Homework 11:

18.15, 18.34 and 18.25

Note: For problem 18.25, the small-signal equivalent circuit is

\[ i_L = i_{C2} = sC_2V_{gs} \]
\[ V_g = -s^2LC_2V_{gs} \]
\[ V_i = V_g - V_{gs} \]
\[ i_{C2} + g_mV_{gs} = \left( \frac{1}{R_L} + sC_1 \right)V_i \]

Please write the expression of oscillation frequency, and the transconductive gain \( g_m \) needed to support the oscillation.
Solution:

18.15

Figure 1 shows the circuit and some of the analysis for the purpose of determining the transfer function of the RC circuit. The current $I$ is given by

$$I = \frac{V_1}{R} (1 + s CR) + s CV_1$$

For $V_o$ we now can write

$$V_o = V_1 (1 + s CR) + \frac{I}{sC}$$

$$\Rightarrow V_o = \frac{V_1 (1 + s CR) + \frac{V_1}{sCR} (1 + s CR) + V_1}{s^2 + s \left( \frac{3}{CR} \right) + \frac{1}{(CR)^2}}$$

The loop gain $L(s)$ can now be found as:

$$L(s) = \frac{s \left( 1 + \frac{R_2}{R_1} \right)}{s^2 + s \frac{3}{CR} + \frac{1}{(CR)^2}}$$

$$L(j\omega) = \frac{j\omega \left[ 1 + \frac{R_2}{R_1} \right] \sqrt{CR}}{(CR)^2 - \omega^2} + \frac{j \frac{3\omega}{CR}}{(CR)^2 - \omega^2}$$

Zero phase shift will occur at $\omega = \omega_0$:

$$\omega_0 = \frac{1}{CR}$$

At $\omega = \omega_0$, we have

$$|L(j\omega)| = \frac{1}{3} \left( 1 + \frac{R_2}{R_1} \right)$$

For oscillations to begin, we need

$$\frac{1}{3} \left( 1 + \frac{R_2}{R_1} \right) \geq 1$$

$$\frac{R_2}{R_1} \geq 2$$

18.34 Output levels = ±0.7 V

Threshold levels = ±\frac{10}{10 + 60} \times 0.7 = 0.1 V

$$i_{p, max} = \frac{12 - 0.7}{10} - \frac{0.7}{10 + 60} = 1.12 \text{ mA}$$
18.25:

\[ V_g = -s^2 LC \cdot V_{gs}, \quad \text{and} \quad V_s = V_g - V_{gs} = -(s^2 LC_2 + 1)V_{gs} \]

Use node equation: 

\[ sC_2 V_{gs} + g_m V_{gs} = \left( \frac{1}{R_L} + sC_1 \right) V_s \]

\[ sC_2 V_{gs} + g_m V_{gs} = \left( \frac{1}{R_L} + sC_1 \right) (s^2 LC_2 + 1)V_{gs} \]

\[ sC_2 + g_m + \left( \frac{1}{R_L} + sC_1 \right) (s^2 LC_2 + 1) = 0 \]

\[ s^3 LC_1 C_2 + s^2 \frac{LC_2}{R_L} + s(C_1 + C_2) + \left( g_m + \frac{1}{R_L} \right) = 0 \]

For \( s = j\omega \)

\[ -j \omega^3 LC_1 C_2 - \omega^2 \frac{LC_2}{R_L} + j \omega(C_1 + C_2) + \left( g_m + \frac{1}{R_L} \right) = 0 \]

\[ j \omega \left[ (C_1 + C_2) - \omega^2 LC_1 C_2 \right] + \left( g_m + \frac{1}{R_L} \right) - \omega^2 \frac{LC_2}{R_L} = 0 \]

In order for the imaginary part to be equal to zero (zero phase shift), at \( \omega = \omega_0 \)

\( (C_1 + C_2) - \omega_0^2 LC_1 C_2 = 0 \), that is \( \omega_0 = \sqrt{\frac{C_1 + C_2}{LC_1 C_2}} \)

For the real part to be equal to zero (to satisfy the resonance condition)

\[ \left( g_m + \frac{1}{R_L} \right) - \omega_0^2 \frac{LC_2}{R_L} = 0; \]

\[ g_m = \omega_0^2 \frac{LC_2}{R_L} - \frac{1}{R_L} = \frac{C_1 + C_2}{LC_1 C_2} \frac{LC_2}{R_L} - \frac{1}{R_L} = \frac{C_2}{R_L C_1} \]