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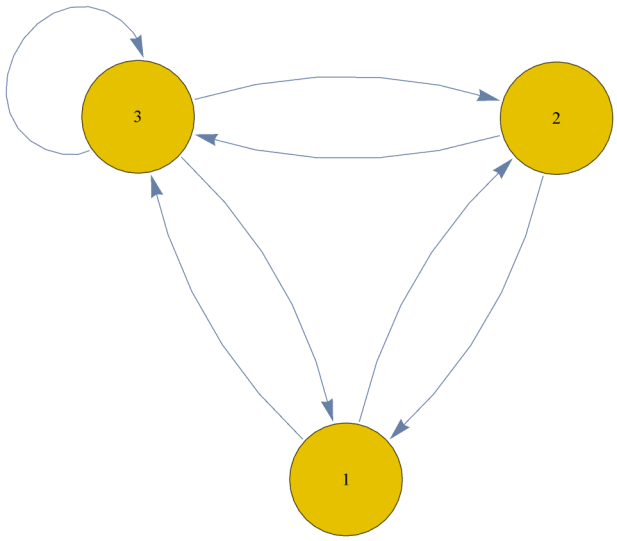
In[1]:=
$$m = \begin{pmatrix} 0 & .75 & 0.25 \\ .25 & 0 & .75 \\ .25 & .25 & .5 \end{pmatrix}$$

Out[1]= {{0, 0.75, 0.25}, {0.25, 0, 0.75}, {0.25, 0.25, 0.5}}

In[2]:=
```
proc = DiscreteMarkovProcess[{1, 0, 0}, m];
```

In[3]:=
```
Graph[proc]
```

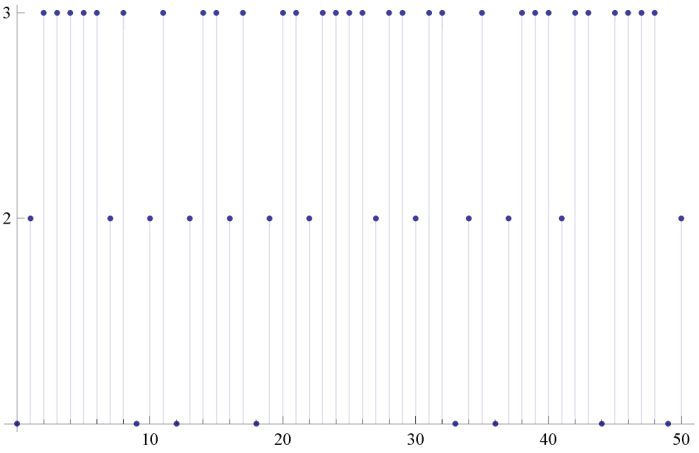
Out[3]=



In[5]:=
```
data = RandomFunction[proc, {0, 50}]
ListPlot[data, Filling → Axis, Ticks → {Automatic, {1, 2, 3}}]
```

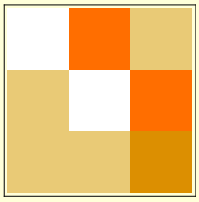
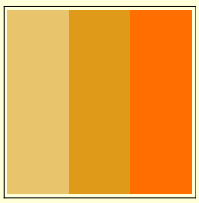
Out[5]= TemporalData[1]

Out[6]=



In[7]:=
```
MarkovProcessProperties[proc]
```

Out[7]=

Basic Properties	
InitialProbabilities	
TransitionMatrix	
HoldingTimeMean	{0, 0, 1.}
HoldingTimeVariance	{0, 0, 2.}
Structural Properties	
CommunicatingClasses	{1, 2, 3}
RecurrentClasses	{1, 2, 3}
TransientClasses	None
AbsorbingClasses	None
PeriodicClasses	None
Periods	{}
Irreducible	True
Primitive	True
Aperiodic	True
Limiting Properties	
LimitTransitionMatrix	
Reversible	False

In[8]:=
```
MatrixPower[m, 2] // MatrixForm
```

Out[8]//MatrixForm=

$$\begin{pmatrix} 0.25 & 0.0625 & 0.6875 \\ 0.1875 & 0.375 & 0.4375 \\ 0.1875 & 0.3125 & 0.5 \end{pmatrix}$$

In[9]:=
```
MatrixPower[m, 3] // MatrixForm
```

Out[9]//MatrixForm=

$$\begin{pmatrix} 0.1875 & 0.359375 & 0.453125 \\ 0.203125 & 0.25 & 0.546875 \\ 0.203125 & 0.265625 & 0.53125 \end{pmatrix}$$

In[10]:=
```
MatrixPower[m, 4] // MatrixForm
```

Out[10]//MatrixForm=

$$\begin{pmatrix} 0.203125 & 0.253906 & 0.542969 \\ 0.199219 & 0.289063 & 0.511719 \\ 0.199219 & 0.285156 & 0.515625 \end{pmatrix}$$

In[11]:=
```
MatrixPower[m, 5] // MatrixForm
```

Out[11]//MatrixForm=

$$\begin{pmatrix} 0.199219 & 0.288086 & 0.512695 \\ 0.200195 & 0.277344 & 0.522461 \\ 0.200195 & 0.27832 & 0.521484 \end{pmatrix}$$

In[12]:=
```
MatrixPower[m, 10] // MatrixForm
```

Out[12]//MatrixForm=

$$\begin{pmatrix} 0.200001 & 0.279984 & 0.520015 \\ 0.2 & 0.280005 & 0.519996 \\ 0.2 & 0.280004 & 0.519997 \end{pmatrix}$$

In[13]:=
```
PDF[proc[0], 1]
PDF[proc[0], 2]
PDF[proc[0], 3]
```

Out[13]= 1

Out[14]= 0

Out[15]= 0

In[16]:=
```
PDF[proc[1], 1]
PDF[proc[1], 2]
PDF[proc[1], 3]
```

Out[16]= 0.

Out[17]= 0.75

Out[18]= 0.25

In[19]:=
```
PDF[proc[2], 1]
PDF[proc[2], 2]
PDF[proc[2], 3]
```

Out[19]= 0.25

Out[20]= 0.0625

Out[21]= 0.6875

In[22]:=
```
PDF[proc[5], 1]
PDF[proc[5], 2]
PDF[proc[5], 3]
```

Out[22]= 0.199219

Out[23]= 0.288086

Out[24]= 0.512695

In[25]:=
```
PDF[proc[∞], 1]
PDF[proc[∞], 2]
PDF[proc[∞], 3]
```

Out[25]= 0.2

Out[26]= 0.28

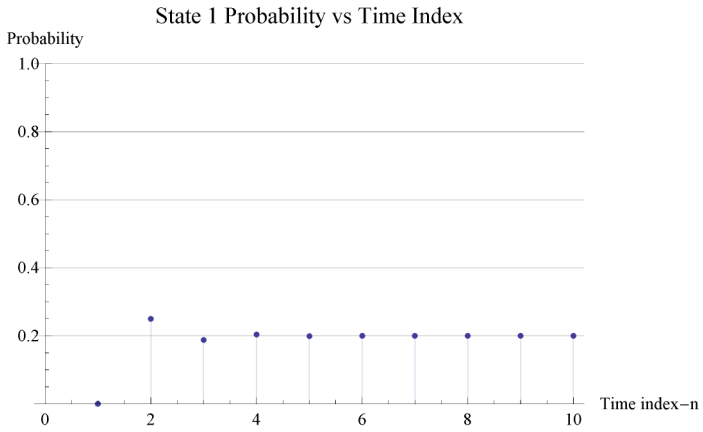
Out[27]= 0.52

In[28]:=
```
PDF[proc[n], 1]
```

Out[28]=
$$(0. + 1.\ (0.4\ (-0.25)^n + 0.4\ (-0.25)^n + 0.2 \times 1.^n)) \Big/ \left(0. + 1.\ (0.4\ (-0.25)^n + 0.4\ (-0.25)^n + 0.2 \times 1.^n) + 1.\ \left(-2.64702 \times 10^7\ (-0.25)^n + 2.64702 \times 10^7\ (-0.25)^n + 0.28 \times 1.^n\right) + 1.\ \left(2.64702 \times 10^7\ (-0.25)^n - 2.64702 \times 10^7\ (-0.25)^n + 0.52 \times 1.^n\right)\right)$$

In[29]:=
```
DiscretePlot[PDF[proc[n], 1], {n, 1, 10},
 PlotRange → {0, 1}, AxesLabel → {"Time index-n", "Probability"},
 PlotLabel → "State 1 Probability vs Time Index", GridLines → {None, Automatic}]
```

Out[29]=

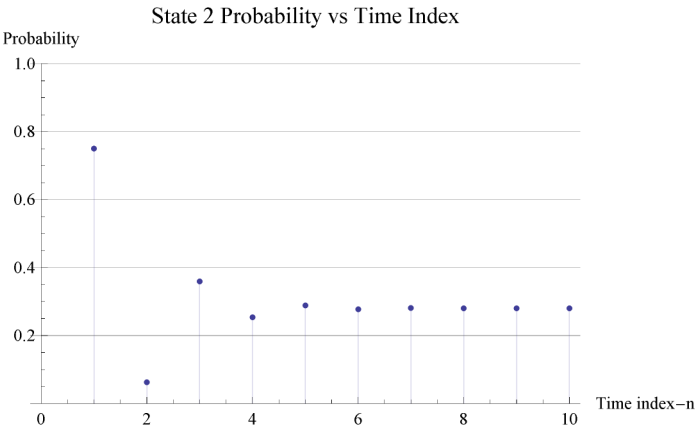


In[30]:=
```
PDF[proc[n], 2]
```

Out[30]=
$$\left(0. + 1.\ \left(-2.64702 \times 10^7\ (-0.25)^n + 2.64702 \times 10^7\ (-0.25)^n + 0.28 \times 1.^n\right)\right) \Big/ \left(0. + 1.\ (0.4\ (-0.25)^n + 0.4\ (-0.25)^n + 0.2 \times 1.^n) + 1.\ \left(-2.64702 \times 10^7\ (-0.25)^n + 2.64702 \times 10^7\ (-0.25)^n + 0.28 \times 1.^n\right) + 1.\ \left(2.64702 \times 10^7\ (-0.25)^n - 2.64702 \times 10^7\ (-0.25)^n + 0.52 \times 1.^n\right)\right)$$

In[31]:=
```
DiscretePlot[PDF[proc[n], 2], {n, 1, 10},
 PlotRange → {0, 1}, AxesLabel → {"Time index-n", "Probability"},
 PlotLabel → "State 2 Probability vs Time Index", GridLines → {None, Automatic}]
```

Out[31]=



In[32]:=
```
PDF[proc[n], 3]
```

Out[32]=
$$\left(0.+1.\left(2.64702\times10^{7}\,(-0.25)^{n}-2.64702\times10^{7}\,(-0.25)^{n}+0.52\times1.^{n}\right)\right)\Big/\left(0.+1.\,(0.4\,(-0.25)^{n}+0.4\,(-0.25)^{n}+0.2\times1.^{n})+1.\left(-2.64702\times10^{7}\,(-0.25)^{n}+2.64702\times10^{7}\,(-0.25)^{n}+0.28\times1.^{n}\right)+1.\left(2.64702\times10^{7}\,(-0.25)^{n}-2.64702\times10^{7}\,(-0.25)^{n}+0.52\times1.^{n}\right)\right)$$

In[33]:=
```
DiscretePlot[PDF[proc[n], 3], {n, 1, 10},
 PlotRange → {0, 1}, AxesLabel → {"Time index-n", "Probability"},
 PlotLabel → "State 3 Probability vs Time Index", GridLines → {None, Automatic}]
```

Out[33]=