

Homework #9  
EECS 360

1.

$$\text{Let } x_a(t) = \sum_{k=-\infty}^{\infty} \text{tri}\left(\frac{t-T_0 k}{w}\right) \text{ where } w=0.2\text{ms and } T_0=0.5\text{ms}$$

- $x_a(t)$  is input to an ideal lowpass filter with a bandwidth of 4.5 kHz that produces an output signal  $y_a(t)$ . Find  $y_a(t)$  and  $Y_a(f)$ .
- $x_a(t)$  is input to an ideal bandpass filter with a bandwidth of 0.5 kHz centered at 2 kHz. that produces an output signal  $y_b(t)$ . Find  $y_b(t)$  and  $Y_b(f)$ .
- What is the total power in the in the frequency range  $|f| \leq 5000$  ?

2.

- $x(t) = 10000 \text{sinc}^2(10000t)$ ,  $x(t)$  is input to an ideal lowpass filter with a bandwidth of B kHz find the smallest B such that the filter introduces no distortion.
- Find the % energy in the frequency range  $|f| \leq 5000$  ?
- What are the barriers to building the filter described in part a)

$$3. \text{ Let } x(t) = \sum_{k=-\infty}^{\infty} \text{rect}\left(\frac{t-T_0 k}{w}\right) \text{ where } w=.1\mu\text{s and } T_0=2\mu\text{s}$$

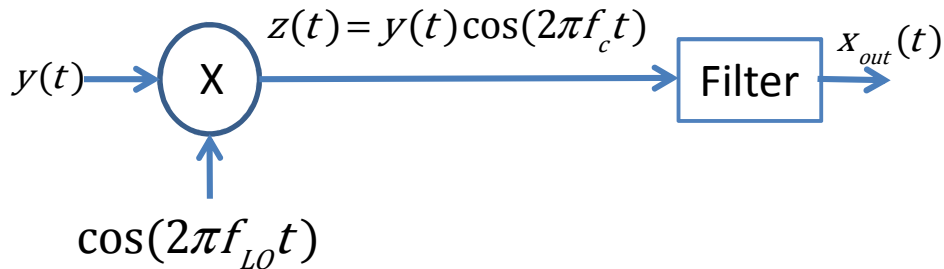
Design (specify) a system (filter) to convert  $x(t)$  to  $y(t) = \cos(2\pi f_a t)$  where  $f_a = 1.5$  MHz.

4. A system transfer function is  $H(f) = \text{tri}\left(\frac{f}{40000}\right) e^{-j2\pi\alpha f}$  where  $\alpha = 1/80000$ . The input to  $H(f)$  is  $x(t) = \cos(2\pi 20000t)$ . Find the system output in the time and frequency domains, i.e.,  $y(t)$  and  $Y(f)$ .

5. A multipath communications channel can be modeled as a filter in the time domain using an impulse response,  $h_c(t)$ . Let  $h_c(t) = a_0 \delta(t) + a_1 \delta(t - 0.1\mu\text{s})$ . The received signal is the filter output given an input signal of  $x(t)$ .

- Find the model for the multipath communications channel in the frequency domain, i.e.,  $H_c(f)$ .
- For  $a_0=1$  and  $a_1=-1$  and a transmitted signal of  $x(t) = \cos(2\pi f_c t)$  where  $f_c = 10$  Mhz find the received signal.

6. Let  $x(t)$  and  $g(t)$  be voice signals with a bandwidth of 3kHz, assume  $X(f)=\text{tri}(f/3000)$  and  $G(f)=\text{rect}(f/6000)$ . Let  $y(t) = x(t)\cos(2\pi f_c t) + g(t)\cos(2\pi(f_c + 20,000)t)$  be a transmitted signal where  $f_c=1500\text{kHz}$ . The received signal  $y(t)$  is processed as shown below.



- Plot  $Y(f)$
- Plot  $Z(f)$
- Design (specify) the Local Oscillator frequency  $f_{LO}$  and the filter such that  $x_{out}(t) = Cx(t)$ , where the constant  $C \neq 0$ .

7. A series R, L, C circuit is modeled by the following differential equation with  $x(t)$ =input voltage and  $y(t)$ =output voltage=voltage across the capacitor.

$$LC \frac{d^2 y(t)}{dt^2} + RC \frac{dy(t)}{dt} + y(t) = x(t)$$

- Find  $H(f)$ .
- Plot  $20\text{Log}_{10}(|H(f)|)$  with  $R=100$  Ohm,  $L=0.001$  Henry,  $C=0.000001$  Farad. (You are encouraged to use Matlab to do this plot.)
- For  $R=100$  Ohm,  $L=0.001$  Henry,  $C=0.000001$  Farad, given  $x(t)=\cos(2\pi 1760t)$  find A in  $y(t)=A\cos(2\pi 1760t+\phi)$
- Is 1760 Hz close to the 3 dB bandwidth for this system?

Confirm your results with:

[http://www.ittc.ku.edu/~frost/EECS\\_360/Mathematica-360/Series-RLC-Transfer-Functions.cdf](http://www.ittc.ku.edu/~frost/EECS_360/Mathematica-360/Series-RLC-Transfer-Functions.cdf)

8. A series R, L, C circuit is modeled by the following differential equation with  $x(t)$ =input voltage and  $y(t)$ =output voltage=voltage across the resistor.

$$LC \frac{d^2 y(t)}{dt^2} + RC \frac{dy(t)}{dt} + y(t) = RC \frac{dx}{dt}$$

- Find  $H(f)$ .
- Plot  $20\text{Log}_{10}(|H(f)|)$  with  $R=500$  Ohm,  $L=0.4$  Henry,  $C=0.0000007$  Farad. (You are encouraged to use Matlab to do this plot.)
- For  $R=500$  Ohm,  $L=0.4$  Henry,  $C=0.0000007$  Farad, given  $x(t)=\cos(2\pi 217t)$  find A in  $y(t)=A\cos(2\pi 217t+\phi)$  and given  $x(t)=\cos(2\pi 417t)$  find B in  $y(t)=B\cos(2\pi 417t+\phi)$ , i.e., find the amplitudes of the output cosine. What is the 3 dB bandwidth for this system? Confirm your results with:

[http://www.ittc.ku.edu/~frost/EECS\\_360/Mathematica-360/Series-RLC-Transfer-Functions.cdf](http://www.ittc.ku.edu/~frost/EECS_360/Mathematica-360/Series-RLC-Transfer-Functions.cdf)

- d) Use [http://www.ittc.ku.edu/~frost/EECS\\_360/Mathematica-360/Series-RLC-Transfer-Functions.cdf](http://www.ittc.ku.edu/~frost/EECS_360/Mathematica-360/Series-RLC-Transfer-Functions.cdf) and find the impact of keeping L and R fixed and changing C.
- e) Use [http://www.ittc.ku.edu/~frost/EECS\\_360/Mathematica-360/Series-RLC-Transfer-Functions.cdf](http://www.ittc.ku.edu/~frost/EECS_360/Mathematica-360/Series-RLC-Transfer-Functions.cdf) and find the impact of keeping L and C fixed and changing R.