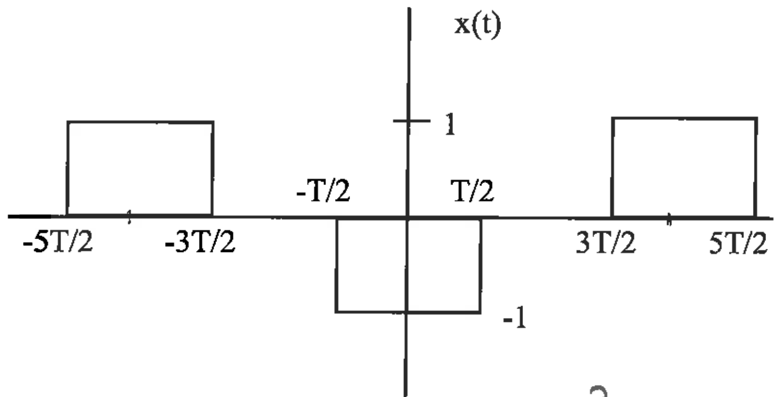
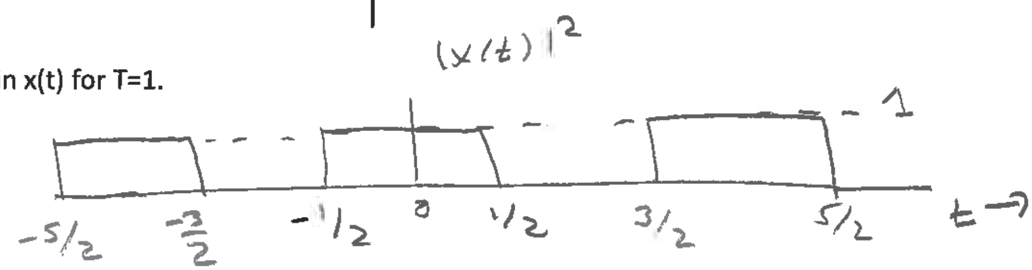
1. Given x(t) as shown below:



a) Find the total energy in x(t) for T=1.



$$\int_{-\infty}^{\infty} |x(t)|^2 \, dt = 3$$

b) Find the Fourier Transform X(f) of x(t) for T=1.

$$x(t) = -\text{rect}(t) + \text{rect}(t+2) + \text{rect}(t-2)$$

$$X(f) = -\text{sinc}(f) + e^{+j2\pi 2f}\,\text{sinc}(f) + e^{-j2\pi 2f}\,\text{sinc}(f)$$

$$= 2\cos(2\pi 2f)\,\text{sinc}(f) - \text{sinc}(f)$$

c) If T decreases will the bandwidth of x(t) 1) increase or 2) decrease. Circle the correct answer.

2. Let $y(t) = 2(1 + x(t))\cos(2\pi f_c t)$ where f_c=150 kHz and
$x(t) = \cos(2\pi f_m t)$ where f_m=10 kHz

$$X(f) = \frac{1}{2}\delta(f+f_m) + \frac{1}{2}\delta(f-f_m)$$

a) Plot the Fourier Transform for y(t), label all frequencies and amplitudes.

$$y(t) = 2\cos 2\pi f_c t + 2x(t)\cos 2\pi f_c t$$

$$Y(f) = \delta(f+f_c) + \delta(f-f_c) + \frac{1}{2}\delta(f+f_c-f_m) + \frac{1}{2}\delta(f+f_c+f_m) + \frac{1}{2}\delta(f-f_c-f_m) + \frac{1}{2}\delta(f-f_c+f_m)$$

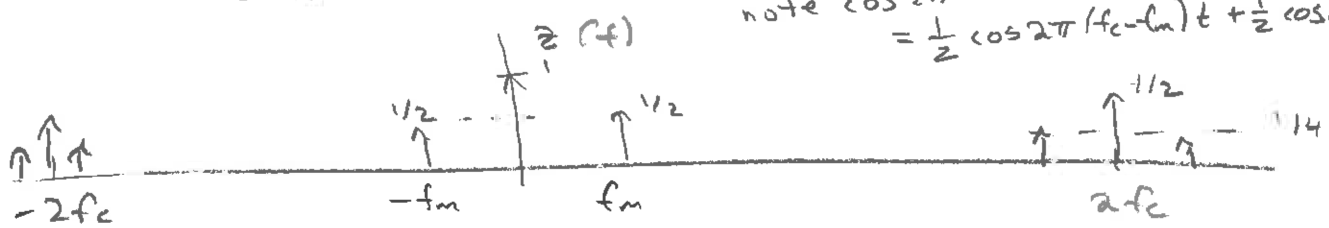
b) Define $z(t) = y(t)\cos(2\pi f_c t)$ plot the spectrum of z(t), label all frequencies and amplitudes.

$$z(t) = [2(1+x(t))\cos 2\pi f_c t]\cos 2\pi f_c t$$

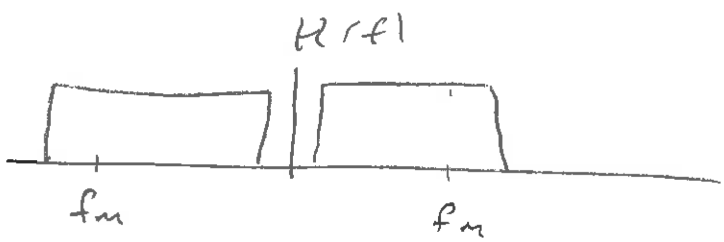
$$= [2(1+x(t))][1/2 + \frac{1}{2}\cos 2\pi 2 f_c t]$$

$$= 1 + x(t) + (1+x(t))\cos 2\pi 2 f_c t$$

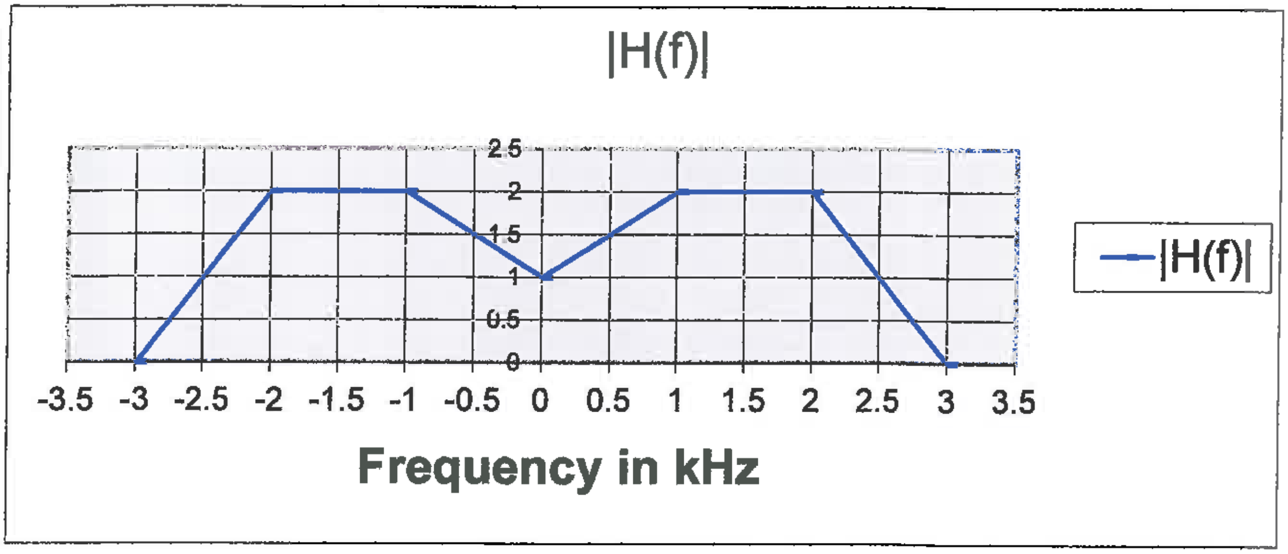
note $\cos 2\pi f_m t \cos 2\pi f_c t = \frac{1}{2}\cos 2\pi(f_c-f_m)t + \frac{1}{2}\cos(2\pi(f_c+f_m))t$

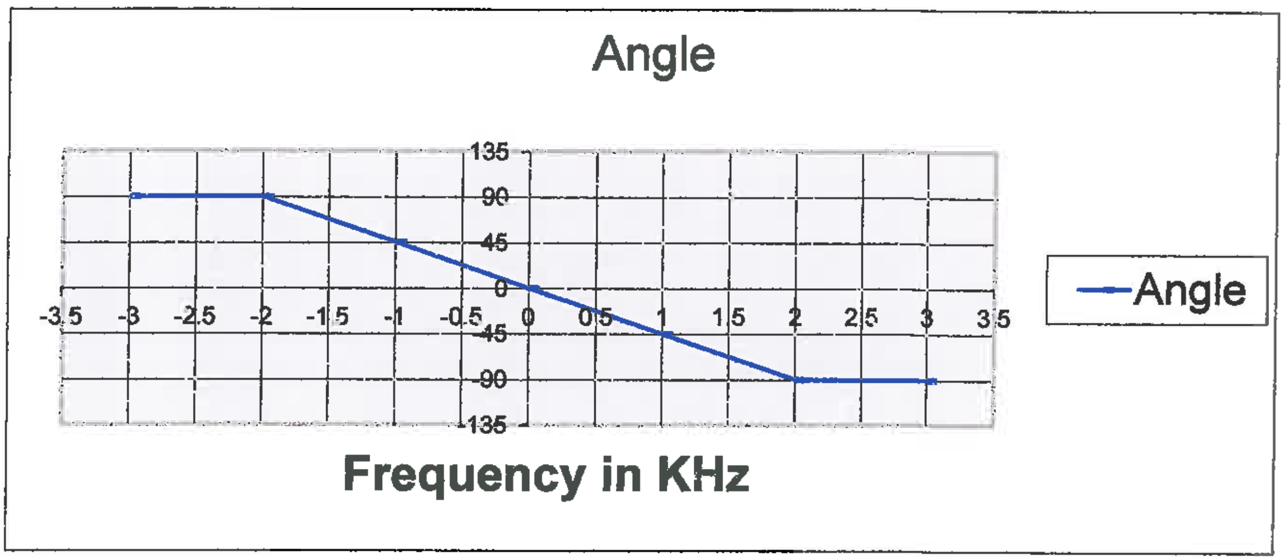


c) Describe a system that can be used to recover x(t) from z(t)



3. A system transfer function for a system is given as:





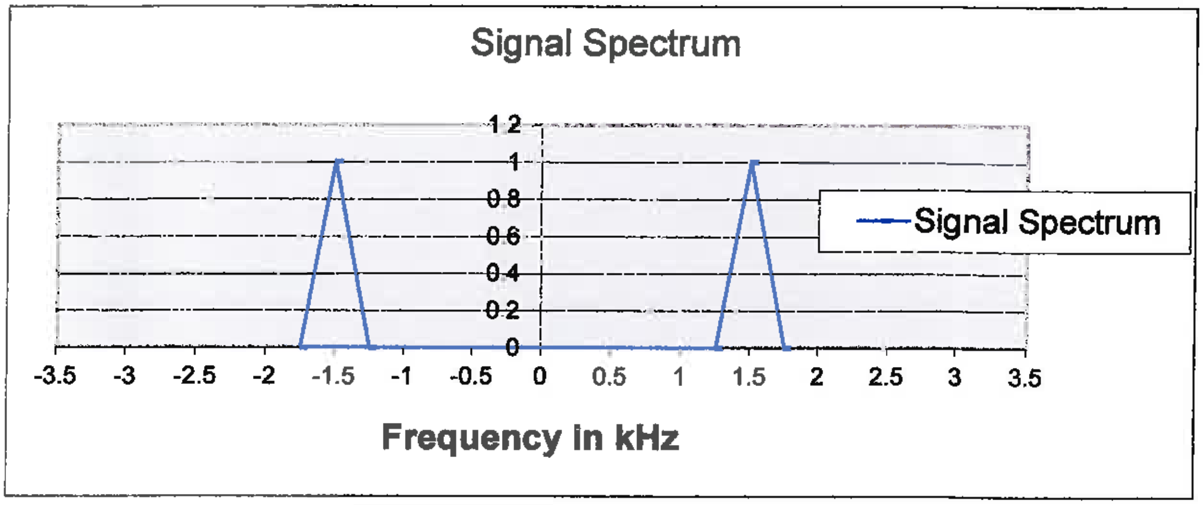
a) If the system input is the input x(t) is

$$x(t) = 5\cos(2\pi 500t) + 3\cos(2\pi 1500t) + \cos(2\pi 2500t)$$

indicate what kind of distortion if any is present.
Circle one of the following and justify your answer.
No distortion
Amplitude only distortion
Phase only distortion
Amplitude and phase distortion

$|H(f)|$ not constant from 500 to 2500 Hz
$\angle H(f)$ not linear from 500 to 2500 Hz

b) Indicate what kind of distortion if any is present if the input signal x(t) has the spectrum X(f) given below.



Circle one of the following and justify your answer.

No distortion

Amplitude only distortion

Phase only distortion

Amplitude and phase distortion

$|H(f)|$ constant for 1.25 kHz to 1.75 kHz

$\angle H(f)$ linear from 1.25 kHz to 1.75 kHz

c) Find the system output, y(t), if the input x(t) is

$$x(t) = 5\cos(2\pi 500t) + 3\cos(2\pi 1500t) + \cos(2\pi 2500t)$$

@ 500 Hz $|H(f)| = 1.5$ $\quad \angle -22.5°$

1500 Hz $|H(f)| = 2$ $\quad \angle -67.5°$

2500 Hz $|H(f)| = 1$ $\quad \angle -90°$

$$y(t) = (1.5)\,5\cos(2\pi 500t - 22.5°)$$
$$+ (2)\,3\cos(2\pi 1500t - 67.5°)$$
$$+ (1)\cos 2\pi(2\pi 2500t - 90°)$$

$$= 7.5\cos(2\pi 500t - 22.5°)$$
$$+ 6\cos(2\pi 1500t - 67.5°)$$
$$+ \cos(2\pi 2500t - 90°)$$

4. A continuous time LTI system is defined by

y(t) = x(t) + ax(t-τ)

a) Find the transfer function (frequency response) H for this system.

$$Y(f) = X(f) + a\, e^{-j2\pi f\tau} X(f)$$

$$H(f) = 1 + a\, e^{-j2\pi f\tau} = 1 + a\cos 2\pi f\tau - ja\sin 2\pi f\tau$$

$$= \alpha + j\beta$$

b) Find the amplitude response |H| for this system.

$$|H(f)| = \sqrt{\alpha^2 + \beta^2} = \sqrt{(1 + a\cos 2\pi f\tau)^2 + (a\sin 2\pi f\tau)^2}$$

c) Find the phase response for this system.

$$\angle H(f) = \tan^{-1}(\beta/\alpha)$$

$$= \tan^{-1}\left(\frac{a\sin(2\pi f\tau)}{(1 + a\cos 2\pi f\tau)}\right)$$