Homework 7 Solutions

5.56 (a) Refer to Fig. P5.56(a): The MOSFET is operating in saturation. Thus

\[ I_D = \frac{1}{2} k_a V_{OV}^2 \]

\[ 10 = \frac{1}{2} \times 500 \times V_{OV}^2 \Rightarrow V_{OV} = 0.2 \text{ V} \]

\[ V_{GS} = V_t + V_{OV} = 0.8 + 0.2 = 1 \text{ V} \]

\[ V_1 = 0 - V_{GS} = -1 \text{ V} \]

(c) Refer to Fig. P5.56(c). The MOSFET is operating in saturation. Thus

\[ 1 = \frac{1}{2} \times 0.5 \times V_{OV}^2 \Rightarrow V_{OV} = 2 \text{ V} \]

\[ V_{GS} = 0.8 + 2 = 2.8 \text{ V} \]

\[ V_3 = -2.8 \text{ V} \]

(d) Refer to Fig. P5.56(d). The MOSFET is operating in saturation. Thus

\[ 10 = \frac{1}{2} \times 500 \times V_{OV}^2 \Rightarrow V_{OV} = 0.2 \text{ V} \]

\[ V_{GS} = 0.8 + 0.2 = 1 \text{ V} \]

\[ V_4 = 1 \text{ V} \]

(f) Refer to Fig. P5.56(f). To simplify our solution, we observe that this circuit is similar to that in Fig. P5.56(d) with the 10-μA current source replaced with a 400-kΩ resistor. Thus \( V_G = V_4 = +1 \text{ V} \) and, as a check, \( I_D = \frac{5 - 1}{400} = 0.01 \text{ mA} = 10 \mu\text{A} \).

(h) Refer to Fig. P5.56(h). Our work is considerably simplified by observing that this circuit is similar to that in Fig. P5.56(a) with the 10-μA current source replaced with a 400-kΩ resistor. Thus \( V_8 = V_1 = -1 \text{ V} \) and, as a check, \( I_D = \frac{-1 + 5}{400} = 0.01 \text{ mA} = 10 \mu\text{A} \).
5.57 (a) Refer to the circuit in Fig. P5.57(a). Transistor $Q_1$ is operating in saturation. Assume that $Q_2$ also is operating in saturation,

$$V_{GS2} = 0 - V_2 = -V_2$$
and

$$V_2 = -2.5 + I_D \times 1$$
$$\Rightarrow I_D = V_2 + 2.5$$

Now,

$$I_D = \frac{1}{2} k_n(V_{GS2} - V_I)^2$$

Substituting $I_D = V_2 + 2.5$ and $V_{GS2} = -V_2$,

$$V_2 + 2.5 = \frac{1}{2} \times 1.5(-V_2 - 0.9)^2$$
$$\Rightarrow \frac{2}{1.5} (V_2 + 2.5) = V_2^2 + 1.8 V_2 + 0.81$$
$$V_2^2 + 0.467 V_2 - 2.523 = 0$$
$$\Rightarrow V_2 = -1.84 \text{ V}$$

Thus,

$$I_D = V_2 + 2.5 = -1.84 + 2.5 = 0.66 \text{ mA}$$
and

$$V_{GS2} = 1.84 \text{ V}$$

Since $Q_1$ is identical to $Q_2$ and is conducting the same $I_D$, then

$$V_{GS1} = 1.84 \text{ V}$$
$$\Rightarrow V_1 = 2.5 - 1.84 = 0.66 \text{ V}$$

which confirms that $Q_1$ is operating in saturation, as assumed.

(b) Refer to the circuit in Fig. P5.57(b). From symmetry, we see that

$$V_4 = 2.5 \text{ V}$$

Now, compare the part of the circuit consisting of $Q_2$ and the 1-kΩ resistor. We observe the similarity of this part with the circuit between the gate of $Q_2$ and ground in Fig. P5.57(a). It follows that for the circuit in Fig. P5.57(b), we can use the solution of part (a) above to write

$$I_{D2} = 0.66 \text{ mA} \quad \text{and} \quad V_{GS2} = 1.84 \text{ V}$$

Thus,

$$V_3 = V_4 - V_{GS2} = 2.5 - 1.84 = 0.66 \text{ V}$$

Since $Q_1$ is conducting an equal $I_D$ and has the same $V_{GS}$,

$$I_{D1} = 0.66 \text{ mA} \quad \text{and} \quad V_{GS1} = 1.84 \text{ V}$$
$$\Rightarrow V_3 = V_4 + V_{GS1} = 2.5 + 1.84 = 4.34 \text{ V}$$

We could, of course, have used the circuit symmetry, observed earlier, to write this final result.
Handout problem 1:
In the following PMOS circuit, please find the resistance value $R_2 = ?$

Solution:

Since $I_1 = 1mA$ is given, we can immediately find that $V_{SD} = I_1 \cdot 1k\Omega = 1V$, and $V_{SG} = 9 - 5 = 4V$. In this case, $V_{SD} < V_{SG} - |V_t| = 4 - 2 = 2V$ the FET is in triode mode.

\[
I_D = K \left[ 2(V_{SG} - |V_t|)V_{SD} - V_{SD}^2 \right] = \frac{1}{3} \left[ 2 \times 2 \times 1 - 1 \right] = 1mA
\]

\[
V_D = 9 - V_{SD} = 9 - 1 = 8V, \ I_2 = I_1 + I_D = 1 + 1 = 2mA
\]

\[
R_2 = \frac{V_D}{I_2} = \frac{8}{2} = 4k\Omega
\]
Handout problem 2
In the following circuit, the PMOS gate voltage is known to be 7V, please find the value of the resistor \( R \) = ?

Solution:

\[
I_1 = \frac{10V - 7V}{1k\Omega} = 3mA, \quad I_2 = I_1 = 3mA, \quad V_D = V_G - I_2 \cdot 1k\Omega = 7 - 3 = 4V
\]

\[
V_{SG} = 10 - 7 = 3\, V, \quad V_{SD} = 10 - 4 = 6\, V
\]

\[
V_{SD} > V_{SG} - |V_t| = 3 - 2 = 1\, V, \text{ therefore, the FET operates in saturation mode.}
\]

\[
I_D = K (V_{SG} - |V_t|)^2 = 1 \times (3 - 2)^2 = 1mA
\]

\[
I_3 = I_2 + I_D = 3 + 1 = 4mA
\]

\[
R = \frac{V_D}{I_3} = \frac{4V}{4mA} = 1k\Omega
\]
Handout problem 3
In the following circuit with a depletion-type NMOS, the gate voltage is known to be -1V, please find the value of the resistor \( R = \) ?

Solution:

\[ V_G = -1V = V_{GS}, \quad I_3 = \frac{V_G - (-10)}{9} = \frac{-1+10}{9} = 1mA, \quad I_2 = I_3 = 1mA \]

\[ V_D = V_G + I_2 \cdot 2k\Omega = -1 + 2 = 1V \]

\[ V_{DS} < V_{GS} - V_t = -1 + 3 = 2V, \text{ so the FET operates in triode mode} \]

\[ I_D = K \left[ 2(V_{GS} - V_t) - V_{DS}^2 \right] = \frac{1}{3} \left[ 2(-1 + 3) \times 1 - 1 \right] = 1mA, \]

\[ I_1 = I_D + I_2 = 2mA \]

\[ R = \frac{10 - V_D}{I_1} = \frac{10 - 1}{2} = 4.5k\Omega \]