The Impact of Interference on an OFDM System with AMC, Hybrid ARQ, and a Finite Queue on End-to-End Performance

Z. Hijaz and V. S. Frost
Information and Telecommunication Technology Center
Department of Electrical Engineering and Computer Science, University of Kansas
Lawrence, KS, USA
E-mail: zhijaz@ku.edu, frost@ittc.ku.edu

Abstract—Several technologies have been added to wireless communication systems that increase reliability of these systems and afford them higher data rates. These new technologies when combined with the multiple layers of a communications network make the analysis of these systems increasingly complex. The work here analyzes the impact of placing a modulated interfering transmission in the guard band of a wireless packet based (infrastructure) network on the end user TCP performance. Wireless networks, such as LTE, employ at the physical layer orthogonal frequency division multiplexing (OFDM) and adaptive modulation and coding (AMC) while at the data-link layer they use hybrid auto-repeat-request (HARQ) and a finite length queue; they also utilize cross-layer optimization to improve performance. These protocols allow the wireless network the ability to adapt to an ever changing fading environment. The following work shows the effect of a modulated interfering signal in the guard band of an OFDM system on the end-to-end TCP performance. In some cases the impact from the interference is indistinguishable from the effects of channel fading.

Keywords—Transport protocol (TCP); Interference; Orthogonal frequency division multiplexing (OFDM); Fading.

I. INTRODUCTION

The performance of the transmission control protocol (TCP) of the internet protocol suite can be utilized to quantify the end user’s experience as part of a wireless communications network (see Fig. 1). Interference at the physical layer of a wireless packet based (infrastructure) network degrades the user experience. The work here places a modulated interfering signal in the guard band of an OFDM system when the signal is subject to a fixed channel and interference at the physical layer. The work in [3] validated a method for estimating the average packet error rates (PER) of a coded OFDM system when the signal is subject to interference with a shaped spectral density, i.e., where each sub-carrier in the OFDM system has a different signal-to-noise ratio (SNR). The analysis in [4] showed that the impact of interference generated by a covert link on the data-link layer of a packet based (infrastructure) wireless network with HARQ was indistinguishable from fading. A method for finding the average PER of a system with AMC and HARQ was presented in [5]. The work in [6] provides a method for analyzing the end-to-end TCP performance for a system with AMC and truncated-ARQ (TARQ). The principle difference

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between the previous work in [6-9] and the work here is the inclusion of interference at the physical layer, HARQ at the data-link layer, and LTE resource block (RB) structure as opposed to HiperLAN. Therefore, the work here utilizes aspects of the aforementioned works to analyze the TCP performance of a wireless network with OFDM, AMC, and HARQ with a finite length queue when subjected to interference from a signal with a shaped spectral density placed in the guard band of the OFDM target signal.

The system model and assumptions are given in the following section. Section III presents the analysis for calculating the throughput, packet loss rate, and end-to-end delay of the target downlink system at the TCP layer. Systems that utilize AMC specify a modulation and coding scheme (MCS) for each AMC mode. We utilize the PER as a function of SNR described in [3] (given below) in place of those from [6-9] to facilitate the analysis of a target system with a 12 sub-carrier RB like LTE. We then extend the analysis [6-9] to include the impact of a modulated interfering signal on a target system with HARQ as opposed to TARQ. Section V offers conclusions and next steps.

**II. SYSTEM DESIGN AND ASSUMPTIONS**

The system considered is seen in Fig. 1. Here a wired network distributes information to the base stations serving the users in each cell. Packets from the wired network arrive at the base station queue and are then transmitted through a wireless link to the user equipment (UE). Packets and frames are utilized as the units of transmission at the data-link and physical layers, respectively. Here the queue at base station, which serves the UE, operates in first-in-first-out (FIFO) mode. We assume a single transmit antenna at the base station and a single receive antenna at the UE.

Implementation of AMC is performed by selecting one of the available MCSs that meets a performance standard while also maximizing throughput over the wireless channel. The selection of a particular MCS is based upon the SNR at the receiver which is sent to the transmitter through a feedback channel. Cross-layer design as in [6-9] is implemented through the utilization of channel state information (CSI), the received SNR, to select the MCS at the physical layer. Type II hybrid-ARQ with Chase Combining (CC) is used at the data-link layer to recover from packet errors caused by the wireless link. The TCP Reno mechanism is utilized at the transport layer with the triple-duplicate acknowledgement (ACK) for congestion control [10].

**A. The Physical Layer**

The interaction between the target system and interfering signal occurs at the physical layer. The model here considers a modulated interfering signal and a target transmitter/receiver pair; the down-link signal from the target transmitter (base station) to the target receiver (UE) and a signal from the transmitter of the interfering signal to the target receiver. We assume both channels experience Nakagami-m fading as it encompasses a large class of channels [8]. The two channels fade independently and share the same m parameter. As in [6-9], the channel model considered here is frequency flat and does not vary from the time the channel is measured until the subsequent frame is received. LTE utilizes OFDM on the downlink and specifies 12 sub-carriers per RB [1]. One or more RB may be assigned per UE. The work here analyzes the impact of the interfering signal on the RB at the edge of the target OFDM signal.

We consider an interfering system with knowledge of target downlink system specifications and place the interfering signal with a shaped spectral density ($\text{sinc}^2$) in the guard band of the target signal as in Fig. 2. Here we assume that the interfering signal utilizes 4-QAM signaling, has the bandwidth of one target signal sub-carrier (e.g., 30 kHz), and has the same symbol time as the target signal. However the methodology developed here can be easily extended to other placement and modulation schemes for the interfering signal.

**B. Data-link Layer**

AMC pairs a specific signal modulation, $M_t$-ary quadrature amplitude modulation (M-QAM) in this case, with a coding rate that meets or exceeds a PER requirement. The selection of a particular MCS is based upon the CSI. The target system is designed without knowledge or consideration of the interfering signal. We assume the feedback channel to be ideal and have no latency. Each packet includes bits for error detection; error detection is assumed to be perfect.

The work here considers a system similar to an LTE system. LTE specifies 16 MCS and turbo codes for error correction [1] with 4, 16, and 64-QAM signaling. The LTE “like” system considered here uses 6 MCS and convolutional codes as in [6-9] in order to simplify the analysis. We consider coding rates of $\frac{1}{2}, \frac{9}{16},$ and $\frac{3}{4}$. The bits from a packet are distributed evenly amongst the sub-carriers of a RB.

**C. System Queue**

In [6, 8] a finite state Markov chain (FSMC) channel model is adopted in order to analyze the performance of the queuing system located at the base station in Fig. 1. A channel is in state $n$ when the received SNR lies between $\gamma \in (\gamma_n, \gamma_{n+1})$. By adopting a slowly varying wireless channel, transitions occur to only the adjacent channel state.

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Fig. 1. Target and interfering networks.

Fig. 2. Placement of the interfering signal in the guard band of the target downlink signal spectrum.
The queue operates in FIFO mode and is of finite length. Arriving packets are dropped if the queue is full.

The system considered here is comprised of both wired and wireless channels. However, performance is dominated by the wireless channel. Therefore, our analysis will focus on the wireless link.

D. TCP Layer

Following [6], we utilize a fixed point procedure to analyze the TCP performance which couples the TCP with AMC. The TCP Reno “congestion control” mechanism is utilized at the TCP layer of both the target server and client as in [10]. We assume only triple duplicate ACK based congestion control for the analysis.

III. PERFORMANCE ANALYSIS

As in [6-9] this work considers streams of bits from the physical layer are fed to the data-link layer and mapped to packets. Here we analyze the performance of the target downlink system when fading, error control coding, signal modulation, queuing, congestion control, and interference are considered on the end-to-end TCP performance of the target system.

A. Packet Error Rate

The pdf of the instantaneous received SNR of a target signal with no interference in Nakagami-m fading is characterized by [12, eq. (12.21)]. The thresholds used in the AMC mode selection \( \{y_{n}\}_{n=0}^{N} \) are calculated using the approach presented in [7, 8] and the pdf of the instantaneous received SNR of a target signal with no interference given in [12, eq. (12.21)]. The pdf of the instantaneous received SNR when the target and the interfering signals experience independent Nakagami-m fading was developed in [13]. A form of the resulting pdf [4] is given as

\[
f_{r}(y) = \frac{1}{2\sqrt{\pi}\gamma_{m}(m)} m^{2m} \left( \frac{1}{\gamma_{c}} \right)^{m} \int_{0}^{\infty} \frac{e^{-m(y-\gamma_{c})}}{2\gamma_{c}} \left( m \left( \gamma_{c} + \frac{1}{\gamma_{c}} \right) \right)^{1-m} \left( K_{m-1} \left( \frac{m(y-\gamma_{c})}{2\gamma_{c}} \right) \right) d\gamma,
\]

where \( \gamma_{c} \) represents the average SNR and \( K_{m} \) is the modified Bessel function of the second kind [14] and \( \gamma_{c} \) is the average in-band transmitted SNR of the interfering signal. Note that the pdf from [13] can also be used to deal with cases where the target and interfering channels experience different fading, i.e., different \( m \).

The impact of the modulated interfering signal on each sub-carrier in a RB must be taken into account. The influence of the interfering signal on a sub-carrier decreases as the spectral distance between the interfering signal and each sub-carrier increases. For each sub-carrier, \( i \), a different normalization factor is needed and can be found as in [15] by

\[
\rho_{i} = \frac{1}{\gamma_{c}} \int_{f_{i}}^{f_{i} + \frac{B}{2}} \text{sinc}^{2} \left( \pi \frac{f - f_{i}}{\gamma_{c}} \right) df.
\]

where \( f_{s} \) is the sub-carrier spacing (we assume \( f_{s} = 15 \text{ kHz} \) as in LTE), \( B \) is the sub-carrier bandwidth, and \( B_{c} \) is the bandwidth of the interfering signal. With an interfering signal power of \( P_{c} \), the in-band interference for sub-carrier \( i \) is \( N_{i} = \rho_{i} P_{c} \), which is the contribution of the interfering signal to the average SNR per sub-carrier \( i \) in the OFDM signal with a total of \( I \) sub-carriers. An exact closed form expression for the PER of a coded packet distributed over multiple carriers is not available. Therefore, we approximate the PER of a RB by using the average in-band transmitted SNR over all sub-carriers in a RB and setting it equal to \( \gamma_{c} \), i.e.,

\[
\tilde{\gamma}_{c} = P_{c} \frac{1}{I} \sum_{i=1}^{I} \rho_{i}.
\]

This approximation enables us to directly extend the analysis in [7] to the analysis of OFDM with HARQ and AMC in the presence of an interferer [4]. The validity of using this approximation on predicting the impact of the interference on the OFDM RB was verified in [3]. Other approximations to find the PER of OFDM systems [16] do not seem to fit the system under study here. However, the proposed methodology may not apply when the total SNR at the target receiver is low; a situation that can result from low target signal power (low target SNR) or high interfering signal power (high interfering signal SNR).

Now following [7], the probability that mode \( n \) will be selected with the interfering signal taken into account is given by

\[
\Pr_{c} = \int_{0}^{y_{c} + 1} f_{r}(y) dy.
\]

Here the channel does not appreciably change from the time the channel is measured until after the first packet is transmitted. Thus, the SNR of the first packet transmission utilizing mode \( n \) lies in the SNR region for mode \( n \), i.e., for the first packet transmission the \( \text{Prob} \{ y \in [y_{n}, y_{n+1}] \} = 1 \). The SNR of all other subsequent packet retransmissions are i.i.d. and can have any value \( y \in [0, \infty) \), i.e., from the time the channel is measured until the packet is retransmitted is long enough such that the retransmission experiences a new i.i.d. SNR in the range of \( y \in [0, \infty) \). Therefore, the average PER for a given mode in the presence of the interfering signal for a RB is defined here to be

\[
\overline{\text{PER}}_{cn}(K) = \frac{1}{\Pr_{c}(y)} \int_{0}^{\infty} \int_{0}^{y_{c} + 1} \prod_{k=1}^{K} \text{PER}_{n}(y_{k}) dy_{1} ... dy_{K},
\]

where \( K \) represents the maximum number of packet transmissions allowed by the HARQ mechanism. Note that (5) is different from [5, eq. (8)].

An approximation for the form of the PER as a function of SNR for coded links was found in [6-9] as

\[
\text{PER}_{n}(y) \approx \begin{cases} 1, & \text{if } 0 < y < y_{pm} \\ 0, & \text{if } y \geq y_{pm} \end{cases},
\]

where \( n \) represents the mode, \( y \) is the received SNR, and \( a_{n}, g_{n}, \text{ and } y_{pm} \) are found through simulation and curve fitting. However the curves in [6-9] are not suitable for this system. Therefore additional simulation and curve fitting was
performed for a system that more closely matched the 12 sub-carrier RB considered here. The parameters $a_n$, $g_n$, and $\gamma_n$ for a 12 sub-carrier RB using the specified modulation and coding are given in Table I.

<table>
<thead>
<tr>
<th>Table I</th>
<th>Transmission Modes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mode 1</td>
</tr>
<tr>
<td>Modulation (MC)</td>
<td>OPSK</td>
</tr>
<tr>
<td>Coding Rate $R_c$</td>
<td>%</td>
</tr>
<tr>
<td>$\gamma_c$ (bits/second)</td>
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</tr>
<tr>
<td>$g_c$</td>
<td>124.8950</td>
</tr>
<tr>
<td>$\rho_c$</td>
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<tr>
<td>$\lambda_c$ (BPS)</td>
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</tr>
<tr>
<td>$\mu_c$ (BPS)</td>
<td>2.01570</td>
</tr>
</tbody>
</table>

The average PER with the interfering signal given all $M$ modes, where $R_c$ is the modulation and coding rate of the $n$th mode, of AMC transmission can be found as the total number of packets transmitted in error over the total number of packets transmitted which is given by [7, eq. (8)]

$$PER_c(K) = \frac{\sum_{n=1}^{N} R_n Pr_c(n)PER_{en}(K)}{\sum_{n=1}^{N} R_n Pr_c(n)}$$  \hspace{1cm} (7)

B. TCP Layer Performance

We use a recursive algorithm from [6] to find the average TCP performance of the target system with interference. The initial value for the TCP sending rate, $\beta$, is set to the maximum server state $c_5 = bR_5$. The TCP segment sending rate is approximated by [10]

$$\beta = \frac{1}{RTT} \sqrt{\frac{3}{2F}}$$  \hspace{1cm} (8)

where the RTT is the average end-to-end round-trip-time and $b$ is the number of packets/segments served by a frame per server time unit ($T_f$) and confirmed by an ACK. The segment loss rate, $\varphi$, is expressed as [9]

$$\varphi = 1 - (1 - P_d)(1 - P_l).$$  \hspace{1cm} (9)

where $P_d$ is the packet dropping probability at the queue and is calculated according to [8] and $P_l$ is set equal to the average PER from (7). Unlike the analysis in [6] here both $P_d$ and $P_l$ include the effects from the interfering signal. In [6, eq. (15)] the TCP throughput is defined as

$$\eta = \frac{1-\varphi}{RTT} \sqrt{\frac{3}{2F}}.$$  \hspace{1cm} (10)

Following [18], the RTT is approximated as

$$RTT \approx 2T_0 + T_{wl} + T_{wf},$$  \hspace{1cm} (11)

where $2T_0$ is the average 2-way delay over the wireline connection, $T_0$ is the average waiting time of a segment in the base station queue, $T_c$ is the average transmission time per segment over the wireless channel, and $T_{wf}$ is the feedback delay as in Fig. 1. It is assumed that $T_0$ and $T_{wf}$ are known constants and $T_{wl} := T_0 + T_c$. This leaves $T_{wf}$ to be determined by the wireless link model as in [6]. As in [6, eq. (22)], the average delay per segment is found by

$$T_{wl} = \frac{N_{wl}}{\beta(1 - \beta)}.$$  \hspace{1cm} (12)

The average number of segments in the wireless link is found according to [8, eq. (21)]

$$N_{wl} = \sum_{u \in U, c \in C} u \cdot P(U = u, C = c) + \sum_{u \in U, c \in C} \min\{u, c\} \cdot P(U = u, C = c),$$  \hspace{1cm} (13)

where the stationary distribution of the queue and server states, $P(U = u, C = c)$, is calculated according to the method outlined in [8]. However, in this work interference is accounted for in the calculation of $N_{wl}$. The algorithm from [8] starts by setting the ensemble average packet arrival rate, $\lambda$, equal to the TCP segment sending rate, $\lambda$. The stationary distribution is found from the procedure outlined in [8] which is then utilized to find $P_d$ and $N_{wl}$. The average delay per segment is then calculated using (12) and the RTT is found according to (11). Finally, an updated value for the TCP segment sending rate is calculated by (8). This process is repeated until the sending rate converges. Typically, this algorithm converges after several iterations. However, for a minority of high SNR cases an additional step is needed for convergence; after a predetermined number of iterations, e.g. 100, the new TCP segment sending rate is adjusted according to

$$\beta_{new} = \frac{(1 - x)\beta \cdot \beta_1 > \lambda}{(1 + x)\beta \cdot \beta_1 \leq \lambda}. $$  \hspace{1cm} (14)

where $\beta_{new}$ represents the updated TCP sending rate on the $j$th iteration and we set the value of $x = 0.01*floor(j/100)$. Here the floor() function rounds the resulting fraction down to the nearest integer.

IV. NUMERICAL RESULTS

The TCP throughput, RTT, and packet loss rate results presented in this section consider the physical layer interaction between target and interfering signals, AMC, HARQ, and the finite queue at the base station. The system parameters $T_p=10$ ms, $T_w=3$ ms, $T_f=50$ ms and $f_d=1$ Hz, the Doppler spread, are taken from [6]; we set $b=4$ packets/frame and $K=3$ attempts.

Results of four different $m$ fading parameters are presented. Measurement of channel characteristics at 900 MHz in an urban environment were reported in [19] where the most likely values of $m$ ranged from 0.5 to 3.5 with an average of $m=1.56$. While in [20] an average $m=2.38$ was reported at 870.9 MHz in an urban environment. However, $m=1.56$ and 2.38 are incompatible with the method of calculation used here. Thus values of $m=0.5, 1.0, 2.0, and 3.0$ were selected which covers the range of $m$ measured in [19] and [20]. For the cases where the interference power is varied we fix the average target SNR at 26.15 dB; at this SNR an uncoded 16-QAM system in Rayleigh fading has a BER-10^{-3}.

Fig. 3 gives throughput of the target system with an interferer SNR of 5 dB; throughput of the target system with no interference and $m=1.0$ is included for comparison. Fig. 4 shows the RTT and Fig. 5 shows packet loss rate for the same parameters. We can see that the performance of the target system improves while the target SNR increases as expected as $m$ increases. There is only a small difference in performance between the scenarios with $m=1.0$ with and
without interference. It is clear from Figs. 3, 4, and 5 that a change in \( m \) of 1.0 has a greater impact on performance than adding an interfering signal with SNR of 5 dB. We can see that the throughput reaches the maximum sending rate at the same point the RTT achieves a minimum. Here the maximum TCP sending is limited by the maximum server rate (1800 packets/second) and the minimum RTT is limited to twice the delay over the wireline (\( 2T_0 \)) plus the feedback delay of the ACK over the wireless channel (\( T_{wf} \)) for a total of 103 milliseconds.

Next, the average target SNR is fixed at 26.15 dB while the interfering SNR ranges from 1 to 30 dB. This allows us to see the impact of increasing the interference power on the target system. Fig. 6 shows that for scenarios with \( m = 0.5 \) and 1.0 the maximum TCP sending rate is never achieved while for cases with \( m = 2.0 \) and 3.0 the maximum TCP sending rate is no longer met after approximately 17 and 20 dB interfering SNR, respectively.

Fig. 7 allows us to more directly compare the impact of increasing the interference power from 5 to 10 dB to an increase of \( m \) from 1.0 to 2.0 and from 2.0 to 3.0. By comparing the throughput of the case with \( m = 1.0 \) and an interfering SNR of 10 dB to the systems with \( m = 2.0 \) and interfering SNR of 5 dB we can see that decreasing \( m \) from 2.0 to 1.0 reduces the performance more than an increase of 5 dB in the interference. Specifically, when the target SNR is 15 dB, the TCP throughput with \( m = 1.0 \) with 5 dB interference and without interference is 135 and 140 packets/s, respectively, whereas when \( m = 2.0 \) and 5 dB interference the throughput is 355 packets/s.

Fig. 8 shows the sensitivity of interference versus fading. We define the throughput from (10), \( \eta \), as a function of the target SNR, interference SNR, and the fading parameter \( m \), \( \eta(SNR_{TAR},SNR_{INT},m) \), and define the change in throughput, \( \Delta \eta \), for two different cases as

\[
\Delta \eta_{m_1,m_2}(SNR_{TAR}) = \eta(SNR_{TAR},NI,m_1) - \eta(SNR_{TAR},NI,m_1 - m_2),
\]

\[
\Delta \eta_{m_{int}}(SNR_{TAR}) = \eta(SNR_{TAR},NI,m) - \eta(SNR_{TAR},SNR_{INT},m),
\]

where \( NI \) is the no interference case. The two definitions of \( \Delta \eta \) allow us to directly compare the impact of changing the interference power with the impact of changes in fading environment for a range of target SNR. We observe from Fig. 8 that the change in throughput caused by a change in \( m \) is consistently greater than the change in throughput which results from a change in interference power.

V. CONCLUSIONS

The impact of placing a modulated interfering signal in the guard band of a wireless packet based (infrastructure) network, like LTE, on end-to-end TCP performance has been analyzed. These systems employ OFDM, AMC, and HARQ with a finite length queue. Target system performance was
measured in terms of the TCP throughput. It was assumed that both interfering and target signals were subject to independent Nakagami-\(m\) fading with a shared \(m\) fading parameter. Results were presented for different fading environments, i.e., different \(m\). The analysis provides a blueprint for predicting the end-to-end TCP performance of 4th generation networks impacted by interference with a shaped spectrum. The results show that the impact of placing a modulated interfering signal with a bandwidth of one subcarrier of the target OFDM signal in the guard band of the OFDM signal can be indistinguishable from fading at the TCP layer. The work presented considers an important aspect of 4th generation wireless networks; the impact of interference on multi-carrier systems’ end-to-end performance where the interferer has a shaped spectrum. Designing a signal waveform that has a lower impact on the target system performance and exploits cognitive aspects of the target system are important questions for future work.

![Fig. 7. Throughput of target system for interferer SNR 5 and 10 dB, Nakagami-\(m\) fading.](image)

![Fig. 8. Change in throughput (\(\Delta \eta\)) of target system, \(\Delta \eta_{1,0.5}\) compared to \(\Delta \eta_{1,0.0\, \text{dB}}\).](image)

**REFERENCES**


