Homework #8 Solutions
7.25, 7.29, 7.31, 7.33 (a) (b), and

7.25 (a) $I_D = \frac{1}{2}k_n(V_{GS} - V_t^2)$

$= \frac{1}{2} \times 5(0.6 - 0.4)^2 = 0.1 \text{ mA}$

$V_{DS} = V_{DD} - I_D R_D = 1.8 - 0.1 \times 10 = 0.8 \text{ V}$

(b) $g_m = k_n V_{OV} = 5 \times 0.2 = 1 \text{ mA/V}$

(c) $A_v = -g_m R_D = -1 \times 10 = -10 \text{ V/V}$

(d) $\lambda = 0.1 \text{ V}^{-1}, \quad V_A = \frac{1}{\lambda} = 10 \text{ V}$

$r_o = \frac{V_A}{I_D} = \frac{10}{0.1} = 100 \text{ k}\Omega$

$A_v = -g_m(R_D \parallel r_o)$

$= -1(10 \parallel 100) = -9.1 \text{ V/V}$

7.29 Given $\mu_n C_{ox} = 250 \ \mu\text{A/V}^2$,

$V_t = 0.5$ V,

$L = 0.5 \ \mu$m

For $g_m = 2 \text{ mA/V}^2$ and $I_D = 0.25$ mA,

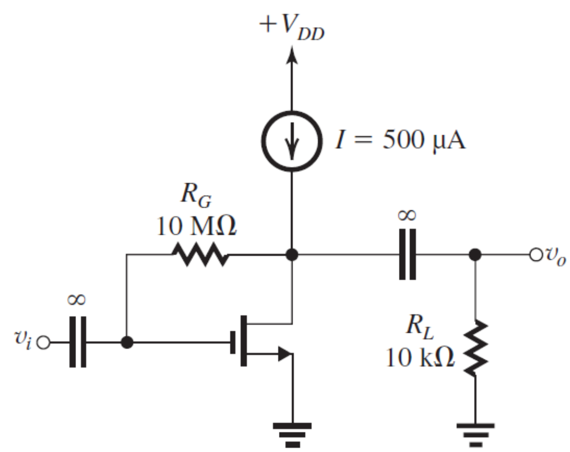
$g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_D} \Rightarrow \frac{W}{L} = 32$

$\therefore W = 16 \ \mu\text{m}$

$V_{OV} = \frac{2I_D}{g_m} = 0.25 \text{ V}$

$\therefore V_{GS} = V_{OV} + V_t = 0.75 \text{ V}$

7.31



$V_t = 0.5$ V

$V_A = 50$ V

Given $V_{DS} = V_{GS} = 1$ V. Also, $I_D = 0.5$ mA.

$V_{OV} = 0.5 \text{ V}, \ g_m = \frac{2I_D}{V_{OV}} = 2 \text{ mA/V}$

$r_o = \frac{V_A}{I_D} = 100 \text{ k}\Omega$

$\frac{v_o}{v_i} = -g_m(R_G \parallel R_L \parallel r_o) = -18.2 \text{ V/V}$

For $I_D = 1$ mA:

V_{OV} increases by $\sqrt{\frac{1}{0.5}} = \sqrt{2}$ to

$\sqrt{2} \times 0.5 = 0.707$ V.

$V_{GS} = V_{DS} = 1.207$ V

$g_m = 2.83 \text{ mA/V}, \ r_o = 50 \text{ k}\Omega$ and

$\frac{v_o}{v_i} = -23.6 \text{ V/V}$

7.33 (a) Open-circuit the capacitors to obtain the bias circuit shown in Fig. 1, which indicates the given values.

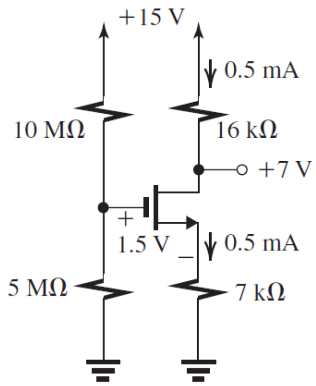


Figure 1

From the voltage divider, we have

$$V_G = 15\frac{5}{10+5} = 5\text{ V}$$

From the circuit, we obtain

$$V_G = V_{GS} + 0.5 \times 7$$

$$= 1.5 + 3.5 = 5\text{ V}$$

which is consistent with the value provided by the voltage divider.

Since the drain voltage (+7 V) is higher than the gate voltage (+5 V), the transistor is operating in saturation.

From the circuit

$V_D = V_{DD} - I_D R_D = 15 - 0.5 \times 16 = +7$ V, as assumed

Finally,

$V_{GS} = 1.5$ V, thus $V_{OV} = 1.5 - V_t = 1.5 - 1$
$= 0.5$ V

$$I_D = \frac{1}{2}k_n V_{OV}^2 = \frac{1}{2} \times 4 \times 0.5^2 = 0.5\text{ mA}$$

which is equal to the given value. Thus the bias calculations are all consistent.

(b) $g_m = \dfrac{2I_D}{V_{OV}} = \dfrac{2 \times 0.5}{0.5} = 2$ mA/V

$$r_o = \frac{V_A}{I_D} = \frac{100}{0.5} = 200\text{ k}\Omega$$

(c) See Fig. 2 below.

(d) $R_{in} = 10\text{ M}\Omega \parallel 5\text{ M}\Omega = 3.33\text{ M}\Omega$

$$\frac{v_{gs}}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{3.33}{3.33 + 0.2}$$

$= 0.94$ V/V

$$\frac{v_o}{v_{gs}} = -g_m(200 \parallel 16 \parallel 16)$$

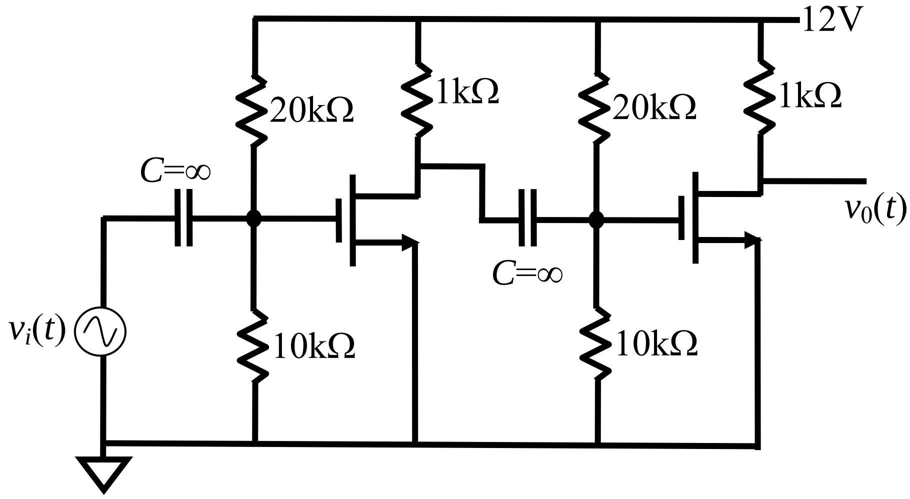
$= -2 \times 7.69 = -15.38$ V/V

$$\frac{v_o}{v_{sig}} = \frac{v_{gs}}{v_{sig}} \times \frac{v_o}{v_{gs}} = -0.94 \times 15.38$$

$= -14.5$ V/V

Handout problem 1:

In the following MOSFET amplifier circuit, two transistors are identical with $V_t = 2$V and $k_n = 2$mA/V^2. Please find the small-signal voltage gain $v_0(t)/v_i(t) = ?$



Solution:

This is an amplifier with 2 identical stages.

(a) DC analysis:

$V_G = V_{GS} = 12 \times \frac{10}{30} = 4V$

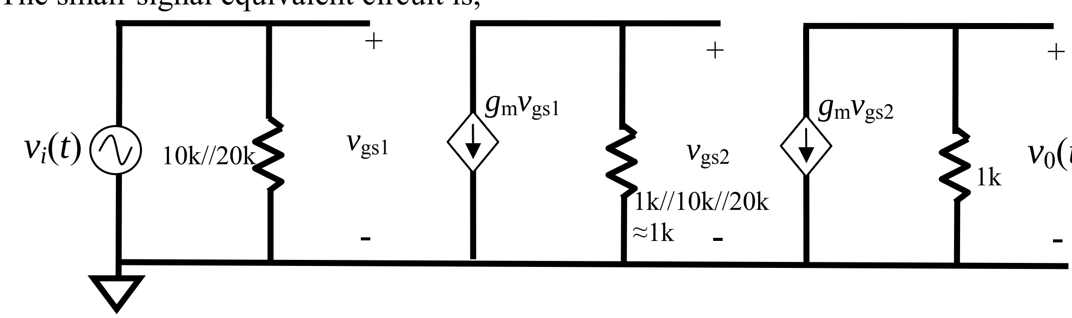
Then $I_D = \frac{k_n}{2}(V_{GS} - V_t)^2 = (4-2)^2 = 4mA$, $V_{DS} = 12 - 1 \times 4 = 8V$

$V_{DS} > V_{GS} - V_t = 4 - 2 = 2V$, and therefore the MOSFET is operating in saturation

Small-signal trans-conductive gain of each stage is, $g_m = k_n(V_{GS} - V_t) = 2 \times 2 = 4mA/V$

(b) small-signal analysis

The small-signal equivalent circuit is,

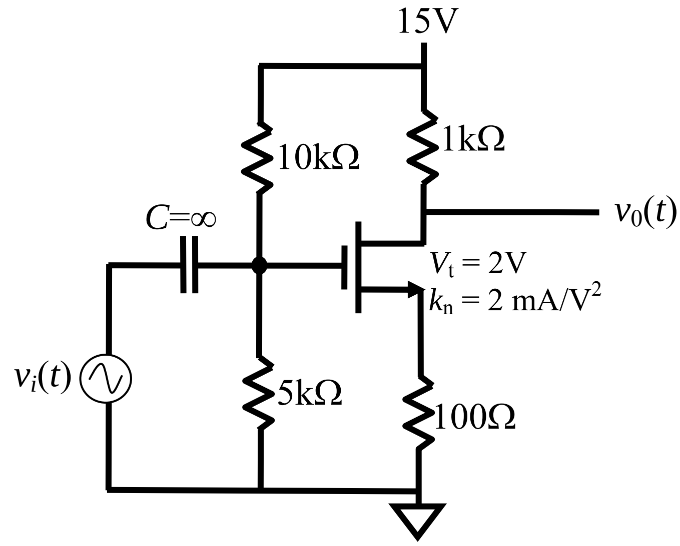


$v_{gs1} = v_i$, $v_{gs2} = -1 \times g_m v_{gs1} = -g_m v_i$, and $v_0 = -1 \times g_m v_{gs2} = g_m^2 v_i$

Therefore, the small-signal voltage gain is

$\frac{v_0}{v_i} = g_m^2 = 16$

Handout problem 2:
In the following MOSFET amplifier circuit, please find the small-signal voltage gain $v_0(t)/v_i(t) = ?$



Solution:
(a) DC analysis:

$$V_G = 15 \times \frac{5}{15} = 5V$$

Assume saturation,

$$I_D = \frac{k_n}{2}(V_{GS} - V_t)^2,$$

$$V_{GS} = 5 - 0.1 I_D.$$

Combine these two equations,

$$V_{GS} = 5 - 0.1\frac{k_n}{2}(V_{GS} - V_t)^2,$$

that is, $V_{GS} = 5 - 0.1(V_{GS} - 2)^2$

$$V_{GS}^2 + 6V_{GS} - 46 = 0$$

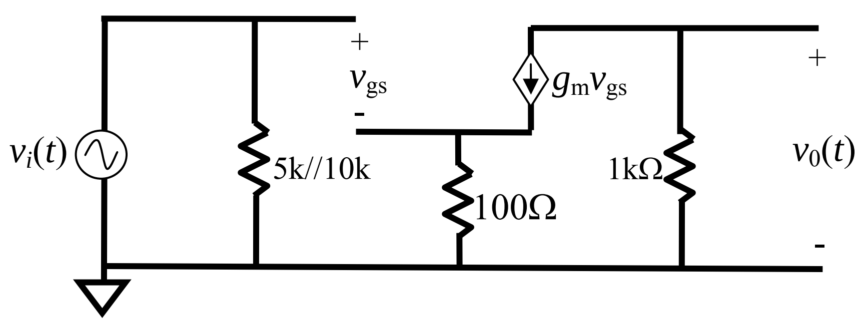
The solution is, $V_{GS} = 4.42$

Then $I_D = 1 \times (4.42 - 2)^2 = 5.86mA$, $V_{DS} = 15 - 1 \times 5.86 - 0.1 \times 5.86 = 8.551V$

$V_{DS} > V_{GS} - V_t = 4.42 - 2 = 2.42$ and therefore the MOSFET is operating in saturation

Small-signal trans-conductive gain is, $g_m = k_n(V_{GS} - V_t) = 2 \times 2.42 = 4.84mA/V$

(b) small-signal analysis
The small-signal equivalent circuit is,



$v_{gs} = v_i - 0.1 \cdot g_m \cdot v_{gs}$, that is, $v_{gs} = \dfrac{v_i}{1 + 0.1g_m}$

$$v_0 = -1 \cdot g_m v_{gs} = \frac{-g_m v_i}{1 + 0.1g_m}$$

Therefore, the small-signal voltage gain is

$$\frac{v_0}{v_i} = \frac{-g_m}{1 + 0.1g_m} = -\frac{4.84}{1 + 0.484} = -3.26$$