EECS 861

Homework #3

1. Show that Var[X]=E[X^2]-(E[X])^2

2. 1. X is a discrete random variable with

   P(X=-2) = a, P(X= -1) = 0.4, P(X=1) = 0.4, P(X= 2) = 0.1

   a. Find "a"

   Given "a" find:

   b. Find P(X> 3)

   c. Find P(X=0)

   d. Find E[X]

   e. Find E[X^2]

   f. Find Var[X]

3. Using 200 samples {x_1.....x_{200}} of a discrete random variable X is given in http://www.ittc.ku.edu/~frost/EECS_861/EECS_861_HW_Fall_2022/data_HW3_Prob_2.csv

   a. Given this data what is an estimate for p_k = P(X=k) for k=-2, -1, 1, 2?

   b. Find the sample mean of X using $\overline{X} = \frac{1}{200}\sum_{i=1}^{200} x_i$

   c. Estimate the mean of X using $\widehat{X} = \sum_{k=-2}^{2} k p_k$

   d. Find the sample mean square of X using $\overline{X}^2 = \frac{1}{200}\sum_{i=1}^{200} x_i^2$

   e. Estimate the mean square of X using $\widehat{X}^2 = \sum_{k=-2}^{2} k^2 p_k$

   f. Is the pmf given in problem 2 a "good" probabilistic model for this data?

4. X is a random variable with $f_X(x) = 0.4\delta(x) + 0.6u(x)e^{-x}$ where u(x)= unit step function

   a. Sketch $f_x(x)$.

   b. Verify that the total probability is 1.

   c. What is P(X= 0)?

   d. What is P(X= 2)?

   e. What is P(-2<X< 1)?

   f. Find E[X]

   g. Var[X]

5. X is a Gaussian random variable X with $\mu_X = 0$ and σ_X=1.7

   a. What is P(-0.5<X<.5)?

   b. Plot $P(X < x_i)$ for x_i= -4.0,-1.0, -0.4, -0.3, -0.2, -0.1, 0.0, 0.1, 0.2, 0.3, 0.4, 1.0, 4.0

   c. Confirm your answers using

https://www.mathportal.org/calculators/statistics-calculator/normal-distribution-calculator.php

d. Assuming X is a Uniform random variable [-2, 2] repeat part b.

e. Is a Uniform random variable [-2, 2] “good” probabilistic model for the data given in Homework 2-Problem 2?

6. X and Y have the following joint distribution function

□	X = -2	X = 0	X = 2
Y = -2	1 / 8	1 / 8	0
Y = 0	0	0	1 / 8
Y = 2	1 / 4	1 / 8	1 / 4

a. Find P(X=0).

b. Find P(Y=2).

c. Find P(X=0|Y=2).

d. Find ρ_{XY}.

e. Are X and Y SI random variables?

7. Show (from Chapter 2: Problem 2.18)

a. $E\{a + bX\} = a + bE\{X\}$

b. $E\{aX + bY\} = aE\{X\} + bE\{Y\}$

c. Variance of $aX + bY = a^2 \operatorname{Var}[X] + b^2 \operatorname{Var}[Y] + 2ab \operatorname{Covar}[X, Y]$