

Written By Dr. Victor S. Frost
 Dan F. Servey Distinguished Professor
 Electrical Engineering and Computer Science
 University of Kansas

Continuous Time Markov Chain

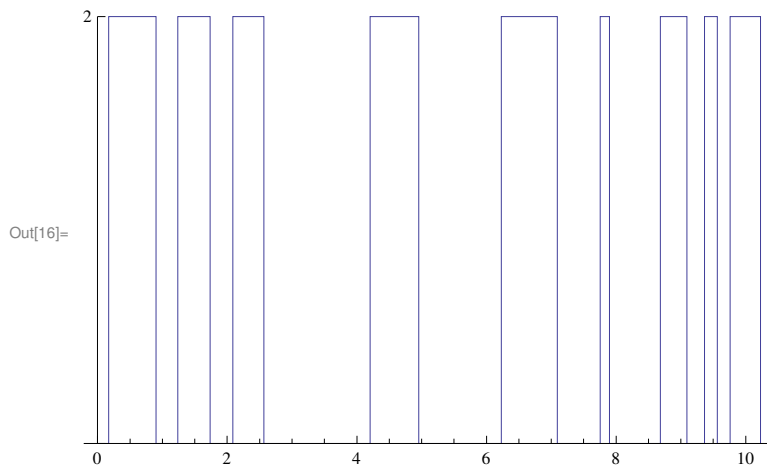
```
In[14]:= P1 = ContinuousMarkovProcess[{1, 0},  $\begin{pmatrix} -1. & 1. \\ 2 & -2 \end{pmatrix}$ ];  

  data = RandomFunction[P1, {0, 10}]  

  ListLinePlot[data, InterpolationOrder -> 0,  

  PlotRange -> {1, 2}, Ticks -> {Automatic, {1, 2}}]
```

Out[15]= TemporalData[1]

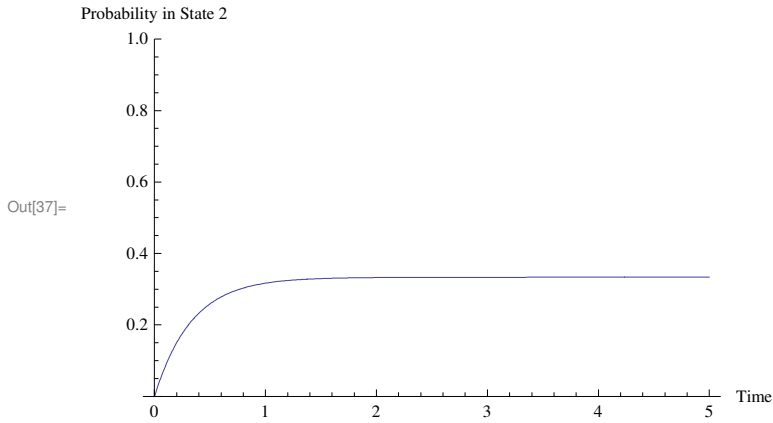


```
In[17]:= PDF[P1[t], k] // PiecewiseExpand
```

$$\text{Out[17]= } \begin{cases} \frac{0.333333 (-1.+1. \times 2.71828^{3. t})}{-1.04673 \times 10^{-16} + 1. \times 2.71828^{3. t}} & k == 2 \\ \frac{0.666667 (0.5+1. \times 2.71828^{3. t})}{-1.04673 \times 10^{-16} + 1. \times 2.71828^{3. t}} & k == 1 \\ 0 & \text{True} \end{cases}$$

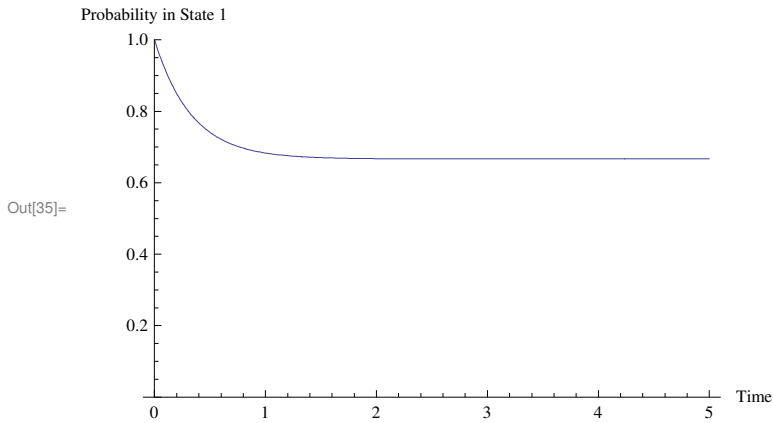
```
In[37]:= Plot[
$$\frac{0.3333333333333335 \cdot (-1. + 1. \times 2.718281828459045^{3. \cdot t})}{-1.0467283057891839 \cdot e^{-16} + 1. \times 2.718281828459045^{3. \cdot t}}$$
, {t, 0, 5},  

  AxesLabel -> {"Time", "Probability in State 2"}, PlotRange -> {0, 1}]
```



```
In[35]:= Plot[
$$\frac{0.6666666666666666 \cdot (0.5000000000000001 \cdot + 1. \times 2.718281828459045^{3. \cdot t})}{-1.0467283057891839 \cdot e^{-16} + 1. \times 2.718281828459045^{3. \cdot t}}$$
, {t, 0, 5},  

  AxesLabel -> {"Time", "Probability in State 1"}, PlotRange -> {0, 1}]
```



```
In[18]:= PDF[ $\mathcal{P}1[\infty]$ , 1]  

  PDF[ $\mathcal{P}1[\infty]$ , 2]
```

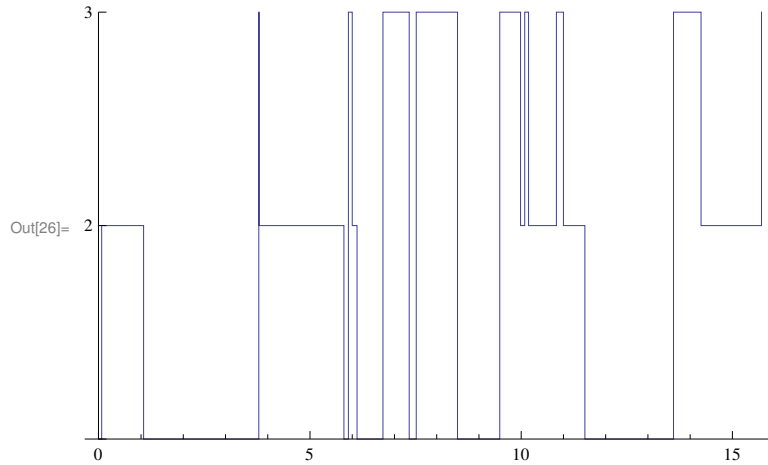
Out[18]= 0.666667

Out[19]= 0.333333

```
In[20]:=  $\mathcal{P} = \text{ContinuousMarkovProcess}[\{1, 0, 0\}, \begin{pmatrix} -1 & .5 & .5 \\ 1 & -2 & 1 \\ 1.5 & 1.5 & -3 \end{pmatrix}]$ ;
```

```
In[25]:= data = RandomFunction[ $\mathcal{P}$ , {0, 15}]
ListLinePlot[data, InterpolationOrder -> 0,
  PlotRange -> {1, 3}, Ticks -> {Automatic, {1, 2, 3}}]
```

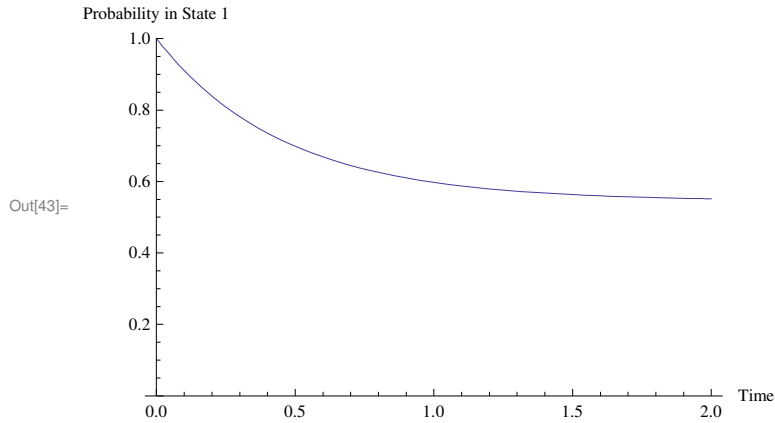
```
Out[25]= TemporalData[ $\{1\}$ ]
```



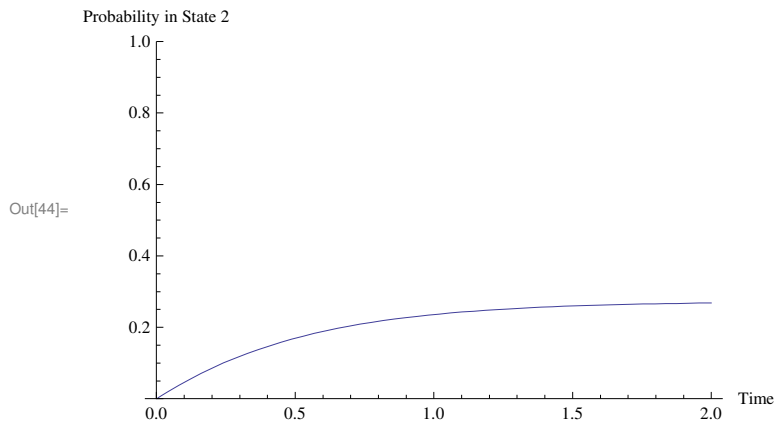
```
In[27]:= PDF[ $\mathcal{P}[t], k$ ] // PiecewiseExpand
```

$$\text{Out[27]= } \begin{cases} \frac{7.10375 \times 10^{15} (1. \times 2.71828^{2.13397 t} + 1.81165 \times 2.71828^{3.86603 t} - 2.81165 \times 2.71828^{6. t})}{1. \times 2.71828^{2.13397 t} - 27.0448 \times 2.71828^{3.86603 t} - 1.09853 \times 10^{17} \times 2.71828^{6. t}} & k = 3 \\ -\frac{5.20031 \times 10^{15} (1. \times 2.71828^{2.13397 t} - 6.76119 \times 2.71828^{3.86603 t} + 5.76119 \times 2.71828^{6. t})}{1. \times 2.71828^{2.13397 t} - 27.0448 \times 2.71828^{3.86603 t} - 1.09853 \times 10^{17} \times 2.71828^{6. t}} & k = 2 \\ -\frac{1.90345 \times 10^{15} (1. \times 2.71828^{2.13397 t} + 25.2331 \times 2.71828^{3.86603 t} + 31.4797 \times 2.71828^{6. t})}{1. \times 2.71828^{2.13397 t} - 27.0448 \times 2.71828^{3.86603 t} - 1.09853 \times 10^{17} \times 2.71828^{6. t}} & k = 1 \\ 0 & \text{True} \end{cases}$$

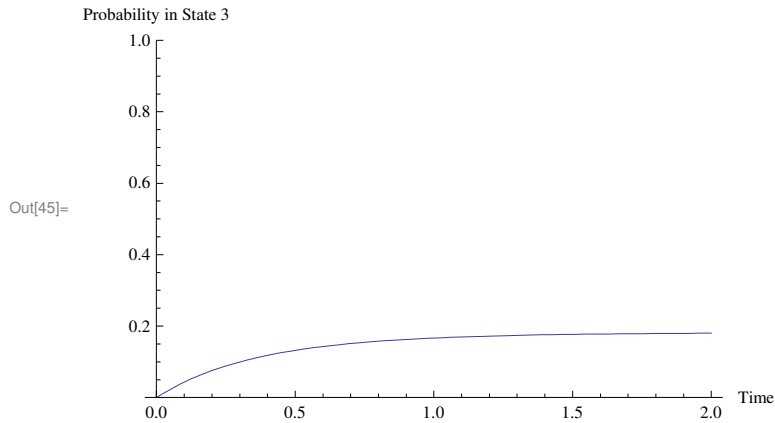
```
In[43]:= Plot[- (1.9034451701858438`*^15 (1.` × 2.718281828459045`^2.1339745962155594` t +
    25.233096782319073` × 2.718281828459045`^3.866025403784439` t +
    31.479716138782855` × 2.718281828459045`^5.999999999999997` t)) /
    (1.` × 2.718281828459045`^2.1339745962155594` t - 27.044751621435047` ×
    2.718281828459045`^3.866025403784439` t -
    1.0985317501251058`*^17 × 2.718281828459045`^5.999999999999997` t),
    {t, 0, 2}, AxesLabel → {"Time", "Probability in State 1"},
    PlotRange → {0, 1}]
```



```
In[44]:= Plot[- (5.200308914369307`*^15 (1.` × 2.718281828459045`^2.1339745962155594` t -
    6.761187905358761` × 2.718281828459045`^3.866025403784439` t +
    5.76118790535876` × 2.718281828459045`^5.999999999999997` t)) /
    (1.` × 2.718281828459045`^2.1339745962155594` t - 27.044751621435047` ×
    2.718281828459045`^3.866025403784439` t -
    1.0985317501251058`*^17 × 2.718281828459045`^5.999999999999997` t),
    {t, 0, 2}, AxesLabel → {"Time", "Probability in State 2"},
    PlotRange → {0, 1}]
```



```
In[45]:= Plot[(7.103754084555151`*^15 (1.` × 2.718281828459045`^2.1339745962155594` t +
  1.811654839115953` × 2.718281828459045`^3.866025403784439` t -
  2.8116548391159526` × 2.718281828459045`^5.999999999999997` t)) /
(1.` × 2.718281828459045`^2.1339745962155594` t - 27.044751621435047` ×
  2.718281828459045`^3.866025403784439` t -
  1.0985317501251058`*^17 × 2.718281828459045`^5.999999999999997` t),
{t, 0, 2}, AxesLabel → {"Time", "Probability in State 3"},
PlotRange → {0, 1}]
```



```
In[28]:= PDF[ $\mathcal{P}[\infty]$ , 1]
PDF[ $\mathcal{P}[\infty]$ , 2]
PDF[ $\mathcal{P}[\infty]$ , 3]
```

Out[28]= 0.545455

Out[29]= 0.272727

Out[30]= 0.181818