1. A received signal $X(t)$ contains pulses of amplitude +1 with width 10 ms plus bandlimited white Gaussian noise $N(t)$. The noise PSD is

$$S_N(f) = \begin{cases} 2.5 \times 10^{-3} & |f| < 500 \text{Hz} \\ 0 & \text{elsewhere} \end{cases}$$

$X(t)$ is sampled at a rate of 1000 samples/sec. Samples are collected in pulse time sync with the pulses.

a) Design a pulse detector.

b) For your pulse detector and given the parameters above calculate the probability of detection and false alarm.

c) How many pulses are in this record? Describe the approach you used to arrive at that number. This file contains a record of collected samples.

http://www.ittc.ku.edu/~frost/EECS_861/EECS_861_HW_Fall_2017/Find_pulses-2017-a.csv

d) How many pulses are in this record? Describe the approach you used to arrive at that number. This file contains a record of collected samples.

http://www.ittc.ku.edu/~frost/EECS_861/EECS_861_HW_Fall_2017/Find_pulses-2017-b.csv

2. Let $f_X(x; \theta) = \frac{1}{\sqrt{10 \pi}} e^{-(x-\theta)^2/10}$. Given $x_1, \ldots, x_N$ be $N$ statistically independent samples from $f_X(x)$ is $\bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i$ a biased estimator for $\theta$; yes or no and justify.

3. We want to estimate a received signal $X$ from $K$ observations $Y$ where is modeled as $Y = X + N$. Here $K = 25$ and the $\bar{y} = \frac{1}{25} \sum_{i=1}^{25} y_i = 16$.

$N$ is Gaussian with $E[N] = 0$ and $\text{Var}[N] = \sigma_N^2$

$X$ is Gaussian with $E[X] = 10$ and $\text{Var}[X] = \sigma_X^2$

$N$ and $X$ are statistically independent. For the following 3 cases:

Case 1: $\sigma_N^2 = 0.10 \sigma_X^2 = 10$

Case 2: $\sigma_N^2 = 5 \sigma_X^2 = 5$

Case 3: $\sigma_N^2 = 10 \sigma_X^2 = 0.10$

Find

a. The MAP estimator for $X$.

b. The Mean Square (MS) estimator for $X$.

c. The Maximum Likelihood (ML) estimator for $X$. 

4. Find the unconstrained Weiner filter to estimate \( S(t) \) from \( Y(t) = S(t) + N(t) \), where \( N(t) \) and \( S(t) \) are statistically independent and

\[
S_s(f) = \frac{8}{1+(8\pi f)^2}
\]
and
\[
S_n(f) = 1
\]

5. Let the desired signal, \( S[k] \), be characterized by a moving average process given by
\[
S[k] = X[k] + 0.75X[k-1] + 0.75 X[k-2]
\]
where \( X[k] \) is a white Gaussian random process with zero mean and variance = 0.1
The observed signal is \( Y[k] = S[k] + N[k] \) where \( N(k) \) is Gaussian with \( R_{NN}[n] = 0.5 \) for \( n=0 \) and \( R_{NN}[n] = 0 \) elsewhere.
   a. Find \( R_{SS}[k] \).
   b. Find the optimum realizable Wiener filter \( h[k] \) where
\[
\hat{S}[n] = \sum_{k=0}^{\infty} h[k] Y[n-k]
\]
   c. Apply the optimum realizable Wiener filter \( h[k] \) to this received signal
   
http://www.ittc.ku.edu/~frost/EECS_861/EECS_861_HW_Fall_2017/received_signal.csv
   d. Calculate the resulting mean square error given the desired signal
   
http://www.ittc.ku.edu/~frost/EECS_861/EECS_861_HW_Fall_2017/desired_signal.csv
   e. Repeat parts c and d using \( h[k] = 1/3, 1/3, 1/3 \), that is a three sample average and compare the resulting mean square errors.
   (Note: The desired signal and the processed signal needs to be aligned to calculate the mean square error, i.e., there is a delay going through \( h[k] \))

6. Let \( s_0[k] = -2.0, -2.0, -2.0 \) for \( k = 0, 1, 2 \) & \( s_0(k) = 0 \) elsewhere and \( s_1[k] = -s_0[k] \)
Assume
\[
P(s_1) = 0.5 = P(s_0)
\]
\( Y[k] = S[k] + N[k] \) for \( k = 0, \ldots, 2 \) where
\( S[k] \) & \( N[k] \) are statistically independent
\( N[k] \) is white Gaussian noise with a zero mean and unit variance, i.e., \( \sigma_N = 1 \).
   a. Find the MAP decision algorithm.
   b. Find the probability of error.
   c. Apply the MAP decision algorithm for the follow observations
\( y(k) = -0.4, 0.1, 0.1 \) for \( k = 0, \ldots, 2 \)
   d. Repeat a)-c) for \( s_0[k] = -1.0, -3.175, -1.0 \)
7. A decision is based on 2 samples, $Y_1$ and $Y_2$. $Y = \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix}$ is a multivariate Gaussian random vector with $E[Y | H_0] = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$ and $E[Y | H_1] = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ with $\Sigma = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$

[Hint see Homework 4 Problem 8]

a) Design the optimum detector.
b) Find $P_e$. 