EECS 861

Homework #1

1. Plot

a. $x_1(t) =$ 10 cos(2 $\pi$1000t),

b. $x_2(t) =$ 10 cos(2 $\pi$1000(t-62.5$\mu$s))

c. $x_3(t) =$ 10 cos(2 $\pi$1000t- $\frac{\pi}{8}$)

d. Compare these three signals and explain their similarities and differences.

e. What is the energy and power in $x_1(t)$.

2. Plot

a. x(t) = rect($\frac{t-2}{2}$) + rect($\frac{t+2}{2}$)

b. What is the energy and power in $x(t)$.

3. For

$x_1(t) =$ sinc(500t),
$x_2(t) =$ sinc(1000 $t$)
$x_3(t) =$ sinc(5000 $t$)
(here sinc(x)=$\frac{\sin(\pi x)}{\pi x}$)

a. Plot $x_1$(t), $x_2$(t), and $x_3$(t)

b. Rank order the signals from lowest bandwidth to highest bandwidth.

4. A bit is transmitted as

x(t) = Acos(2$\pi$1000t) if bit = "1" for $T_b$

or

x(t) = -Acos(2$\pi$1000t)= -Acos(2$\pi$1000t-$\pi$) if bit = "0" for $T_b$

In another form x(t) = ±Acos(2 $\pi$1000t) rect$\left(\frac{t-T_b}{\frac{T_b}{2}}\right)$

For A=2 and $T_b$=100ms

a. Find the energy in x(t), that is the energy/bit = $E_b$.

b. What is the bit rate in b/s?

(This problem provides the basis for Binary Phase Shift Keying-BPSK.)

5. For $f_1$=2000 and $f_2$=2150

a. Find $\int_{-0.01}^{0.01} \cos(2\pi f_1 t)\cos(2\pi f_2 t)\,dt$

b. What property describes the relationship between cos (2 $\pi f_1$ t) and cos (2 $\pi f_2$ t).

c. Let $x_1(t) =$ rect$\left(\frac{t}{20\text{ ms}}\right)$cos(2 $\pi f_1$t) and $x_2(t) =$ rect$\left(\frac{t}{20\text{ ms}}\right)$cos(2 $\pi f_2$t), find $\int_{-\infty}^{\infty} x_1(t)\,x_2(t)\,dt$ and relate the result to part a.

6. For x(t) = 1+6cos(100$\pi$t) +4sin(300$\pi$t)

a. What is the fundamental frequency in Hz?

b. Find the complex Fourier series for x(t).

[Hint: no integration is required for this problem, convert to complex exponential form and note sin($\alpha$)=cos($\alpha$-$\pi$/2) and -cos($\alpha$)=cos($\alpha$-$\pi$)]

c. Plot the double-sided phase and amplitude spectrum for x(t).

d. What is the bandwidth of x(t)?

7. A signal x(t) = $e^{-1000|t|}$ is the input to a linear time invariant system with an impulse response of h(t)=$\delta$(t-0.01). Find and sketch the system output y(t).

8. A linear time-invariant system with input signal x(t) produces an output signal y(t) =2x(t-1ms), find the system transfer function and impulse response.

9. The transfer function for the voltage across the capacitor in an RC filter is

$$H(f) = \frac{1}{1+j2\pi RCf}$$

Set R=1000Ω and C= 100pf

a. Plot 10 $\text{Log}_{10}$ $|H(f_1)|^2$ )= 20 $\text{Log}_{10}$(|H(f)|)

b. Plot Phase of H(f) (or Angle H(f))

c. Given an input signal $x(t) = \cos(2\pi f_1 t) + 2\cos(2\pi 2 f_1 t)$ with $f_1$=1.6MHz, find the system output y(t).

10. A filter has an impulse response of h(t) = 100sinc(100t)

a. Is this an ILPF? Yes or NO

b. What the filter bandwidth?

c. With an input signal x(t)=2$\delta$(t- $\tau$) + $\delta$(t- 2$\tau$) where $\tau$=10ms input to this filter, plot the output signal y(t) in the time domain.

11. Solve the following $\int_{-\infty}^{\infty} \delta(\tau - t)\, u(t)\, e^{-10\tau}\, d\tau$

12. Consider a linear time invariant system with a impulse response of h(t), and input signal x(t) given below. The input signal x(t) given below produces and output of y(t). Here x(t)= rect(t+.5)+u(t)$e^{-t}$ and h(t)=rect(t).

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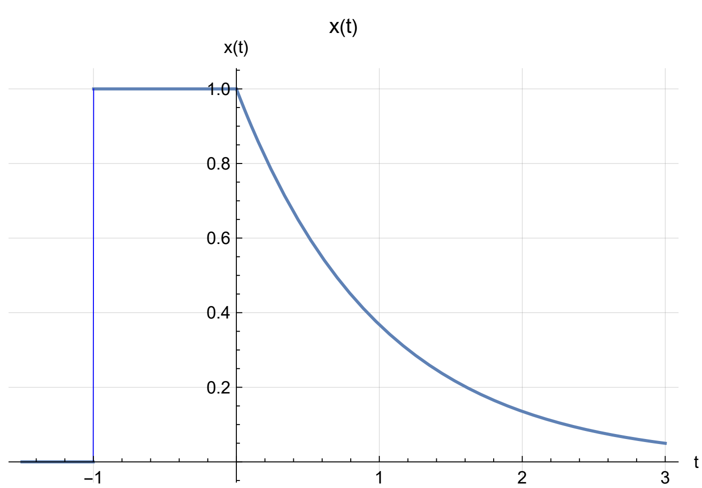

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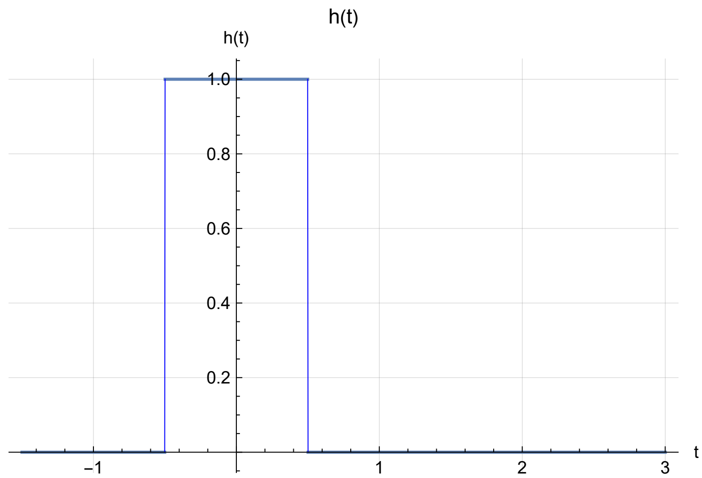
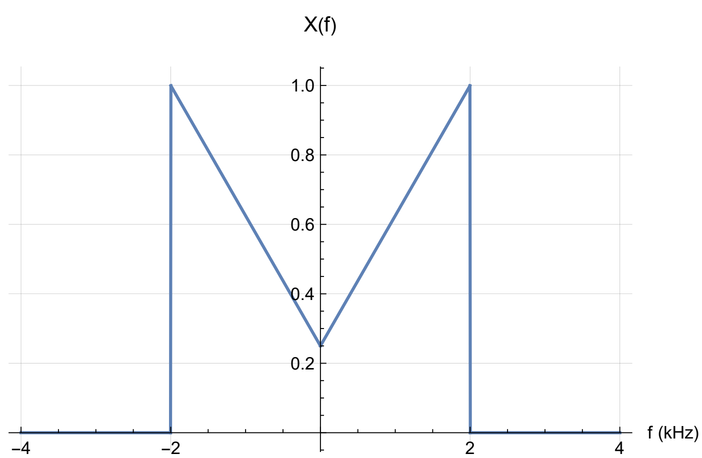



a. What is the range of time such that y(t) =0

b. Find and plot the convolution of x(t) and h(t).

13. A signal x(t) has a Fourier Transform given as




a. The signal x(t) is sampled at a rate of $f_s$=5000 samples/sec to create $x_s(t)$. Sketch the spectrum of the sampled signal $x_s(t)$.

b. The signal x(t) is sampled at a rate of 5000 samples/sec. Describe, e.g., draw a block diagram, a system to recover x(t) from $x_s(t)$.

c. What is the minium value of $f_s$ where x(t) can be recovered from $x_s(t)$?

14. Let s(t) = $x(t)\cos(2\pi f_0 t)+z(t)\sin(2\pi f_0 t)$ with $f_0$=100khz and X(f) = $\text{rect}\left(\frac{f}{20000}\right)$ and Z(f)= $\Lambda\left(\frac{f}{10000}\right)$ as shown below.

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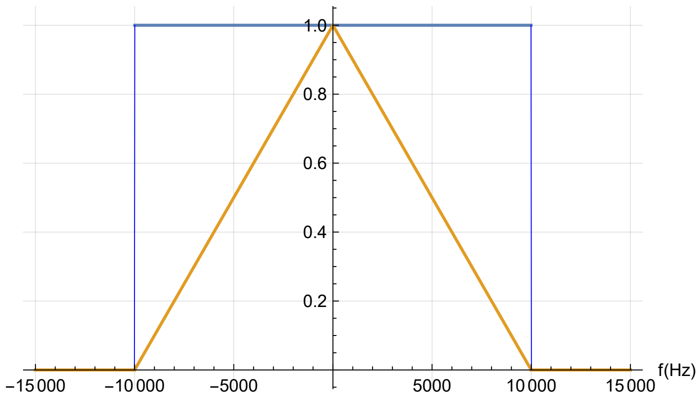
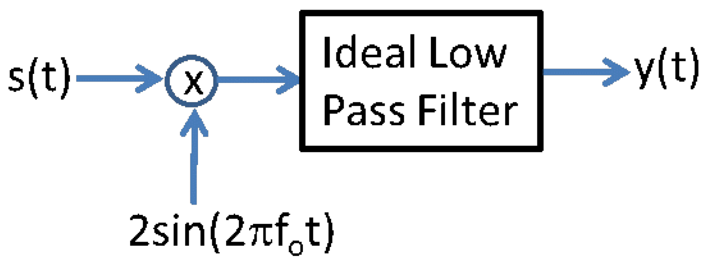



a. Plot the amplitude spectrum of s(t).




b. Find the output y(t) in terms of x(t) and z(t) of the system above. The bandwidth of the ILPF is 11 kHz. [Hint: use the trigonometry identities for $\sin^2(\theta)$ and $\cos^2(\theta)$]

(The solution to this problem provides the basis for quadrature modulation.)

15. Use the Discrete Fourier Transform of a Two-Tone Signal at http://demonstrations.wolfram.-com/DiscreteFourierTransformOfATwoToneSignal/ to answer the following questions. Set the noise amplitude to 0.1 for this problem. Note the frequency of the fix tome is 770 Hz and the sample rate is 8000 samples/sec. Use the dBV scale.

a. For N = 512 what is the record length and Δf.

b. Set the sine frequency = 1050 and the sine amplitude =0.6. Describe and explain the change as N changes from 512 to 1024 to 4096.

c. Set the sine frequency = 1050 and the sine amplitude =0.6 and N=512. Describe and explain the change as the window is changed from rect to Hanning to Blackman.