EECS 562

Homework 1

**1.** Let

a. $z_1$ = 4+j3, Find Re($z_1$), Im($z_1$), $|z_1|$, $|z_1|^2$ and $\alpha$ and $\beta$ in $z_1=\alpha e^{j\beta}$

b. Repeat a for $z_1$ = −4 − j4

c. Find $x(t) = \text{Re}[z_1 e^{-j2\pi f_c t}]$ where $f_c$ = 1000 Hz

**2.** Let $z_1$ = 0.92+j0.38, $z_2$ = -0.38+ j0.92, $z_3$ = -0.92-j0.38, $z_4$ = 0.38-j0.92

a. Plot $z_i$ for i=1..4 putting the real part of $z_i$ on the x-axis and the imagery part of $z_i$ on the y-axis.

b. For $f_c$ = 100 MHz find $x_i(t) = \text{Re}[z_i e^{-j2\pi f_c t}]$ for i=1..4

(This problems leads to the mathematical model for quadrature phase shift keying -QPSK; for each symbol time sending one of these 4 signals to send 2 bits)

**3.** Let $x_1(t)$ = 10 cos(2 $\pi$1000(t-62.5$\mu$s)) and $x_2(t)$ = 10 cos(2 $\pi$1000t- $\frac{\pi}{8}$), $x_1(t)$=$x_2(t)$ TRUE or FALSE

**4.**

For

$x_1(t)$ = 100 sinc(100$\pi$t),

$x_2(t)$ = 200 sinc(200 $\pi$t)

$x_3(t)$ = 500 sinc(500 $\pi$t)

a. Find the Fourier transform of $x_1(t)$, i.e. find $X_1(f)$.

b. Plot $x_1$(t), $x_2$(t), and $x_3$(t)

c. Rank order the signals from lowest bandwidth to highest bandwidth.

**5.** Find the power and energy in 4cos(2$\pi$1000t) + 8sin(2$\pi$2000t)

**6.** A bit is transmitted as

x(t) = Acos(2 $\pi f_c$t) if bit = “1” for $T_b$

or

x(t) = −Acos (2 $\pi f_c$t)= -Acos(2$\pi$1000t-$\pi$) if bit = “0” for $T_b$

For $A = 2 \times 10^{-4}$ and $T_b$=1$\mu$s and $f_c$ = 100 MHz

a. Find the energy and power in x(t).

b. What is the bit rate in Mb/s?

(This problem provides the basis for Binary Phase Shift Keying-BPSK.)

**7.** Let $x_1(t)$ = cos(2 $\pi f_1$ $t$) rect($\frac{t-0.05}{0.01}$) and $x_2(t)$ = cos(2 $\pi f_2$ $t$) rect($\frac{t-0.05}{0.01}$)

For $f_1$=2000 and $f_2$=2150

a. Find $\int_{-\infty}^{\infty} \cos(2\pi f_1 t)\sin(2\pi f_1 t)\,dt$

b. Find $\int_{0}^{0.01} \cos(2\pi f_1 t)\cos(2\pi f_2 t)\,dt$

c. Define the orthogonality property of signals

For x(t) = 4cos(2$\pi$1000t) + 8sin(2$\pi$2000t)

a. What is the fundamental frequency?

b. Find the complex Fourier series for x(t).

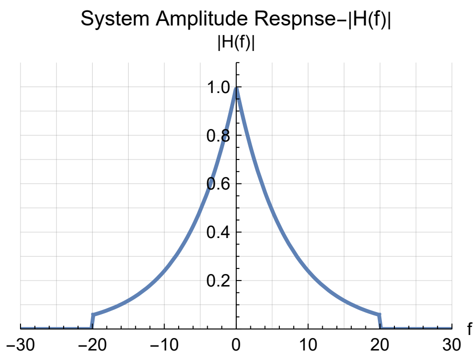
[Hint: no integration is required for this problem, convert and note sin($\alpha$)=cos($\alpha$-$\pi$/2) and -cos($\alpha$)=cos($\alpha$-$\pi$)]

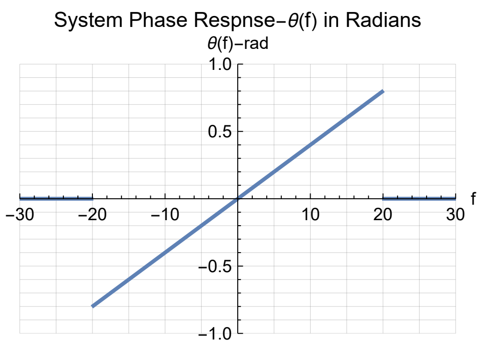
c. Plot the double-sided phase and magnitude spectrum for x(t).

d. What is the bandwidth of x(t)?

**9.** A system with H(f)= $|H(f)|\, e^{-j\theta(f)}$

where amplitude $|H(f)|= \text{rect}\left(\frac{f}{40}\right) e^{-\left|\frac{f}{7}\right|}$ and phase $\theta$(f)=$-\frac{f}{25}$ response is given below.




a. The signal x(t)=cos(10$\pi$t)+cos(30$\pi$t) is the input to the system with the frequency response H(f) given above. Find the system output, y(t).

b. A signal g(t) with a bandwidth of $B_g$ rad/sec is the input to a system with H($\omega$) with a bandwidth of $B_{sys}$ rad/sec, the system output is y(t). If $B_{sys}$>>$B_z$ then y(t)≈g(t). TRUE or FALSE. Circle TRUE or FALSE.

**10.** A signal x(t) = $e^{-100|t|}$ is the input to a linear time invariant system with an impulse response of h(t)=$\delta$(t-0.01). Find and sketch the system output y(t).

**11.** A filter has an impulse response of h(t) = sinc(1000t)

a. Is this an ILPF? Yes or NO

b. What the filter bandwidth?

c. With an input signal x(t)=2$\delta$(t) + $\delta$(t- $\tau$) where $\tau$=1ms input to this filter, plot the output signal y(t) in the time domain.

d. What is y(0) and y($\tau$)? How are they related to x(0) and x($\tau$)?

**12.** Solve the following.

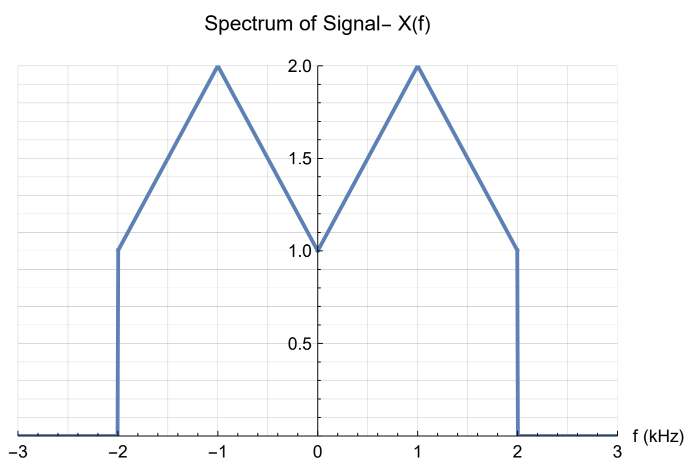
a. $\int_{-\infty}^{\infty} \delta(\tau)\, e^{-5\tau}\, d\tau$

b. $\int_{-\infty}^{\infty} \delta(\tau - .01)\, u(\tau)\, e^{-5\tau}\, d\tau$

c. $\int_{-\infty}^{\infty} \delta(\tau - 1)\, u(\tau)\, e^{-5\tau}\, d\tau$

d. $\int_{-\infty}^{\infty} \delta(\tau - t)\, u(\tau)\, e^{-5\tau}\, d\tau$

**13.** The spectrum of x(t) is given by:



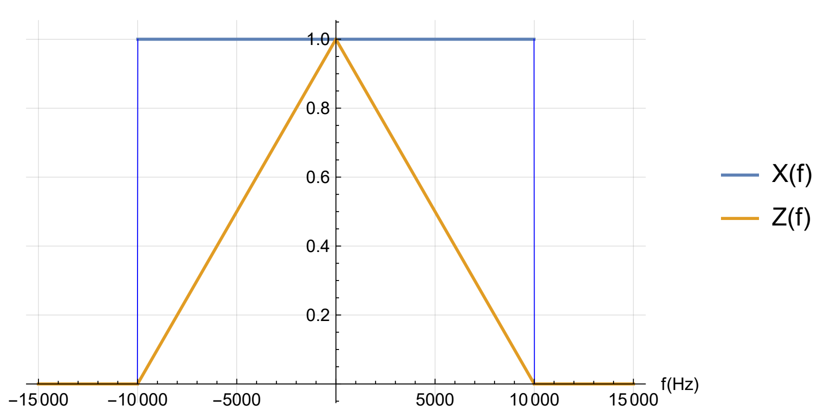

a. The signal x(t) is sampled at 7000 samples/sec to form $x_s$(t). Plot the spectrum of $x_s$(t).

b. For x(t) given above, what is the minimum sample rate required to prevent aliasing?

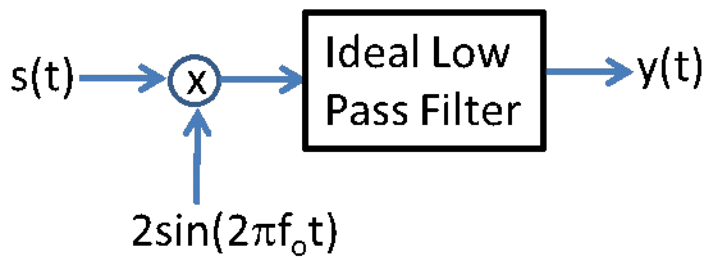
c. If no aliasing is present, describe how x(t) is recovered from $x_s$(t).

**14.** Let s(t) = $x(t)\cos(2\pi f_0 t) + z(t)\sin(2\pi f_0 t)$ with $f_0$=100khz and X(f) = rect$\left(\frac{f}{20000}\right)$ and Z(f)= $\Lambda\left(\frac{f}{10000}\right)$ as shown below.

Out[ ]=




a. Plot the Magnitude spectrum at the input to the LPF.




b. Find the output y(t) in terms of x(t) and z(t) of the system above. The bandwidth of the ILPF is 11 kHz. [Hint: use the trigonometry identities for $\sin^2(\theta)$ and $\cos^2(\theta)$]

(The solution to this problem provides the basis for quadrature modulation.)

**15.** LTE (4G/5G) systems use the DFT. Here a DFT of length 2048 is used with a sample frequency of $f_s$ = 30.72Msamples/s.

a. What is the record length in seconds?
(In LTE this is the symbol time.)
b. What is the spacing in kHz between frequency components of this DFT?
(In LTE this is the carrier spacing)

**16.** Use the DFT to convolve x[n]={1,1,4,4,4,4,1,1} with h[n]={-1,0,-2,0,5,4,3,2}. Use a computer tool, e.g., Matlab, for this problem,

**17.** Use the Discrete Fourier Transform of a Two-Tone Signal at

http://demonstrations.wolfram.com/DiscreteFourierTransformOfATwoToneSignal/ to answer the following questions. Set the noise amplitude to 0.1 for this problem. Note the frequency of the fix tone is 770 Hz and the sample rate is 8000 samples/sec. Use the dBV scale.

a. For N = 512 what is the record length and Δf.
b. Set the sine frequency $f_1$ = 800 and the sine amplitude =0.5. Describe and explain the change as N changes from 512 to 1024 to 4096. Set window to rect.
c. Repeat b with the Blackman window.

**18.** Show that the impulse response $h_R(t)$ for a system defined as $y(t) = \frac{1}{T}\int_t^{t-T} x(\tau)\,d\tau$ is $\frac{1}{T}\text{rect}\left(\frac{t-T/2}{T}\right)$.