Homework #7
7.25, 7.29, 7.31, 7.33 (a) (b), and

7.25  (a) \(I_D = \frac{1}{2} h_n(V_{GS} - V_t^2)\)

\[ = \frac{1}{2} \times 5(0.6 - 0.4)^2 = 0.1 \text{ mA}\]

\(V_{DS} = V_{DD} - I_D R_D = 1.8 - 0.1 \times 10 = 0.8 \text{ V}\)

(b) \(g_m = h_n V_{OV} = 5 \times 0.2 = 1 \text{ mA/V}\)

(c) \(A_v = -g_m R_D = -1 \times 10 = -10 \text{ V/V}\)

(d) \(\lambda = 0.1 \text{ V}^{-1}, \quad V_A = \frac{1}{\lambda} = 10 \text{ V}\)

\[r_o = \frac{V_A}{I_D} = \frac{10}{0.1} = 100 \text{ k\Omega}\]

\[A_v = -g_m (R_D \parallel r_o)\]

\[= -1(10 \parallel 100) = -9.1 \text{ V/V}\]

7.29  Given \(\mu_n C_{ox} = 250 \mu\text{A/V}^2,\)

\(V_t = 0.5 \text{ V},\)

\(L = 0.5 \mu\text{m}\)

For \(g_m = 2 \text{ mA/V}^2\) and \(I_D = 0.25 \text{ mA},\)

\[g_m = \sqrt{2\mu_n C_{ox}} \frac{W}{L} I_D \Rightarrow \frac{W}{L} = 32\]

\[\therefore W = 16 \mu\text{m}\]

\(V_{OV} = \frac{2I_D}{g_m} = 0.25 \text{ V}\)

\[\therefore V_{GS} = V_{OV} + V_t = 0.75 \text{ V}\]

7.31  
\[I = 500 \mu\text{A}\]

\[V_t = 0.5 \text{ V}\]

\(V_A = 50 \text{ V}\)

Given \(V_{DS} = V_{GS} = 1 \text{ V}.\) Also, \(I_D = 0.5 \text{ mA}\).

\(V_{OV} = 0.5 \text{ V}, \quad g_m = \frac{2I_D}{V_{OV}} = 2 \text{ mA/V}\)

\[r_o = \frac{V_A}{I_D} = 100 \text{ k\Omega}\]

\[\frac{v_o}{v_i} = -g_m (R_G \parallel R_L \parallel r_o) = -18.2 \text{ V/V}\]

For \(I_D = 1 \text{ mA}:\)

\(V_{OV}\) increases by \(\sqrt{\frac{1}{0.5}} = \sqrt{2}\) to

\(\sqrt{2} \times 0.5 = 0.707 \text{ V}.\)

\(V_{GS} = V_{DS} = 1.207 \text{ V}\)

\(g_m = 2.83 \text{ mA/V}, \quad r_o = 50 \text{ k\Omega}\)

\[\frac{v_o}{v_i} = -23.6 \text{ V/V}\]
7.33 (a) Open-circuit the capacitors to obtain the bias circuit shown in Fig. 1, which indicates the given values.

\[ g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.5}{0.5} = 2 \text{ mA/V} \]

\[ r_o = \frac{V_A}{I_D} = \frac{100}{0.5} = 200 \text{ k}\Omega \]

(c) See Fig. 2 below.

(d) \( R_{in} = 10 \text{ M}\Omega \parallel 5 \text{ M}\Omega = 3.33 \text{ M}\Omega \)

\[ \frac{v_{gs}}{v_{ss}} = \frac{R_{in}}{R_{in} + R_{ss}} = \frac{3.33}{3.33 + 0.2} = 0.94 \text{ V/V} \]

\[ \frac{v_o}{v_{gs}} = -g_m(200 \parallel 16 \parallel 16) \]

\[ = -2 \times 7.69 = -15.38 \text{ V/V} \]

\[ \frac{v_o}{v_{ss}} = \frac{v_{gs}}{v_{ss}} \times \frac{v_o}{v_{gs}} = -0.94 \times 15.38 = -14.5 \text{ V/V} \]

From the voltage divider, we have

\[ V_G = 15 \times \frac{5}{10 + 5} = 5 \text{ V} \]

From the circuit, we obtain

\[ V_G = V_{GS} + 0.5 \times 7 \]

\[ = 1.5 + 3.5 = 5 \text{ V} \]

which is consistent with the value provided by the voltage divider.

Since the drain voltage (+7 V) is higher than the gate voltage (+5 V), the transistor is operating in saturation.

From the circuit

\[ V_D = V_{DD} - I_D R_D = 15 - 0.5 \times 16 = +7 \text{ V}, \text{ as assumed} \]

Finally,

\[ V_{GS} = 1.5 \text{ V}, \text{ thus } V_{OV} = 1.5 - V_t = 1.5 - 1 \]

\[ = 0.5 \text{ V} \]

\[ I_D = \frac{1}{2} h_v V_{OV}^2 = \frac{1}{2} \times 4 \times 0.5^2 = 0.5 \text{ mA} \]

which is equal to the given value. Thus the bias calculations are all consistent.
Handout problem 1:
In the following MOSFET amplifier circuit, two transistors are identical with $V_t = 2V$ and $K = 2mA/V^2$. Please find the small-signal voltage gain $v_0(t)/v_i(t) = ?$

![MOSFET amplifier circuit diagram]

Solution:
This is an amplifier with 2 identical stages.
(a) DC analysis:
$V_G = V_{GS} = 12 \times \frac{10}{30} = 4V$
Then $I_D = K(V_{GS} - V_t)^2 = 2(4 - 2)^2 = 8mA$, $V_{DS} = 12 - 1 \times 8 = 4V$
$V_{DS} > V_{GS} - V_t = 4 - 2 = 2V$, and therefore the MOSFET is operating in saturation
Small-signal trans-conductive gain of each stage is, $g_m = 2K(V_{GS} - V_t) = 4 \times 2 = 8mA/V$

(b) small-signal analysis
The small-signal equivalent circuit is,

![Small-signal equivalent circuit diagram]

$v_{gs1} = v_i$, $v_{gs2} = -1 \times g_m v_{gs1} = -g_m v_i$, and $v_0 = -1 \times g_m v_{gs2} = g_m^2 v_i$
Therefore, the small-signal voltage gain is
$\frac{v_0}{v_i} = g_m^2 = 64$
Handout problem 2:
In the following MOSFET amplifier circuit, please find the small-signal voltage gain $v_0(t)/v_i(t) = ?$

![Circuit Diagram]

Solution:
(a) DC analysis:

$V_G = 15 \times \frac{5}{15} = 5V$

Assume saturation, $I_D = K(V_{GS} - V_t)^2$, $V_{GS} = 5 - 0.1I_D$. Combine these two equations,

$V_{GS} = 5 - 0.1K(V_{GS} - V_t)^2$, that is, $V_{GS} = 5 - 0.2(V_{GS} - 2)^2$

$V_{GS}^2 + V_{GS} - 21 = 0$

The solution is, $V_{GS} = 4.11$

Then $I_D = 2 \times (4.11 - 2)^2 = 8.9mA$, $V_{DS} = 15 - 1 \times 8.9 - 0.1 \times 8.9 = 5.2V$

$V_{DS} > V_{GS} - V_t = 4.11 - 2 = 2.11$ and therefore the MOSFET is operating in saturation

Small-signal trans-conductive gain is, $g_m = 2K(V_{GS} - V_t) = 4 \times 2.11 = 8.44mA/V$

(b) small-signal analysis

The small-signal equivalent circuit is,
\[ v_{gs} = v_i - 0.1 \cdot g_m \cdot v_{gs}, \] that is, \[ v_{gs} = \frac{v_i}{1 + 0.1g_m}. \]

\[ v_0 = -1 \cdot g_m \cdot v_{gs} = \frac{-g_m v_i}{1 + 0.1g_m}. \]

Therefore, the small-signal voltage gain is

\[ \frac{v_0}{v_i} = \frac{-g_m}{1 + 0.1g_m} = \frac{-8.44}{1 + 0.844} = -4.58 \]