Homework #1

Fundamentals Review Homework for EECS 562

(As needed for plotting you can use Matlab or another software tool for your choice)

1. Plot $x_1(t) = 2\cos(2\pi 50t)$, $x_2(t) = 2\cos(2\pi 50(t - 0.0025))$, and $x_3(t) = 2\cos(2\pi 50t - \frac{\pi}{4})$
   Compare these three signals and explain their similarities and differences.

2. Let $z_1 = 1 + j$ find: Re(z_1), Im(z_1), $|z_1|$ and $|z_1|^2$ and α and β if $z_1 = \alpha e^{j\beta}$

3. Let z(t) be a complex signal
   $z(t) = Ae^{j(2\pi f_c t + \phi)}$ find f(t)=Re($z(t)$), g(t)= Im($z(t)$), and r(t), and $\theta(t)$ *if* z(t)=r(t)$e^{j\theta(t)}$

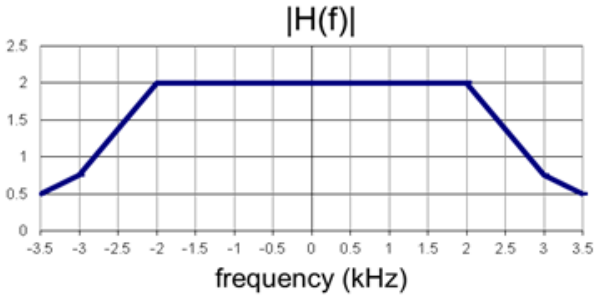
4. Let
   $z_1 = 1 + j$, $z_2 = 1 - j$, $z_3 = -1 + j$, $z_4 = -1 - j$
   Find Re($z_i e^{j2\pi f_c t}$) for $i = 1...4$

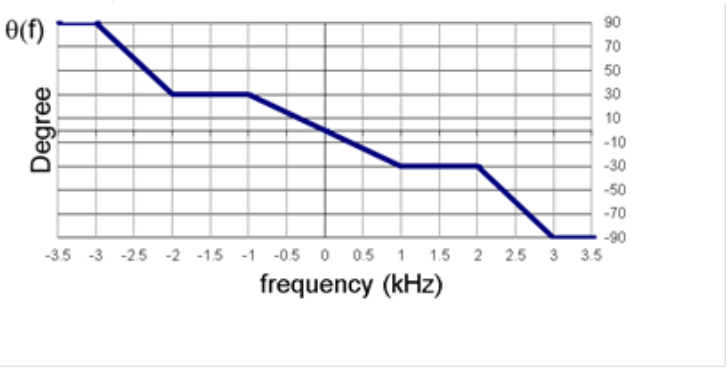
5. Find $\int_0^1 \cos(2\pi t)\sin(2\pi t)dt$ and what property describes the relationship between $\cos(2\pi t)$ and $\sin(2\pi t)$?

6. For $x(t) = 2\cos(100\pi t) - 4\sin(200\pi t)$
   a) What is the fundamental frequency?
   b) Find the complex Fourier series for x(t).
      [Hint: no integration is required for this problem, convert
      $\cos(100\pi t)$ and $\sin(200\pi t)$ into their complex exponential forms and
      note $\sin(\alpha)=\cos(\alpha-\pi/2)$ and $-\cos(\alpha)=\cos(\alpha-\pi)$]
   c) Plot the double sided phase and amplitude spectrum for x(t).
   d) What is the power in x(t)?
   e) What is the bandwidth of x(t)?

7. A bit is transmitted as x(t) = $\cos(2\pi 1000t)$ for a "1" or $-\cos(2\pi 1000t)$ for a "0" for 100 ms.
   Find the energy and power in x(t).
   E_x=_____ P_x=_____

8. An input signal x(t) is processed by a filter with an amplitude |H(f)| and phase θ(f) response given below.





a) For $x_a(t) = 2\cos(2\pi 500t)$ find output signal $y_a(t)$.

b) For $x_b(t) = 4\cos(2\pi 750t)$ find output signal $y_b(t)$.

c) For $x_c(t) = 2\cos(2\pi 500t) + 4\cos(2\pi 750t)$ find output signal $y_c(t)$.

d) For $x_d(t) = 4\cos(2\pi 1500t)$ find output signal $y_d(t)$.

e) For $x_e(t) = 2\cos(2\pi 500t) + 4\cos(2\pi 1500t)$ find output signal $y_e(t)$.

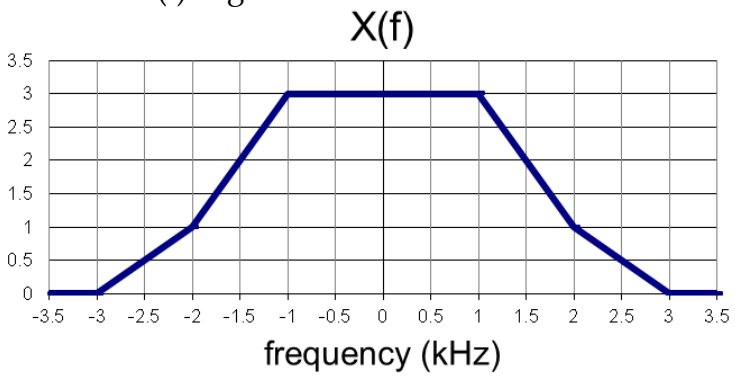
f) For $x_f(t) = 2\cos(2\pi 500t) + 4\cos(2\pi 3500t)$ find output signal $y_f(t)$.

g) Which input signal above, $x_a(t)$… $x_f(t)$ has the largest bandwidth and what is that bandwidth?

h) An input signal x(t) with a bandwidth B is processed by a filter with an amplitude |H(f)| and phase θ(f) response given above. What is the maximum value of B that will result in distortion-less transmission of an input signal x(t) through the filter, H(f)?

9. A linear time-invariant system with input signal x(t) produces an output signal $y(t) = \alpha\, x(t-\tau)$, find the system transfer function and impulse response.

10. The spectrum of x(t) is given below:

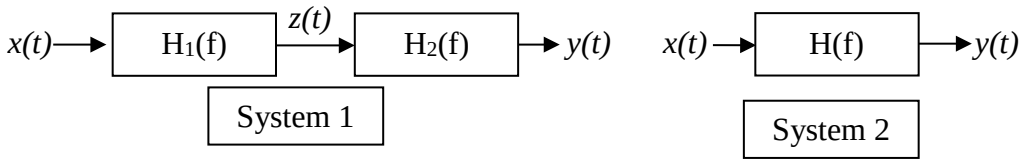


a) The signal x(t) is sampled at 7000 samples/sec to form $x_s(t)$. Plot the spectrum of $x_s(t)$.

b) For x(t) given above, what is the minimum sample rate required to prevent aliasing?

c) If no aliasing is present, describe how x(t) is recovered from $x_s(t)$.

11. Two linear time invariant systems have transfer functions of H_1 and H_2 are configured as:



H_1 and H_2 have the following transfer functions

$$H_1(f) = e^{-j2\pi(0.1)f} \qquad H_2(f) = \frac{1}{\frac{1}{4} + j2\pi f}$$

a) Find H(f) such that the two systems above (System 1 and System 2) are the same, i.e., for the same input x(t) find H(f) such that System 1 and System 2 produce the same output.

b) Plot $|H_2(f)|$

c) Find $h_2(t)$.

d) Find the output signal, y(t), when the input signal is $x(t) = \cos(2\pi t)$.

e) Is the system H(f) casual, **Circle YES or NO**, **Justify your answer.**

12. An ideal bandpass filter H(f) has center frequency of 200 kHz and bandwidth B_h=10 kHz. The input to H(f) is x(t), where

$$x(t) = \sum_{k=-\infty}^{\infty} rect\left(\frac{t - kT_0}{\tau}\right) \text{ where } \tau = 1\mu s \text{ and } T_0 = 5\mu s$$

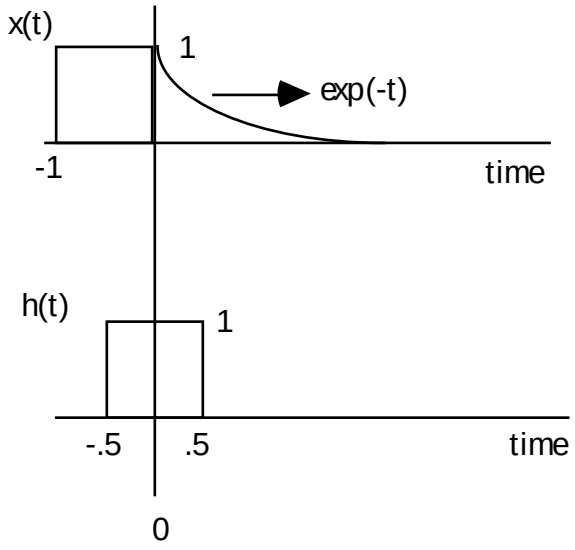
a) Plot $|H(f)|$, label axis.

b) Plot $|X(f)|$, label axis.

c) Find the power in x(t) at the fundamental frequency f_0.

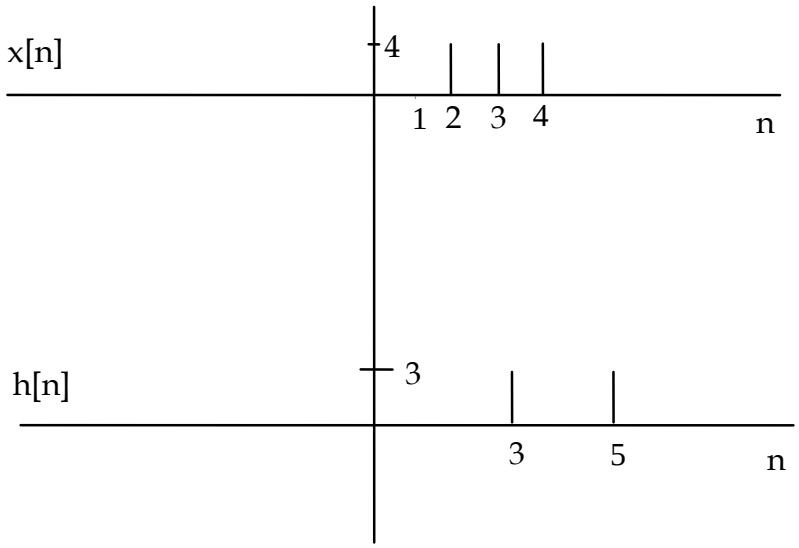
d) For the x(t) and H(f) given above find the system output y(t). [Hint: Examine the results of part a) and b) and note Y(f)=X(f)H(f)]

13. Consider a linear time invariant system with a impulse response of h(t), and input signal x(t) given below. The input signal x(t) given below produces and output of y(t).



a) What is y(-50)?
b) What is y(-0.5)?
c) What is y(0.5)?

14. The signal x[n] is input to a LTI system with impulse response h[n].



Find the discrete time convolution of x[n]*h[n]=y[n].

15. A radar signal has a bandwidth of about 50 MHz. A DFT is use to analyze the frequency content of a radar signal with a frequency resolution of 1 kHz.
    a) To achieve this frequency resolution what is the required record length in seconds?
    b) How many samples are in the record, state any assumptions?

16. Properties of the DFT.
    (For this problem use Matlab or another software tool for your choice)
    a) Let $X_1[n] = \cos(n\pi/2)$, n=1...16. Plot the magnitude of the DFT of $X_1[n]$.
    b) Let $X_2[n]= 0$, n=1..4, $\cos(n\pi/2)$, n=5..16. Plot the magnitude of the DFT of $X_2[n]$. Explain the difference between the results of part a) and part b).

17. Let $s(t)= x(t)\sin(2\pi f_o t)$ where f_o=1 MHz and $X(f) = rect(\frac{f}{20000})$
    a) Plot the amplitude spectrum of s(t).
    b) Find the output y(t) of the following system in terms of x(t). The bandwidth of the ILPF is 11 kHz. [Hints: $\sin^2(\theta) = \frac{1}{2} - \frac{1}{2}\cos(2\theta)$ and then plot the spectrum of the signal at the input to the ILPF]