Homework 4 Solution:

4.69

First find $t_1$ and $t_2$

\[
\frac{2.5}{T} = \frac{0.7}{\frac{T}{4}}
\]

$\Rightarrow t_1 = 0.07 \text{ T}$

$t_2 = \frac{T}{2} - t_1$

$t_2 = 0.43 \text{ T}$

$v_o(\text{ave}) = \frac{1}{T} \times \text{area of shaded triangle}$

\[
= \frac{1}{T} \times (2.5 - 0.7) \times \left( \frac{T}{4} - t_1 \right)
\]

\[
= \frac{1}{T} \times 1.8 \times T \left( \frac{1}{4} - 0.07 \right)
\]

$= 0.324 \text{ V}$

4.71

\[\ddot{v}_o = 10\sqrt{2} - V_D = 13.44 \text{ V}\]

Conduction starts at $\theta = \sin^{-1} \frac{0.7}{10\sqrt{2}} = 2.84^\circ = 0.05 \text{ rad}$

and ends at $\pi - \theta$. Conduction angle $= \pi - 2\theta = 3.04 \text{ rad}$ in each half cycle. Thus the fraction of a cycle for which one of the two diodes conduct:

\[
= \frac{2(3.04)}{2\pi} \times 100 = 96.8\%
\]

Note that during 96.8% of the cycle there will be conduction. However, each of the two diodes conducts for only half the time, i.e., for 48.4% of the cycle.

\[
v_{o,\text{avg}} = \frac{1}{\pi} \int_0^{\pi-\theta} (10\sqrt{2}\sin\phi - 0.7) d\phi
\]

$= 8.3 \text{ V}$

\[
i_{L,\text{avg}} = \frac{8.3}{1 \text{k}\Omega} = 8.3 \text{ mA}
\]
Peak voltage across $R = 10\sqrt{2} - 2V_D$

$= 10\sqrt{2} - 1.4$

$= 12.74 \text{ V}$

$\theta = \sin^{-1} \frac{1.4}{10\sqrt{2}} = 5.68^\circ = 0.1 \text{ rad}$

Fraction of cycle that $D_1$ & $D_2$ conduct is

\[ \frac{\pi - 2\theta}{2\pi} \times 100 = 46.8\% \]

Note that $D_3$ & $D_4$ conduct in the other half cycle so that there is 2 (46.8) = 93.6\% conduction interval.

\[ v_{O,\text{avg}} = \frac{2}{2\pi} \int_0^{\pi-\theta} (10\sqrt{2}\sin\phi - 2V_D) \, d\phi \]

\[ = \frac{1}{\pi} \left[ -12\sqrt{2} \cos \phi - 1.4\phi \right]_0^{\pi-\theta} \]

\[ = \frac{2(12\sqrt{2} \cos \theta) - 1.4(\pi - 2\theta)}{\pi} \]

\[ = 7.65 \text{ V} \]

\[ i_{R,\text{avg}} = \frac{v_{O,\text{avg}}}{R} = \frac{7.65}{1} = 7.65 \text{ mA} \]
4. For the following circuit the breakdown voltage of the Zener diode is $V_{zk} = 10V$. Please determine the transfer function for the limiter shown below. In other words, determine $V_0$ as a function of $V_{in}$.

Solution:
(1) If the Zener diode is in breakdown, the equivalent circuit is,
\[ V_{in} - i_i \times 1k - i_D \times 1k - 10 = -6 \], and
\[ i_i = 10 + i_D \]
Combine them to have,
\[ V_{in} - (10 + i_D) \times 1k - i_D \times 1k - 10 = -6 \]
That is
\[ 2i_D = V_{in} - 14 \]
In order for the diode to be in breakdown $i_D > 0$ is required, so that,
\[ V_{in} > 14V \]
Under this condition, the input/output voltage relation can be found with,
\[ i_D = \frac{V_{in}}{2} - 7 \], and $V_0 = -6 + 10 + i_D \times 1k$
That is,
\[ V_0 = 4 + \frac{V_{in}}{2} - 7 = \frac{V_{in}}{2} - 3 \]
(2) If the Zener diode is forward biased, the equivalent circuit is,
\[ V_{in} - i_1 \times 1k + i_D \times 1k + 0.7 = -6, \]
and \( i_1 = 10 - i_D \)
Combine them,
\[ V_{in} - (10 - i_D) \times 1k + i_D \times 1k + 0.7 = -6 \]
In order for the diode to be in forward bias \( i_D > 0 \) is required, that is,
\[ V_{in} < 3.3 \text{V} \]
Under this condition, the output voltage can be found using,
\[ i_D = \frac{3.3 - V_{in}}{2}, \text{ and } V_0 = -6 - 0.7 - i_D \times 1k \]
That is,
\[ V_0 = -6.7 - \frac{3.3 - V_{in}}{2} = \frac{V_{in}}{2} - 8.35 \]

(3) If the Zener diode is reverse biased, the equivalent circuit is,
\( i_1 = 10 \text{mA} \) and \( V_0 = V_{in} - 10V \)
In order for the diode to be in reverse bias \(-10 < V_D < 0.7 \) is required, that is,
\[ -10 < -(V_{in} - 10V) < 0.7 \]
Or, \( 3.3 < V_{in} < 14 \)

So that the overall transfer function is,
\[ V_0 = \begin{cases} 
\frac{V_{in}}{2} - 8.35V & \text{for } V_{in} < 3.3 \\
\frac{V_{in}}{2} - 10V & \text{for } 3.3 < V_{in} < 14 \\
\frac{V_{in}}{2} - 3V & \text{for } V_{in} > 14 
\end{cases} \]
5. The circuit shown below is a diode limiter. Determine and draw the transfer characteristics ($V_0$ versus $V_i$). Be sure and label all relevant values (slope, intercept points, maximum values, etc.).

\[ V_{zk} = 10V \]
\[ V_{zk} = \infty \]
\[ 5V \]
\[ 1K\Omega \]
\[ 8V \]
\[ -5V \]

Solution:
(1) If zener diode is in breakdown (the regular diode is reverse biased),
\[ V_0 = 5V - 10V = -5V \]
This requires $V_i < -5V = V_0$

(2) If the regular diode is in forward bias (the zener diode is reverse biased),
\[ V_0 = -5V + 0.7V + 8V = 3.7V \]
This requires $V_i > 3.7V = V_0$

(3) If zener diode is in forward bias (the regular diode is reverse biased),
\[ V_0 = 5V + 0.7V = 5.7V \]
This requires $V_i > 5.7V$, which will never happen because the regular diode will be in forward biased condition. So the zener diode will never be in forward bias.

(4) If both diodes are reverse biased, $V_0 = V_i$. This requires $-5V < V_i < 3.7V$
6. In the following circuit, three diodes have identical characteristics with $n = 2$ and therefore $nV_T = 0.05V$. $V_D$ is a DC bias voltage ($V_D$ has a positive value) and $v_s$ is a small voltage signal. Please use small-signal analysis and find the small-signal output voltage as the function of input $v_s$.

Solution:

In DC analysis, the capacitor is an open circuit. Assume diodes D1, D2 and D3 are all forward biased, the DC equivalent circuit is

$$I_1 = 2mA, I_2 = 1mA, I_3 = 6 - I_1 - I_2 = 3mA$$

Since all the diode currents are positive, the assumption of forward bias was correct. The small signal equivalent resistances are:
For the AC analysis, the capacitor is a short circuit. The small signal output is,

\[ v_0 = v_s \frac{r_{d3} + 100}{r_{d1} + r_{d3} + 100} = v_s \frac{116.7}{141.7} = 0.8236v_s \]