Homework 7 Solutions

5.50 Refer to Fig. P5.50. Both Q_1 and Q_2 are operating in saturation at $I_D = 0.5$ mA. For Q_1,

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W_1}{L_1}\,V_{OV1}^2$$

$$0.5 = \frac{1}{2} \times 0.25 \times \frac{W_1}{L_1}(1 - 0.5)^2$$

$$\Rightarrow \frac{W_1}{L_1} = 16$$

$$W_1 = 16 \times 0.25 = 4\ \mu\text{m}$$

For Q_2, we have

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W_2}{L_2}\right)V_{OV2}^2$$

$$0.5 = \frac{1}{2} \times 0.25 \times \frac{W_2}{L_2}(1.8 - 1 - 0.5)^2$$

$$\Rightarrow \frac{W_2}{L_2} = 44.4$$

$$W_2 = 44.4 \times 0.25 = 11.1$$

$$R = \frac{2.5 - 1.8}{0.5} = 1.4\ \text{k}\Omega$$

5.56 (a) Refer to Fig. P5.56(a): The MOSFET is operating in saturation. Thus

$$I_D = \frac{1}{2}k_n V_{OV}^2$$

$$10 = \frac{1}{2} \times 500 \times V_{OV}^2 \Rightarrow V_{OV} = 0.2\ \text{V}$$

$$V_{GS} = V_t + V_{OV} = 0.8 + 0.2 = 1\ \text{V}$$

$$V_1 = 0 - V_{GS} = -1\ \text{V}$$

(c) Refer to Fig. P5.56(c). The MOSFET is operating in saturation. Thus

$$1 = \frac{1}{2} \times 0.5 \times V_{OV}^2 \Rightarrow V_{OV} = 2\ \text{V}$$

$$V_{GS} = 0.8 + 2 = 2.8\ \text{V}$$

$$V_3 = -2.8\ \text{V}$$

(d) Refer to Fig. P5.56(d). The MOSFET is operating in saturation. Thus

$$10 = \frac{1}{2} \times 500 \times V_{OV}^2 \Rightarrow V_{OV} = 0.2\ \text{V}$$

$$V_{GS} = 0.8 + 0.2 = 1\ \text{V}$$

$$V_4 = 1\ \text{V}$$

(f) Refer to Fig. P5.56(f). To simplify our solution, we observe that this circuit is that in Fig. P5.56(d) with the 10-μA current source replaced with a 400-kΩ resistor. Thus $V_G = V_4 = +1$ V and, as a check, $I_D = \dfrac{5-1}{400} = 0.01\ \text{mA} = 10\ \mu\text{A}$.

(h) Refer to Fig. P5.56(h). Our work is considerably simplified by observing that this circuit is similar to that in Fig. P5.56(a) with the 10-μA current source replaced with a 400-kΩ resistor. Thus $V_8 = V_1 = -1$ V and, as a check, $I_D = \dfrac{-1+5}{400} = 0.01\ \text{mA} = 10\ \mu\text{A}$.

5.57 (a) Refer to the circuit in Fig. P5.57(a). Transistor Q_1 is operating in saturation. Assume that Q_2 also is operating in saturation,

$$V_{GS2} = 0 - V_2 = -V_2$$

and

$$V_2 = -2.5 + I_D \times 1$$

$$\Rightarrow I_D = V_2 + 2.5$$

Now,

$$I_D = \frac{1}{2}k_n(V_{GS2} - V_t)^2$$

Substituting $I_D = V_2 + 2.5$ and $V_{GS2} = -V_2$,

$$V_2 + 2.5 = \frac{1}{2} \times 1.5(-V_2 - 0.9)^2$$

$$\frac{2}{1.5}(V_2 + 2.5) = V_2^2 + 1.8\ V_2 + 0.81$$

$$V_2^2 + 0.467\ V_2 - 2.523 = 0$$

$$\Rightarrow V_2 = -1.84\ \text{V}$$

Thus,

$$I_D = V_2 + 2.5 = -1.84 + 2.5 = 0.66\ \text{mA}$$

and

$$V_{GS2} = 1.84\ \text{V}$$

Since Q_1 is identical to Q_2 and is conducting the same I_D, then

$$V_{GS1} = 1.84\ \text{V}$$

$$\Rightarrow V_1 = 2.5 - 1.84 = 0.66\ \text{V}$$

which confirms that Q_1 is operating in saturation, as assumed.

(b) Refer to the circuit in Fig. P5.57(b). From symmetry, we see that

$$V_4 = 2.5\ \text{V}$$

Now, compare the part of the circuit consisting of Q_2 and the 1-kΩ resistor. We observe the similarity of this part with the circuit between the gate of Q_2 and ground in Fig. P5.57(a). It follows that for the circuit in Fig. P5.57(b), we can use the solution of part (a) above to write

$$I_{D2} = 0.66\ \text{mA} \quad \text{and} \quad V_{GS2} = 1.84\ \text{V}$$

Thus,

$$V_5 = V_4 - V_{GS2} = 2.5 - 1.84 = 0.66\ \text{V}$$

Since Q_1 is conducting an equal I_D and has the same V_{GS},

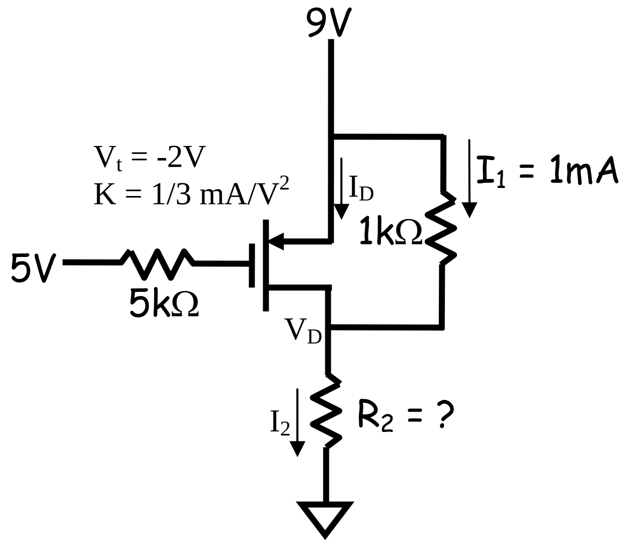
$$I_{D1} = 0.66\ \text{mA} \quad \text{and} \quad V_{GS1} = 1.84\ \text{V}$$

$$\Rightarrow V_3 = V_4 + V_{GS1} = 2.5 + 1.84 = 4.34\text{V}$$

We could, of course, have used the circuit symmetry, observed earlier, to write this final result.

Handout problem 1:
In the following PMOS circuit, please find the resistance value R_2 = ?

Solution:



Since $I_1 = 1mA$ is given, we can immediately find that $V_{SD} = I_1 \cdot 1k\Omega = 1V$, and $V_{SG} = 9 - 5 = 4V$. In this case, $V_{SD} < V_{SG} - |V_t| = 4 - 2 = 2V$ the FET is in triode mode.

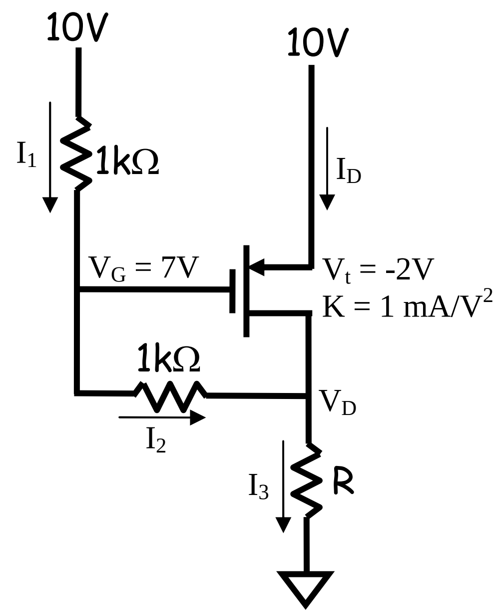
$$I_D = K\left[2(V_{SG} - |V_t|)V_{SD} - V_{SD}^2\right] = \frac{1}{3}[2 \times 2 \times 1 - 1] = 1mA$$

$$V_D = 9 - V_{SD} = 9 - 1 = 8V\ ,\ I_2 = I_1 + I_D = 1 + 1 = 2mA$$

$$R_2 = \frac{V_D}{I_2} = \frac{8}{2} = 4k\Omega$$

Handout problem 2

In the following circuit, the PMOS gate voltage is known to be 7V, please find the value of the resistor R = ?



Solution:

$$I_1 = \frac{10V - 7V}{1k\Omega} = 3mA,\ I_2 = I_1 = 3mA,\ V_D = V_G - I_2 \cdot 1k\Omega = 7 - 3 = 4V$$

$$V_{SG} = 10 - 7 = 3V,\ \ V_{SD} = 10 - 4 = 6V$$

$V_{SD} > V_{SG} - |V_t| = 3 - 2 = 1V$, therefore, the FET operates in saturation mode.

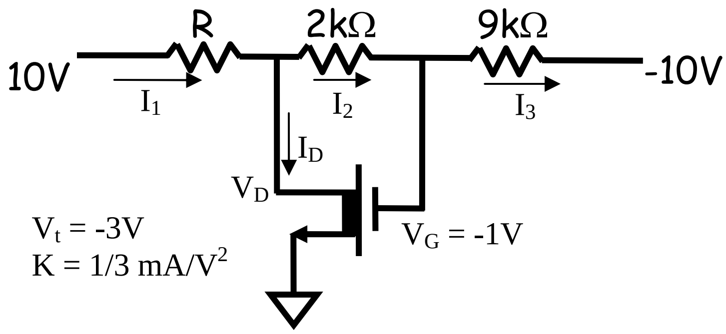
$$I_D = K\left(V_{SG} - |V_t|\right)^2 = 1 \times (3-2)^2 = 1mA$$

$$I_3 = I_2 + I_D = 3 + 1 = 4mA$$

$$R = \frac{V_D}{I_3} = \frac{4V}{4mA} = 1k\Omega$$

Handout problem 3

In the following circuit with a depletion-type NMOS, the gate voltage is known to be -1V, please find the value of the resistor *R* = ?



Solution:

$$V_G = -1V = V_{GS},\ I_3 = \frac{V_G - (-10)}{9} = \frac{-1+10}{9} = 1mA,\ I_2 = I_3 = 1mA$$

$$V_D = V_G + I_2 \cdot 2k\Omega = -1 + 2 = 1V$$

$V_{DS} < V_{GS} - V_t = -1 + 3 = 2V$, so the FET operates in triode mode

$$I_D = K\left[2(V_{GS} - V_t)V_{DS} - V_{DS}^2\right] = \frac{1}{3}\left[2(-1+3_t)\times 1 - 1\right] = 1mA,$$

$$I_1 = I_D + I_2 = 2mA$$

$$R = \frac{10 - V_D}{I_1} = \frac{10-1}{2} = 4.5k\Omega$$