EECS 861

Homework #13

1. We want to estimate a received signal X from K observations of Y where Y is modeled as Y=X+N. Here K = 20 and the

$\overline{y} = \frac{1}{20} \sum_{i=1}^{20} y_i = 18.$

N is Gaussian with E[N]=0 and Var[N] = σ_N^2

X is Gaussian with E[X]=12 and Var[X] = σ_X^2

N and X are statistically independent. For the following 3 cases:

Case 1: $\sigma_N^2 = 0.10$ $\sigma_X^2 = 15$

Case 2: $\sigma_N^2 = 15$ $\sigma_X^2 = 15$

Case 3: $\sigma_N^2 = 15$ $\sigma_X^2 = 0.10$

Find

a. The MAP estimator for X.

b. The Mean Square (MS) estimator for X.

c. The Maximum Likelihood (ML) estimator for X.

2. Find the unconstrained Weiner filter to estimate S(t) from Y(t) =S(t)+N(t), where N(t) and S(t) are statistically independent and

$S_S(f) = \frac{4}{1+(2\pi f)^2}$

and

$S_N(f) = 20$

H(f) = S(f) / (S(f) + N(f))

3. Let the desired signal, S[k], be characterized by a moving average process given by

S[k]= X[k] + 0.8X[k-1] +0.7 X[k-2]

where X[k] is a white Gaussian random process with zero mean and variance = 1/16

The observed signal is Y[k]=S[k] + N[k] where N[k] is Gaussian with R_{NN}[n]=0.4 for n=0 and R_{NN}[n]=0 elsewhere.

a. Find R_{SS}[n].

b. Find the optimum realizable Wiener filter h[k] where

$\hat{S}[n] = \sum_{k=0}^{\infty} h[k] \times Y[n-k]$

c. Apply the optimum realizable Wiener filter h[k] to this received signal

http://www.ittc.ku.edu/~frost/EECS_861/EECS_861_HW_Fall_2023/Received_signal-2023.xls

d. Calculate the resulting mean square error given the desired signal

http://www.ittc.ku.edu/~frost/EECS_861/EECS_861_HW_Fall_2023/Desired_signal-2023.xls

e. Repeat parts c and d using h[k] = 1/3, 1/3, 1/3, that is a three sample moving average and compare the resulting mean square errors.

(Note: The desired signal and the processed signal needs to be aligned to calculate the mean square error, i.e., there is a delay going through h[k])