Homework #1

Fundamentals Review Homework for EECS 562

(As needed for plotting you can use Matlab or another software tool for your choice)

1. Plot $x_1(t) = 20\cos(2\pi 500t)$, $x_2(t) = 20\cos(2\pi 500(t - .25ms))$, and $x_3(t) = 20\cos(2\pi 500t - \frac{\pi}{4})$

   Compare these three signals and explain their similarities and differences.

2. For $x_1(t) = \text{sinc}(1000t)$ and $x_2(t) = \text{sinc}(10000t)$
   a) Plot $x_1(t)$ and $x_2(t)$
   b) The bandwidth of $x_1(t)$ is larger than the bandwidth of $x_2(t)$. TRUE or FALSE

3. Let $z_1 = 2 + 3j$ find: $\text{Re}(z_1)$, $\text{Im}(z_1)$, $|z_1|$ and $|z_1|^2$ and α and β putting z_1 in ploar form $z_1 = \alpha e^{j\beta}$

4. Let z(t) be a complex signal
   $z(t) = Ae^{j(2\pi f_c t + \phi)}$ find f(t)=Re($z(t)$), g(t)= Im($z(t)$), and r(t), and $\theta(t)$ *if* z(t)=r(t)$e^{j\theta(t)}$

5. Let
   $z_1 = 0.94 + 0.34j$, $z_2 = -0.34 + 0.94j$, $z_3 = -0.94 - 0.34j$, $z_4 = 0.34 - 0.94j$
   a) Plot z_i for i=1..4 putting the real part of z_i on the x-axis and the imagery part of z_i on the y-axis.

   b) For f_c = 1 GHz find $\text{Re}(z_i e^{j2\pi f_c t})$ for $i = 1...4$

   (This problems leads to the mathematical model for quadrature phase shift keying - QPSK)

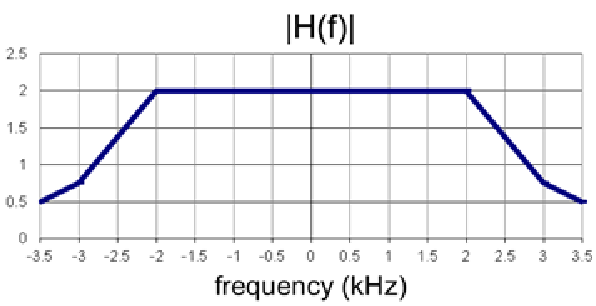
6. For $H(f) = \dfrac{10}{2 + j(2\pi f)}$ find
   a) $|H(1)|^2$
   b) Phase of H(1) (or Angle H(1))
   c) Plot $10\text{Log}_{10}(|H(f)|^2)$
   d) Plot Phase of H(f) (or Angle H(f))

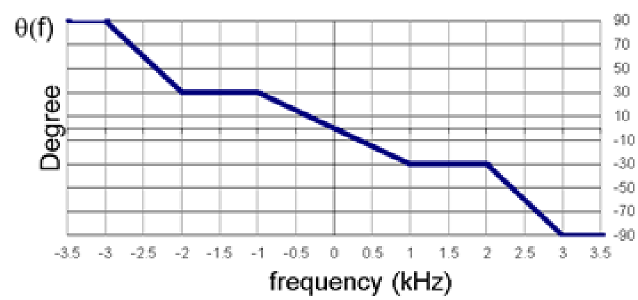
7. Find $\int_0^1 \cos(2\pi t)\sin(2\pi t)dt$ and what property describes the relationship between $\cos(2\pi t)$ and $\sin(2\pi t)$?

8. For $x(t) = 4\cos(100\pi t) + \sin(200\pi t)$
   a) What is the fundamental frequency?
   b) Find the complex Fourier series for x(t) (see eq A2.11).
      [Hint: no integration is required for this problem, convert $\cos(100\pi t)$ and $\sin(200\pi t)$ into their complex exponential forms and note $\sin(\alpha)=\cos(\alpha-\pi/2)$ and $-\cos(\alpha)=\cos(\alpha-\pi)$]
   c) Plot the double sided phase and amplitude spectrum for x(t).
   d) What is the power in x(t)?
   e) What is the bandwidth of x(t)?

9. A bit is transmitted as $x(t) = 4\cos(2\pi 1000t)$ for a "1" or $-4\cos(2\pi 1000t) = 4\cos(2\pi 1000t - \pi)$ for a "0" for 100 ms.
   a) Find the energy and power in x(t).
      E_x=________ P_x=__________
   b) What is the bit rate in b/s?
      (This problem provides the basis for Binary Phase Shift Keying-BPSK.)

10. An input signal x(t) is processed by a filter with an amplitude |H(f)| and phase θ(f) response given below.

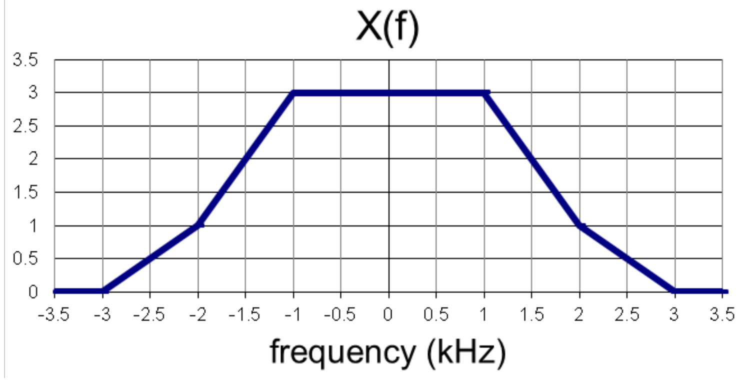




   a) For $x_a(t) = 2\cos(2\pi 500t)$ find output signal $y_a(t)$.
   b) For $x_b(t) = 4\cos(2\pi 750t)$ find output signal $y_b(t)$.
   c) For $x_c(t) = 2\cos(2\pi 500t) + 4\cos(2\pi 750t)$ find output signal $y_c(t)$.
   d) For $x_d(t) = 4\cos(2\pi 1500t)$ find output signal $y_d(t)$.
   e) For $x_e(t) = 2\cos(2\pi 500t) + 4\cos(2\pi 1500t)$ find output signal $y_e(t)$.
   f) For $x_f(t) = 2\cos(2\pi 500t) + 4\cos(2\pi 3500t)$ find output signal $y_f(t)$.
   g) Which input signal above, $x_a(t)$… $x_f(t)$ has the largest bandwidth and what is that bandwidth?
   h) An input signal x(t) with a bandwidth B is processed by a filter with an amplitude |H(f)| and phase θ(f) response given above. What is the maximum value of B that will result in distortion-less transmission of an input signal x(t) through the filter, H(f)?

11. A linear time-invariant system with input signal x(t) produces an output signal y(t) =2x(t-1ms), find the system transfer function and impulse response.

12. The spectrum of x(t) is given below:

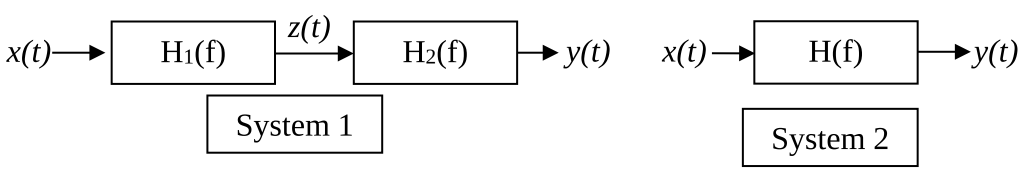


a) The signal x(t) is sampled at 7000 samples/sec to form $x_s(t)$. Plot the spectrum of $x_s(t)$.

b) For x(t) given above, what is the minimum sample rate required to prevent aliasing?

c) If no aliasing is present, describe how x(t) is recovered from $x_s(t)$.

13. Two linear time invariant systems have transfer functions of H_1 and H_2 are configured as:



H_1 and H_2 have the following transfer functions

$$H_1(f) = 2e^{-j2\pi(0.2)f} \qquad H_2(f) = \frac{1}{\frac{1}{8} + j2\pi f}$$

a) Find H(f) such that the two systems above (System 1 and System 2) are the same, i.e., for the same input x(t) find H(f) such that System 1 and System 2 produce the same output.

b) Find the impulse response, $h_2(t)$, corresponding to transfer function $H_2(f)$.

c) Find the output signal, y(t), when the signal z(t) is $z(t) = \cos(2\pi t)$.

14. An ideal bandpass filter H(f) has center frequency of 400 kHz and bandwidth B_h=5 kHz. The input to H(f) is x(t), where

$$x(t) = \sum_{k=-\infty}^{\infty} rect\left(\frac{t - kT_0}{\tau}\right) \quad \text{where } \tau = 1\mu s \text{ and } T_0 = 5\mu s$$

a) Plot |H(f)|, label axis.

b) What is the fundamental frequency f_0 of *x(t)*?

c) Find the complex exponential Fourier Series for *x(t),* i.e., find c_n (see eq A2.11) in

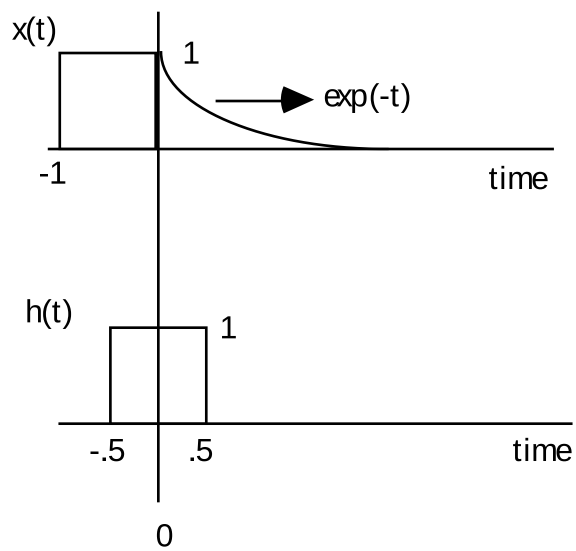
$$x(t) = \sum_{n=-\infty}^{\infty} c_n \exp(j2\pi n f_0 t) \ .$$

d) Plot the amplitude spectrum for *x(t)* that is $|c_n|$, label x-axis in Hz.

e) Find the power in *x(t)* at the fundamental frequency f_0.

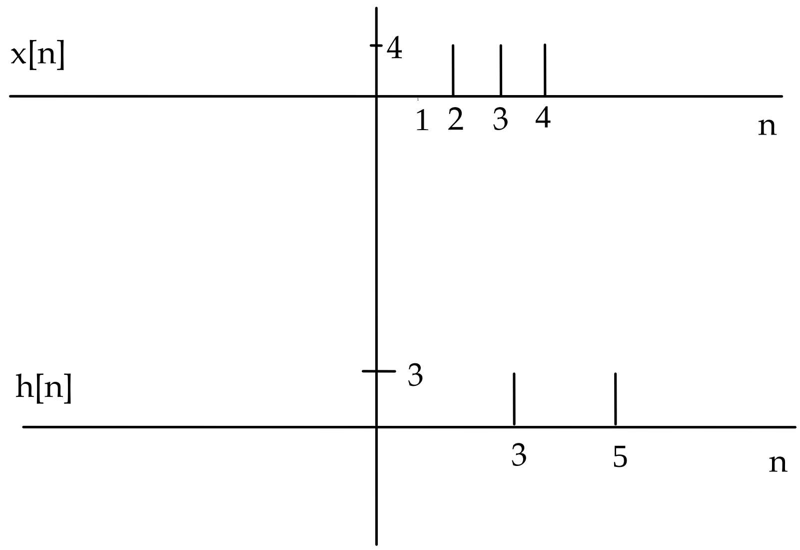
f) For the *x(t)* and H(f) given above find the system output y(t). [Hint: Examine the results of part a) and d) and note Y(f)=X(f)H(f)]

15. Consider a linear time invariant system with a impulse response of h(t), and input signal x(t) given below. The input signal x(t) given below produces and output of y(t).



a) What is y(-5)?
b) What is y(-0.5)?
c) What is y(1)?

16. The signal x[n] is input to a LTI system with impulse response h[n].



Find the discrete time convolution of x[n]*h[n]=y[n].

17. LTE systems use the DFT. Here a DFT of length 2048 is used with a sample frequency of 30.72Msamples/s to process this LTE signal.
   a) What is the record length in seconds?
      (In LTE this is the symbol time.)
   b) What is the spacing in kHz between frequency components of this DFT?
      (In LTE this is the carrier spacing)?

18. Properties of the DFT.
   (For this problem use Matlab or another software tool for your choice)
   a) Let $x_1[n] = \cos(n\pi/2)$, n=1...16. Plot the magnitude of the DFT of $x_1[n]$.
   b) Explain spectral leakage (leads to the requirement for a cyclic prefix in LTE).
   c) Let $x_2[n]$= 0, n=1..4, $\cos(n\pi/2)$, n=5..16. Plot the magnitude of the DFT of $x_2[n]$. Explain the difference between the results of part a) and part b).

19. Let $s(t)= x(t)\sin(2\pi f_o t)$ where f_o=100 MHz and $X(f) = rect(\frac{f}{20000})$
   Find the output y(t) of the following system in terms of x(t). The bandwidth of the ILPF is 11 kHz. [Hints: use the trigonometry identity for sin(α)cos(α) and then plot the spectrum of the signal at the input to the ILPF] (the solution to this problem provides the basis for a quadrature receiver)