EECS 861

Homework #7

1. Determine whether the following functions can be the autocorrelation function for a WSS real values random process (YES or NO):

a. $2\delta(\tau) + \sin(2\pi 100\tau)$
b. $5\,\mathrm{rect}(\frac{\tau-2}{3})$
c. $3\Lambda(\tau)$
d. $4e^{-|t|}$
e. $100\mathrm{sinc}(100\tau)$
f. $4e^{\frac{-\pi\tau^2}{8}}$

2. X(t) and Y(t) are wide sense stationary, independent, zero mean, and jointly Gaussian random processes

$Z(t) = X(t)\cos(2\pi f_c t) + Y(t)\sin(2\pi f_c t)$ with f_c a constant and $R_{XX}(\tau) = R_{YY}(\tau)$

a. Find E[Z(t)]
b. Find $R_{ZZ}(\tau)$

3. Find the E[X(t)] and Var[X(t)] for a wide sense stationary random process with the following autocorrelation functions:

a. $R_{XX}(\tau) = 4e^{-|t|}$
b. $R_{XX}(\tau) = 9 + 4e^{-|t|}$
c. $R_{XX}(\tau) = 16e^{\frac{-\pi\tau^2}{8}}$
d. $R_{XX}(\tau) = 9 + 16e^{\frac{-\pi\tau^2}{8}}$

4. X(t) is a wide sense stationary Gaussian random processes with $R_{XX}(\tau) = 10. * e^{\frac{-\pi * t^2}{2}}$.

a. Find E[X(1)], Var[X(1)], E[X(2)], and Var[X(2)]
b. What is the distribution of X(1), i.e., name of pdf and its parameters?
c. Find P(X(1)>4)
d. What is the covariance matrix for X(1) and X(2)?
e. What is the joint distribution of X(1) and X(2), i.e., name of pdf and its parameters?
f. What is the correlation coefficient between X(1) and X(2)?
g. Find P(X(2)>4|X(1)=3)
h. Approximate P(X(20)>4|X(1)=3)

5. X(t) is a wide sense stationary zero mean, Gaussian random processes with $R_{xx}(\tau) = 16e^{\frac{-\pi\tau^2}{8}}$.
$Z(t) = X^2(t)$. This is the case of Gaussian noise as input to a square law detector.

a. Find E[Z(t)]
b. Find $R_{ZZ}(t_1, t_2)$

c. Is Z(t) a wide sense stationary random processes (YES or NO)?

d. Is Z(t) a Gaussian random processes (YES or NO)?

Hint: If X and Y are jointly Gaussian random variables then

$E[X^2 Y^2] = E[X^2] E[Y^2] + 2 (E[XY])^2$