EECS 861

Homework #8

1. X(t) is a wide sense stationary zero mean, Gaussian random processes with a power spectral density of $S_x(f) = 10\sqrt{2}\, e^{-2 f^2 \pi}$. [Hint: see Homework 7-problem 4]

   a. Find E[X(t)], Var[X(t)], E[X(t+1)], and Var[X(t+1)]

   b. What is the distribution of X(t), i.e., name of pdf and its parameters?

   c. Find P(X(t)>4)

   d. What is the covariance matrix for X(t) and X(t+1)?

   e. What is the joint distribution of X(t) and X(t+1), i.e., name of pdf and its parameters?

   f. What is the correlation coefficient between X(t) and X(t+1)?

   g. Find P(X(t+1)>4|X(t)=3)

   h. Approximate P(X(t+20)>4|X(t)=3)

2. For a random process with a PSD of $S_x(f) = \frac{1}{1000} * \mathrm{sinc}^2\left(\frac{\pi * f}{1000}\right)$

   a. Find the B_{eff}

   b. Find the $B_{3\,\text{dB}}$

   c. Find $B_{\text{first zero}}$ defined as the first frequency where S_X(f) =0

   d. Compare the above definitions of bandwidth.

3. Given the random process from problem 2,

   a. Find the correlation time τ_c

   b. Compare the correlation time to $\frac{1}{2 B_{3\,\text{dB}}}$ and $\frac{1}{2 B_{\text{first zero}}}$

   c. What is the correlation coefficient between X(t) and X(t+2ms)?

4. Determine whether the following functions can be the power spectral density for a WSS real valued random process (YES or NO).

   a. $-\,\mathrm{rect}(f)$

   b. $2e^{+4\pi f^2}$

   c. $100\,\Lambda(100f)$

   d. $10e^{-\pi(f+0.10)}$

   e. $16e^{-|f-2|}$

   f. $5\,\delta(f) + \sin(200\,\pi f)$

   g. $\delta(f) + 4\,\delta(f+20) + 4\,\delta(f-20)$

5. The random process X(t) is WSS. For each of the autocorrelation functions below find and plot the corresponding power spectral density, S_x(f).

   a) $R_{XX}(\tau) = 8\cos(2\pi 1000\tau)$

   b) $R_{XX}(\tau) = 8\,\Lambda(\frac{\tau}{4})$

   c) $R_{XX}(\tau) = 8\,e^{-\left|\frac{\tau}{4}\right|}$

   d) $R_{XX}(\tau) = 8\,e^{-\pi(\frac{\tau}{4})^2}$

6. A power spectral density for a WSS random process X(t) is
$S_X(f) = 0.001 \Lambda \left(\frac{f}{100\,kHz}\right)$

a. Find $E[X^2(t)]$.

b. Find the Average power.

c. Find the % power the band [0, 50 kHz]