the passband. In the analog domain, Butterworth filters are typically implemented using active components like operational amplifiers and passive components like resistors and capacitors.

Butterworth filters were invented in 1930 by Stephen Butterworth, introduced them in his paper titled: "On the Theory of Filter Amplifiers", Wireless Engineer, September 1930. The key goal for this filter was to design a filter with a maximally flat frequency response in the passband — meaning no ripples or irregularities.

The transfer function of a Butterworth filter of order n is given by:

$$H(s) = \frac{1}{1+\left(\frac{s}{\omega_C}\right)^{2n}}$$

where s is the complex frequency variable, n is the filter order, and ω_c is the cutoff frequency.

Butterworth filters have the following properties:

1. Flat Frequency Response: The Butterworth filter is designed to have a maximally flat frequency response in the passband. This means that the amplitude response in the passband is as flat as possible.

2. Roll-off Rate: The roll-off rate of a Butterworth filter is determined by its filter order n. Higher-order filters have steeper roll-off rates, meaning they attenuate frequencies outside the passband more rapidly.

3. Phase Response: Butterworth filters have a linear phase response within the passband. This is an important characteristic in applications where maintaining the phase relationship of signals is critical.

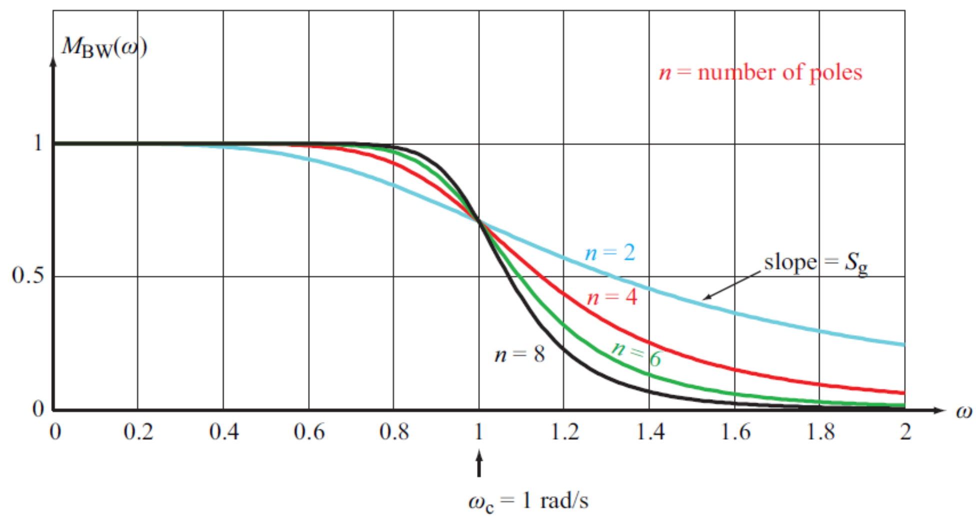


Figure 6-41: Butterworth lowpass filter for $\mathbf{C} = 1$ and $\omega_c = 1$ rad/s; magnitude spectra for various values of the number of poles, n.

Chebyshev filters

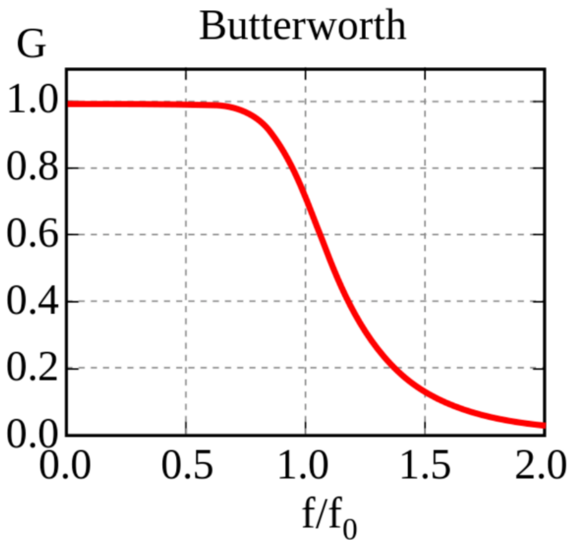
Chebyshev filters are another type of analog filter that is designed to achieve a sharper roll-off in the stopband compared to Butterworth filters. Chebyshev filters were developed in the mid-1950s.

Ripple in Passband: Some configurations of Chebyshev filters have ripple in the passband. The amount of ripple is determined by the ripple factor ε. As ε increases, the roll-off becomes sharper, but the passband ripple also increases.

Ripple in Stopband: Some configurations of Chebyshev filters have ripple in the stopband. Again, the amount of ripple is controlled by the ripple factor ε. Increasing ε sharpens the roll-off but increases stopband ripple.

Sharper Roll-off: Chebyshev filters provide steeper roll-off characteristics compared to Butterworth filters. This makes them suitable for applications where a faster transition between the passband and stopband is required.

Trade-off between Ripple and Roll-off: The designer can adjust the ripple factor (ε) to achieve a trade-off between passband ripple and stopband roll-off. This flexibility allows for the optimization of Chebyshev filters for specific design requirements.


Butterworth
G
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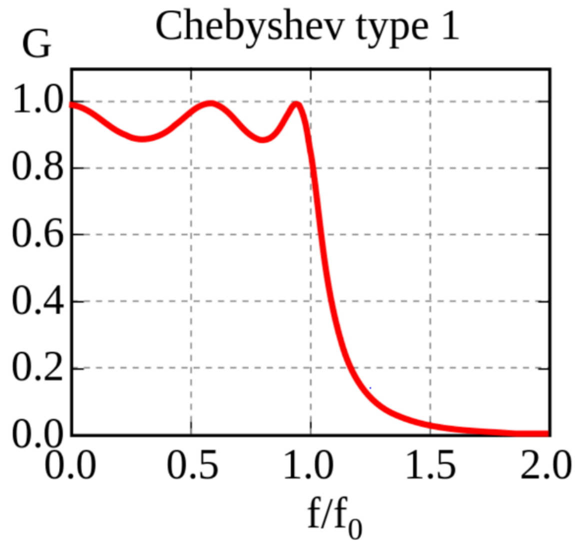
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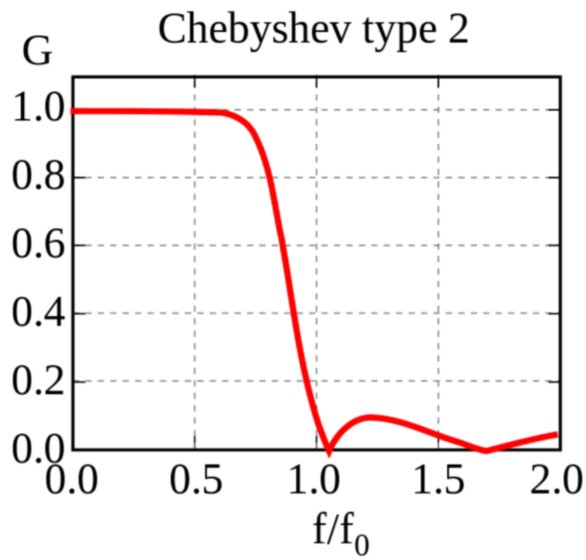
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1.5
2.0
f/f0