## Homework 5 Solution:

$5.2 C_{o x}=9 \mathrm{fF} / \mu \mathrm{m}^{2}, V_{O V}=0.2 \mathrm{~V}$
$L=0.36 \mu \mathrm{~m}, V_{D S}=0 \mathrm{~V}$
$W=3.6 \mu \mathrm{~m}$
$Q=C_{o x} . W . L . V_{O V}=2.33 \mathrm{fC}$
$5.3 k_{n}^{\prime}=\mu_{n} C_{o x}$
$=\frac{\mathrm{m}^{2}}{\mathrm{~V} \cdot \mathrm{~s}} \frac{\mathrm{~F}}{\mathrm{~m}^{2}}=\frac{\mathrm{F}}{\mathrm{V} \cdot \mathrm{s}}=\frac{\mathrm{C} / \mathrm{V}}{\mathrm{V} \cdot \mathrm{s}}=\frac{\mathrm{C}}{\mathrm{s}} \frac{1}{\mathrm{~V}^{2}}$
$=\frac{\mathrm{A}}{\mathrm{V}^{2}}$
Since $k_{n}=k_{n}^{\prime} W / L$ and $W / L$ is dimensionless, $k_{n}$ has the same dimensions as $k_{n}^{\prime}$; that is, $\mathrm{A} / \mathrm{V}^{2}$.
$5.6 k_{n}=5 \mathrm{~mA} / \mathrm{V}^{2}, V_{t n}=0.5 \mathrm{~V}$,
small $v_{D S}$
$i_{D}=k_{n}\left(v_{G S}-V_{t}\right) v_{D S}=k_{n} v_{O V} v_{D S}$
$g_{D S}=\frac{1}{r_{D S}}=k_{n} v_{O V}$


| $\boldsymbol{V}_{G S}$ <br> $(\mathbf{V})$ | $\boldsymbol{V}_{\boldsymbol{O V}}$ <br> $(\mathbf{V})$ | $\mathbf{g}_{D S}$ <br> $(\mathbf{m A / V})$ | $\boldsymbol{r}_{D S}$ <br> $(\boldsymbol{\Omega})$ |
| :---: | :---: | :---: | :---: |
| 0.5 | 0 | 0 | $\infty$ |
| 1.0 | 0.5 | 2.5 | 400 |
| 1.5 | 1.0 | 5.0 | 200 |
| 2.0 | 1.5 | 7.5 | 133 |
| 2.5 | 2.0 | 10 | 100 |

$$
\begin{aligned}
& 5.8 r_{d s}=1 /\left.\frac{\partial i_{D}}{\partial v_{D S}}\right|_{v_{D S}=V_{D S}} \\
& =\left[\frac{\partial}{\partial v_{D S}}\left(k_{n}\left(V_{O V} v_{D S}-\frac{1}{2} v_{D S}^{2}\right)\right)\right]^{-1} \\
& =\left[k_{n}\left(\frac{\partial}{\partial v_{D S}}\right)\left(v_{O V} v_{D S}\right)-1 / 2 \frac{\partial}{\partial v_{D S}}\left(v_{D S}^{2}\right)\right]^{-1} \\
& =\left[k_{n}\left(V_{O V}-\frac{1}{2} \cdot 2 V_{D S}\right)\right]^{-1} \\
& =\frac{1}{k_{n}\left(V_{O V}-V_{D S}\right)}
\end{aligned}
$$

If $V_{D S}=0 \Rightarrow r_{d s}=\frac{1}{k_{n} V_{O V}}$
If $V_{D S}=0.2 V_{O V} \Rightarrow r_{d s}=\frac{1.25}{V_{O V}}$
If $V_{D S}=0.5 V_{O V} \Rightarrow r_{d s}=\frac{1}{k_{n}\left(V_{O V}-0.5 V_{O V}\right)}$
$=1 / k_{n}\left(0.5 V_{O V}\right)=\frac{2}{k_{n} V_{O V}}$
If $V_{D S}=0.8 V_{O V} \Rightarrow r_{d s}=\frac{1}{k_{n}\left(V_{O V}-0.8 V_{O V}\right)}$
$=1 / k_{n}\left(0.2 V_{O V}\right)=\frac{5}{k_{n} V_{O V}}$
If $V_{D S}=V_{O V}$,
$r_{d s}=\frac{1}{0} \Rightarrow \infty$
$5.11 V_{t p}=-0.7 \mathrm{~V}$
(a) $\left|V_{S G}\right|=\left|V_{t p}\right|+\left|V_{O V}\right|$
$=0.7+0.4=1.1 \mathrm{~V}$
$\Rightarrow V_{G}=-1.1 \mathrm{~V}$
(b) For the $p$-channel transistor to operate in saturation, the drain voltage must not exceed the gate voltage by more than $\left|V_{t p}\right|$. Thus
$v_{D \max }=-1.1+0.7=-0.4 \mathrm{~V}$
Put differently, $V_{S D}$ must be at least equal to $\left|V_{O V}\right|$, which in this case is 0.4 V . Thus $v_{D \max }=-0.4 \mathrm{~V}$.
(c) In (b), the transistor is operating in saturation, thus
$I_{D}=\frac{1}{2} k_{p}\left|V_{O V}\right|^{2}$
$0.5=\frac{1}{2} \times k_{p} \times 0.4^{2}$
$\Rightarrow k_{p}=6.25 \mathrm{~mA} / \mathrm{V}^{2}$
For $V_{D}=-20 \mathrm{mV}$, the transistor will be operating in the triode region. Thus
$I_{D}=k_{p}\left[v_{S D}\left|V_{O V}\right|-\frac{1}{2} v_{S D}^{2}\right]$
$=6.25\left[0.02 \times 0.4-\frac{1}{2}(0.02)^{2}\right]$
$=0.05 \mathrm{~mA}$
For $V_{D}=-2 \mathrm{~V}$, the transistor will be operating in saturation, thus
$I_{D}=\frac{1}{2} k_{p}\left|V_{O V}\right|^{2}=\frac{1}{2} \times 6.25 \times 0.4^{2}=0.5 \mathrm{~mA}$
$5.18 V_{t n}=0.5 \mathrm{~V}, \quad k_{n}=1.6 \mathrm{~mA} / \mathrm{V}^{2}$
$I_{D}=0.05=\frac{1}{2} \times 1.6 \times V_{O V}^{2}$
$\Rightarrow V_{O V}=0.25 \mathrm{~V}$ and $V_{D S} \geq 0.25 \mathrm{~V}$
$V_{G S}=0.5+0.25=0.75 \mathrm{~V}$
$I_{D}=0.2=\frac{1}{2} \times 1.6 \times V_{O V}^{2}$
$\Rightarrow V_{O V}=0.5 \mathrm{~V}$ and $V_{D S} \geq 0.5 \mathrm{~V}$
$V_{G S}=0.5+0.5=1 \mathrm{~V}$

