

TABLE A3.1 *Table of Bessel Functions^a*

$n \backslash x$	$J_n(x)$									
	0.5	1	2	3	4	6	8	10	12	
0	0.9385	0.7652	0.2239	-0.2601	-0.3971	0.1506	0.1717	-0.2459	0.047	
1	0.2423	0.4401	0.5767	0.3391	-0.0660	-0.2767	0.2346	0.0435	-0.223	
2	0.0306	0.1149	0.3528	0.4861	0.3641	-0.2429	-0.1130	0.2546	-0.084	
3	0.0026	0.0196	0.1289	0.3091	0.4302	0.1148	-0.2911	0.0584	0.195	
4	0.0002	0.0025	0.0340	0.1320	0.2811	0.3576	-0.1054	-0.2196	0.182	
5	—	0.0002	0.0070	0.0430	0.1321	0.3621	0.1858	-0.2341	-0.073	
6		—	0.0012	0.0114	0.0491	0.2458	0.3376	-0.0145	-0.243	
7			0.0002	0.0025	0.0152	0.1296	0.3206	0.2167	-0.170	
8			—	0.0005	0.0040	0.0565	0.2235	0.3179	0.045	
9				0.0001	0.0009	0.0212	0.1263	0.2919	0.230	
10				—	0.0002	0.0070	0.0608	0.2075	0.300	
11					—	0.0020	0.0256	0.1231	0.270	
12						0.0005	0.0096	0.0634	0.195	
13						0.0001	0.0033	0.0290	0.120	
14						—	0.0010	0.0120	0.065	

^aFor more extensive tables of Bessel functions, see Abramowitz and Stegun (1965, pp. 358–406).

$$x_{\text{FM}}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(2\pi(f_c + nf_m)t)$$

$$|k = 1.38 \times 10^{-23} \lambda = \frac{c}{f_c} \text{ (m) with } c = 3 \times 10^8 \text{ m/s}$$

$$T_e = T_o(F - 1) \text{ with } T_o = 290^\circ$$

$$S_n(f) = \frac{N_0}{2} \forall f \quad N_0 = k(T_a + T_e)$$

$$T_e = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2}$$

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2}$$

$$G_{Ant} = \frac{4\pi A_{eff}}{\lambda^2}$$

For dish (circular) antenna $G_{Ant} = \left(\frac{\pi D}{\lambda}\right)^2$ D=Diameter(m)

$$\text{Path Loss} = L_p = \left(\frac{4\pi r}{\lambda}\right)^2 \quad r = \text{distance (m)}$$

$$P_T = T \times \text{Power}; G_T = T \times \text{Ant Gain}; G_R = R \times \text{Ant Gain}$$

$$\text{Received power} = P_R = \frac{P_T G_T G_R}{L_p}$$

$$(S/N)_{pre} = \frac{P_T G_T G_R}{L_M L_p k(T_a + T_e) B_e} = \frac{P_T G_T G_R}{L_M L_p k(T_a + T_o(F - 1)) B_e}$$

$$(S/N)_{pre} \text{ (dB)} = P_T + G_T + G_R - L_M - L_p - 10 \log(k(T_a + T_o(F - 1))) - 10 \log(B_e)$$

<i>Time Function</i>	<i>Fourier Transform</i>
$\text{rect}\left(\frac{t}{T}\right)$	$T \text{sinc}(fT)$
$\text{sinc}(2Wt)$	$\frac{1}{2W} \text{rect}\left(\frac{f}{2W}\right)$
$\exp(-at)u(t), \quad a > 0$	$\frac{1}{a + j2\pi f}$
$\exp(-a t), \quad a > 0$	$\frac{2a}{a^2 + (2\pi f)^2}$
$\exp(-\pi t^2)$	$\exp(-\pi f^2)$
$\begin{cases} 1 - \frac{ t }{T}, & t < T \\ 0, & t \geq T \end{cases}$	$T \text{sinc}^2(fT)$
$\delta(t)$	1
1	$\delta(f)$
$\delta(t - t_0)$	$\exp(-j2\pi f t_0)$
$\exp(j2\pi f_c t)$	$\delta(f - f_c)$
$\cos(2\pi f_c t)$	$\frac{1}{2}[\delta(f - f_c) + \delta(f + f_c)]$
$\sin(2\pi f_c t)$	$\frac{1}{2j}[\delta(f - f_c) - \delta(f + f_c)]$
$\text{sgn}(t)$	$\frac{1}{j\pi f}$
$\frac{1}{\pi t}$	$-j \text{sgn}(f)$
$u(t)$	$\frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$
$\sum_{i=-\infty}^{\infty} \delta(t - iT_0)$	$\frac{1}{T_0} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_0}\right)$

Notes: $u(t)$ = unit step function
 $\delta(t)$ = Dirac delta function
 $\text{rect}(t)$ = rectangular function
 $\text{sgn}(t)$ = signum function
 $\text{sinc}(t)$ = sinc function

$$\text{Here } \text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

$$\exp(\pm j\theta) = \cos \theta \pm j \sin \theta$$

$$\cos \theta = \frac{1}{2}[\exp(j\theta) + \exp(-j\theta)]$$

$$\sin \theta = \frac{1}{2j}[\exp(j\theta) - \exp(-j\theta)]$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\cos^2 \theta - \sin^2 \theta = \cos(2\theta)$$

$$\cos^2 \theta = \frac{1}{2}[1 + \cos(2\theta)]$$

$$\sin^2 \theta = \frac{1}{2}[1 - \cos(2\theta)]$$

$$2 \sin \theta \cos \theta = \sin(2\theta)$$

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

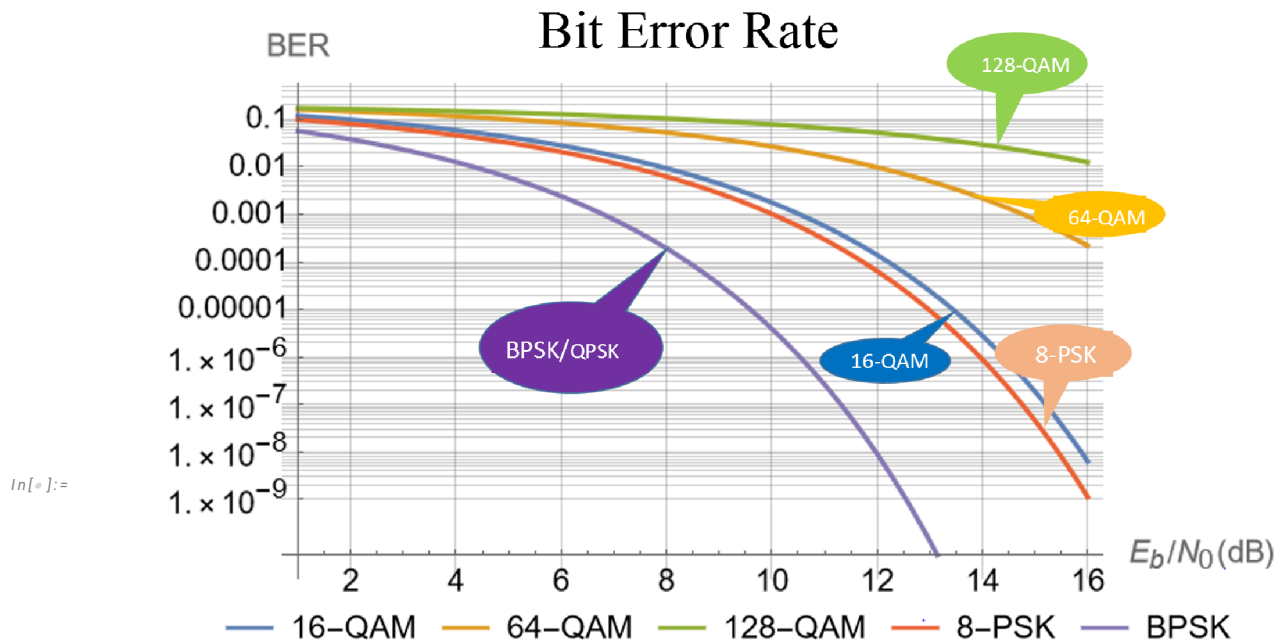
$$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$$

$$\sin \alpha \sin \beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\cos \alpha \cos \beta = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$\sin \alpha \cos \beta = \frac{1}{2}[\sin(\alpha - \beta) + \sin(\alpha + \beta)]$$

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BER Equations

$$\text{BPSK/QPSK } Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \quad \text{Coherent ASK } Q\left(\sqrt{\frac{E_b}{N_0}}\right) \quad \text{Noncoherent ASK } \frac{1}{2} e^{-\frac{E_b}{2N_0}}$$

$$\text{M-PSK } \frac{2 Q\left(\sqrt{\frac{2 \log_2(M) E_b}{N_0} \sin^2\left(\frac{\pi}{M}\right)}\right)}{\log_2(M)}, \quad \text{QAM (Square Grid)} \frac{4(\sqrt{M}-1)}{\sqrt{M} \log_2(M)} Q\left(\sqrt{\frac{3 E_b \log_2(M)}{N_0(M-1)}}\right)$$

Gaussian Probabilities

(1) $P(X > \mu_x + y\sigma_x) = Q(y) = \int_y^\infty \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-z^2}{2}\right) dz$

(2) $Q(0) = \frac{1}{2}$; $Q(-y) = 1 - Q(y)$, when $y \geq 0$

(3) $Q(y) \approx \frac{1}{y\sqrt{2\pi}} \exp\left(\frac{-y^2}{2}\right)$ when $y > 4$

(4) $\operatorname{erfc}(y) \triangleq \frac{2}{\sqrt{\pi}} \int_y^\infty \exp(-z^2) dz = 2Q(\sqrt{2}y)$, $y > 0$.

TABLE D.1 GAUSSIAN PROBABILITIES

y	Q(y)	y	Q(y)	y	Q(y)	Q(y)	y
.05	.4801	1.05	.1469	2.10	.0179	10 ⁻³	3.10
.10	.4602	1.10	.1357	2.20	.0139		
.15	.4405	1.15	.1251	2.30	.0107		
.20	.4207	1.20	.1151	2.40	.0082		
.25	.4013	1.25	.1056	2.50	.0062		
.30	.3821	1.30	.0968	2.60	.0047	10 ⁻³ 2	3.28
.35	.3632	1.35	.0885	2.70	.0035		
.40	.3446	1.40	.0808	2.80	.0026	10 ⁻⁴	3.70
.45	.3264	1.45	.0735	2.90	.0019		
.50	.3085	1.50	.0668	3.00	.0013		
.55	.2912	1.55	.0606	3.10	.0010		
.60	.2743	1.60	.0548	3.20	.00069		
.65	.2578	1.65	.0495	3.30	.00048	10 ⁻⁴ 2	3.90
.70	.2420	1.70	.0446	3.40	.00034		
.75	.2266	1.75	.0401	3.50	.00023	10 ⁻⁵	4.27
.80	.2119	1.80	.0359	3.60	.00016		
.85	.1977	1.85	.0322	3.70	.00010		
.90	.1841	1.90	.0287	3.80	.00007		
.95	.1711	1.95	.0256	3.90	.00005		
1.00	.1587	2.00	.0228	4.00	.00003	10 ⁻⁵	4.78

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Source: K. Sam Shanmugan, *Digital and Analog Communication Systems*, John Wiley & Sons, New York, 1979, pp. 583-84.

