1. Given \( x(t) \) as shown below:

a) Find the total energy in \( x(t) \) for \( T=1 \).

\[
\int_{-\infty}^{\infty} |x(t)|^2 \, dt = 3
\]

b) Find the Fourier Transform \( X(f) \) of \( x(t) \) for \( T=1 \).

\[
X(f) = -\text{sinc}(f) + e^{+j\pi T f} \text{sinc}(f) + e^{-j\pi T f} \text{sinc}(f)
\]

\[
= 2 \cos(\pi T f) \text{sinc}(f) - \text{sinc}(f)
\]

c) If \( T \) decreases will the bandwidth of \( x(t) \) 1) increase or 2) decrease. Circle the correct answer.
2. Let \( y(t) = 2(1 + x(t)) \cos(2\pi f_c t) \) where \( f_c = 150 \text{ kHz} \) and 
\( x(t) = \cos(2\pi f_m t) \) where \( f_m = 10 \text{ kHz} \)

\[
\mathcal{F} \{ y(t) \} = \frac{1}{2} \mathcal{F} \{ f(t + f_m) \} + \frac{1}{2} \mathcal{F} \{ f(t - f_m) \}
\]

a) Plot the Fourier Transform for \( y(t) \), label all frequencies and amplitudes.

\[
y(t) = 2 \cos 2\pi f_c t + 2 \times 1(t) \cos 2\pi f_c t
\]

\[
\mathcal{F} \{ y(t) \} = \delta(f + f_c) + \delta(f - f_c) + \frac{1}{2} \delta(f + f_c - f_m) + \frac{1}{2} \delta(f - f_c + f_m)
\]

b) Define \( z(t) = y(t) \cos(2\pi f_c t) \) plot the spectrum of \( z(t) \), label all frequencies and amplitudes.

\[
z(t) = \left[ 2 \left(1 + x(t) \right) \cos 2\pi f_c t \right] \cos 2\pi f_c t
\]

\[
= \left[ 2 \left(1 + x(t) \right) \right] \left[ \frac{1}{2} + \frac{1}{2} \cos 2\pi 2f_c t \right]
\]

\[
= \left(1 + x(t)\right) + \left(1 + x(t) \right) \cos 2\pi 2f_c t
\]

\[
\mathcal{F} \{ z(t) \} = \frac{1}{2} \cos 2\pi (f_c - f_m) t + \frac{1}{2} \cos 2\pi (f_c + f_m) t
\]

c) Describe a system that can be used to recover \( x(t) \) from \( z(t) \)
3. A system transfer function for a system is given as:

\[
|H(f)|
\]

Frequency in kHz

\[
\text{Angle}
\]

Frequency in KHz

a) If the system input is the input \( x(t) \) is

\[
x(t) = 5 \cos(2 \pi 500t) + 3 \cos(2 \pi 1500t) + \cos(2 \pi 2500t)
\]

indicate what kind of distortion if any is present.
Circle one of the following and justify your answer.
No distortion
Amplitude only distortion
Phase only distortion
Amplitude and phase distortion

\( |H(f)| \) not constant from 500 to 2500 Hz
\( \angle H(f) \) not linear from 500 to 2500 Hz
b) Indicate what kind of distortion if any is present if the input signal \( x(t) \) has the spectrum \( X(f) \) given below.

![Signal Spectrum](image)

**Frequency in kHz**

Circle one of the following and justify your answer.

- **No distortion**
- Amplitude only distortion
- Phase only distortion
- Amplitude and phase distortion

\[ |H(f)| \text{ constant from 1.25 kHz to 1.75 kHz} \]
\[ \angle H(f) \text{ linear from 1.25 kHz to 1.75 kHz} \]

\[ c) \text{ Find the system output, } y(t), \text{ if the input } x(t) \text{ is} \]
\[ x(t) = 5 \cos(2\pi500t) + 3 \cos(2\pi1500t) + \cos(2\pi2500t) \]
\[ \omega_{500 \text{ Hz}} |H(f)| = 1.5 \quad \angle -22.5^\circ \]
\[ \omega_{1500 \text{ Hz}} |H(f)| = 2 \quad \angle -67.5^\circ \]
\[ \omega_{2500 \text{ Hz}} |H(f)| = 1 \quad \angle -90^\circ \]

\[ y(t) = (1.5) 5 \cos(2\pi500t - 22.5^\circ) \]
\[ + (3) 3 \cos(2\pi1500t - 67.5^\circ) \]
\[ + (1) \cos(2\pi2500t - 90^\circ) \]
\[ = 7.5 \cos(2\pi500t - 22.5^\circ) \]
\[ + 6 \cos(2\pi1500t - 67.5^\circ) \]
\[ + \cos(2\pi2500t - 90^\circ) \]
4. A continuous time LTI system is defined by

\[ y(t) = x(t) + ax(t-\tau) \]

a) Find the transfer function (frequency response) \( H \) for this system.

\[
\bar{X}(f) = \bar{X}(f) + a \, e^{-j2\pi f \tau} \bar{X}(f)
\]

\[
H(f) = 1 + a \, e^{-j2\pi f \tau} = 1 + a \cos 2\pi f \tau - ja \sin 2\pi f \tau
\]

\[
= a + j\beta
\]

b) Find the amplitude response \( |H| \) for this system.

\[
|H(f)| = \sqrt{a^2 + \beta^2} = \sqrt{(1 + a \cos 2\pi f \tau)^2 + (a \sin 2\pi f \tau)^2}
\]

c) Find the phase response for this system.

\[
\angle H(f) = \tan^{-1} \left( \frac{\beta}{a} \right)
\]

\[
= \tan^{-1} \left( \frac{a \sin (2\pi f \tau)}{1 + a \cos 2\pi f \tau} \right)
\]