Homework #1  
Fundamentals Review Homework for EECS 562

1. For \( x(t) = 2\sin(100\pi t) - 4\cos(200\pi t) \)
   a) Find the complex Fourier series for \( x(t) \)
   b) Plot the double sided phase and amplitude spectral density for \( x(t) \).

2. a) Plot \( x(t) = 10(\text{rect}(\frac{t+1}{0.125}) - \text{rect}(\frac{t-1}{0.125})) \)
   b) Find the energy and power in \( x(t) \).
   \( E_x = \) \___________ \( P_x = \) \___________
   c) Find \( X(f) \) and plot its amplitude spectrum.

3. An input signal \( x(t) \) is processed by a filter with an amplitude \( |H(f)| \) and phase \( \theta(f) \) response given below.

\[
|H(f)|
\]

\[
\theta(f)
\]

a) For \( x(t) = 10\cos(2\pi 500t) + 7.5\cos(2\pi 750t) \) find output system \( y(t) \).

b) For \( x(t) = 10\cos(2\pi 500t) + 7.5\cos(2\pi 1500t) \) find output system \( y(t) \).

c) For \( x(t) = 10\cos(2\pi 500t) + 7.5\cos(2\pi 3500t) \) find output system \( y(t) \).

\[
\text{An input signal } x(t) \text{ with a bandwidth } B \text{ is processed by a filter with an amplitude } |H(f)| \text{ and phase } \theta(f) \text{ response given above. What is the maximum value of } B \text{ that will result in distortionless transmission of } x(t) \text{ through the filter, } H(f).
\]
4. Find and plot the amplitude spectrum for \( x(t) = 10 \text{sinc}(2Kt) \)
   
   a) For \( K = 0.1 \)
   b) For \( K = 1 \)
   c) For \( K = 10 \)
   d) As \( K \) increases the bandwidth of \( x(t) \) increases, TRUE or FALSE.
   e) Find the energy and power in \( x(t) \).
   f) Is \( x(t) \) an energy or power signal?

5. The spectrum of \( x(t) \) is given below:

   ![Spectrum Plot]

   a) The signal \( x(t) \) is sampled at 8000 samples/sec to form \( x_s(t) \). Plot the spectrum of \( x_s(t) \).

   b) For \( x(t) \) given above, what is the minimum sample rate required to prevent aliasing?

   c) If no aliasing is present, describe how \( x(t) \) is recovered from \( x_s(t) \).
6. Two linear time invariant systems have transfer functions of $H_1$ and $H_2$ are configured as:

\[ x(t) \rightarrow H_1(f) \rightarrow z(t) \rightarrow H_2(f) \rightarrow y(t) \]

System 1

\[ x(t) \rightarrow H(f) \rightarrow y(t) \]

System 2

$H_1$ and $H_2$ have the following transfer functions

\[ H_1(f) = e^{-j2\pi(0.1)f} \quad H_2(f) = \frac{1}{\frac{1}{4} + j2\pi f} \]

a) Find $H(f)$ such that the two systems above (System 1 and System 2) are the same, i.e., for the same input $x(t)$ find $H(f)$ such that System 1 and System 2 produce the same output.

b) Plot $|H_2(f)|$

c) Find $h_2(t)$.

d) Find the output signal, $y(t)$, when the input signal is $x(t) = \cos(2\pi t)$.

e) Is the system $H(f)$ casual, Circle YES or NO, Justify your answer.

7. An ideal bandpass filter $H(f)$ has center frequency of 200 kHz and bandwidth $B_h = 50$ kHz. The input to $H(f)$ is $x(t)$, where

\[ x(t) = \sum_{k=-\infty}^{\infty} \text{rect} \left( \frac{t - kT_0}{\tau} \right) \text{ where } \tau = 2 \mu s \text{ and } T_0 = 10 \mu s \]

a) Sketch $|H(f)|$.

b) Sketch $|X(f)|$.

c) Find the power at the fundamental frequency $f_0$.

d) For the $x(t)$ and $H(f)$ given above find the system output $y(t)$. 
8. Consider a linear time invariant system with a impulse response of $h(t)$, and input signal $x(t)$ given below. The input signal $x(t)$ given below produces an output of $y(t)$.

![Diagram of x(t) and h(t)]

a) What is $y(-10)$?
b) What is $y(-0.5)$?
c) What is $y(0.5)$?

9. The signal $x[n]$ is input to a LTI system with impulse response $h[n]$.

![Diagram of x[n] and h[n]]

a) Find $X(z)$ and $H(z)$.
b) Find the discrete time convolution of $x[n] * h[n] = y[n]$. 

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10. A video signal has a bandwidth of about 5 MHz. A DFT is use to analyze the frequency content of a video signal with a frequency resolution of 1 kHz.
   a) To achieve this frequency resolution what is the required record length in seconds?
   b) How many samples are in the record, state any assumptions?

11. Properties of the DFT.
   a) Let $X_1[n] = \cos(n\pi/2)$, $n=1\ldots16$. Plot the magnitude of the DFT of $X_1[n]$. (Use Matlab or other tool)
   b) Let $X_2[n]= 0$, $n=1..4, \cos(n\pi/2)$, $n=5..16$. Plot the magnitude of the DFT of $X_2[n]$. (Use Matlab or other tool)
   c) Explain the difference between the results of part a) and part b).

12. Let of $s_I(t)= x(t)\cos(2\pi f_0t)$ and $s_q(t)= z(t)\sin(2\pi f_0t)$, and $s(t) = s_I(t) + s_q(t) = x(t)\cos(2\pi f_0t) + z(t)\sin(2\pi f_0t)$, assume that both $x(t)$ and $z(t)$ have a 10kHz bandwidth and that $f_0=1$ MHz. Assume $X(f)=rect\left(\frac{f}{20000}\right)$ and $Z(f)=tri\left(\frac{f}{10000}\right)$.

   a) Sketch the amplitude spectrum of $s_I(t)= x(t)\cos(2\pi f_0t)$ and $s_q(t)= z(t)\sin(2\pi f_0t)$.
   b) Sketch the amplitude spectrum of $s(t) = s_I(t) + s_q(t) = x(t)\cos(2\pi f_0t) + z(t)\sin(2\pi f_0t)$.
   c) Find the output $y(t)$ of the following system in terms of $x(t)$ and $z(t)$. The bandwidth of the ILPF is 11 kHz:

   ![Diagram](image)

   d) Discuss the spectral occupancy of $s(t)$, $s_I(t)$, and $s_q(t)$? What property of signals can be used to explain the result of part c) assuming that an ideal low pass filter is an approximation for an integrator?