1. A zero mean WSS random process, X(t) has the following autocorrelation function

$R_{XX}(\tau) = e^{-100|\tau|}$

    a. Find E[X(t)]

    b. Find the variance of X(t).

    c. Find the effective bandwidth, B_{eff} of X(t).

    d. Find the correlation time τ_c in ms

Let

$$Y = \frac{1}{T} \int_0^T X(\eta)\, d\eta$$

    e. For T = 5 ms find the variance of Y and the ratio of Var[X]/Var[Y].

    f. For T = 2000 ms find the variance of Y and the ratio of Var[X]/Var[Y]; then compare the Var[Y] to the large $B_{eff}T$ approximation of the of Var[Y].

    g. For T = 2000 ms find P(Y> 0.05)

2. Let X(t) = A cos(2 π1000t) where A is a random variable with E[A]=m and Var[A] = σ^2.

    a. Find E[X(t)]

    b. Find $\langle X(t)\rangle_T = \frac{1}{T} \int_{-T/2}^{T/2} X(t)\, dt$

    c. Find $\underset{T\to\infty}{Lim}\langle X(t)\rangle_T$

    d. Compare the results from part a and part b.

    e. Repeat a. and d. with E[A]=0 and Var[A] = σ^2

3. Explain the concepts of strict sense stationarity and ergodicity.

4. Chapter 3: Problem 3.26

X(t) and Y(t) are two independent WSS random processes with the power spectral density functions shown below-Figure 3.34. Let Z(t) = X(t)Y(t).
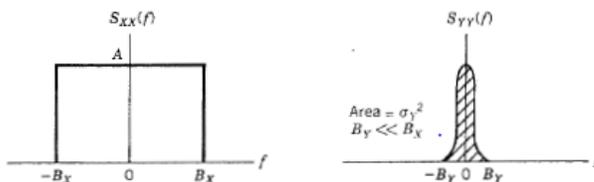Sketch the Power Spectral Density of Z(t), and find $S_Z(0)$.

Figure 3.34 Psd functions for Problem 3.26.

5. A zero mean Gaussian WSS random process, X(t) has the following PSD

    $S_x(f) = \frac{1}{500} \Lambda(\frac{f}{500})$

    a. Plot $R_{XX}(\tau)$.

    b Find E[X(t)]

    c. Find Var[X(t)]

    d. Are the random variables X(t) and X(t-2ms) uncorrelated (Yes or No); justify?

e. Are the random variables X(t) and X(t-2ms) statistically independent (Yes or No); justify?

f. What is P(X(t)>2| X(t-2ms)=3.1]?

g. What is the bivariate pdf for the random variables X(t) and X(t-1ms), Specify the pdf, mean vector and covariance matrix.

6. Given the random process given in problem 5, i.e., X(t) has the following PSD

$S_x(f) = \frac{1}{500} \Lambda(\frac{f}{500})$. The average of 10 samples with are collected at a sample rate of 500 samples/sec is formed as $Y = \frac{1}{10} \sum_{k=1}^{10} X(t - k\Delta t)$ where $\Delta t = 2$ ms

a. Find the variance of Y.

b. How long to you need to sample X(t) (that observe the signal X(t)) at a sample rate of 500 samples/sec such that the varaince of the time average Var[Y]=0.01