Abstract:

Starting from basic physics of optical reflection and refraction of optical interfaces, this chapter introduces concept and major properties of optical fiber, which is the basic transmission medium for fiber-optic communication. These include the concept and classification of propagation modes along the fiber, single mode condition, numerical aperture, mechanisms and specifications of optical attenuation and dispersion, as well as nonlinearities of optical fiber. While dispersion specifies mode-dependent or wavelength-dependent propagation speed of optical signal propagating in an optical fiber, which are both linear effects, nonlinear effects such as stimulated Raman scattering, stimulated Brillouin scattering, and power-dependent refractive index known as Kerr effect nonlinearity may also affect wave propagation in optical fiber. Although standard multi-mode and single-mode fibers are most often used in optical communication systems, a variety of specialty fibers have also been developed for special application.

Keywords:

Optical fiber; reflection and refraction; geometric optics; fiber modes; optical attenuation; fiber dispersion, fiber nonlinearity; specialty optical fiber
Introduction

Optical wave is a special category of electromagnetic waves which can propagate in free space as well as been guided dielectric waveguides. Optical fiber is enabled by the optical field confinement mechanism of the waveguide. Low absorption of the materials that construct the optical waveguide is another important factor to enable long distance transmission of lightwave along the waveguide. The total internal reflection at the core/cladding interface of an optical waveguide can be achieved either by a discrete change of refractive index at the interface known as the step index profile, or by a gradual index change from the core to the cladding known as the graded index profile.

For long distance optical transmission, optical fiber is often used, and the basic structure of an optical fiber is shown in Figure 2.1 (a), which has a central core, a cladding surrounding the core, and an external coating to protect and strengthen the fiber. Silicon dioxide (SiO₂) is the basic material to make optical fibers because of its ultra-low absorption in the optical communications wavelength windows. Historically, the fabrication process developed by Corning in 1970s that reduced the fiber loss to less than 10dB/km made long distance fiber-optic communication feasible, and was commonly regarded as the starting point for industrial applications of fiber-optic communication systems. In addition to optical transmission over long distance, guided-wave mechanism also enables the design of photonic devices for the processing of optical signals. In photonic devices, optical waveguides are often created on planar dielectric substrates, known as the planar lightwave circuits (PLC). One example of a PLC is illustrated in Figure 2.1 (b), in which the higher index core of the waveguide is buried inside the lower index cladding, and they are both deposited on a substrate which provides the mechanical support. Waveguides based on PLC are usually created through photolithography and etching [Okamoto, 2005], and can be made into various structures to form photonic devices with the desired functionalities. As the sizes of photonic devices are often in the orders of millimeters or centimeters, the material loss requirement for PLC is not as stringent as that for optical fibers. Thus a variety of materials, such as polymer, silicon (Si), Indium Phosphate (InP) and Gallium Nitride (GaN), have been used to make PLC to enhance their functionality, reduce the fabrication cost, and to allow photonic integration.
Figure 2.1 Basic structure of an optical fiber (a) and a planar optical waveguide coupler (b).

Although both optical fiber and PLC are based on total internal reflection between the core and the cladding, we will focus on the basic guiding mechanisms and applications of optical fibers in this chapter, while integrated optical circuits based on PLC will be discussed later in chapter 7. This chapter starts with the introduction of wave reflection, refraction, and the condition of total internal reflection at an optical interface which is the basic requirement for wave guiding. We will then discuss propagation modes and mode classification in an optical fiber. Although single-mode optical fiber is the major fiber type for long distance optical communications, multi-mode fibers with relatively bigger core diameters are also widely used, especially for local area and access optical networks, as well as fiber to the home applications, because of their much relaxed tolerance to misalignment and the simplicity of handling. In addition, multi-mode fibers can potentially be used for mode multiplexing in which each mode carries an independent information channel in a communication system. Basic properties of optical fibers, including attenuation, dispersion and nonlinearity, will also be discussed. While attenuation describes the reduction of optical power level along the fiber, dispersion is a differential delay which introduces waveform distortion in a transmission system. In a multi-mode fiber, in principle different spatial modes in the same fiber would travel in different speeds, which is responsible for the so called "model dispersion". At the same time, within each spatial mode, different frequency components of an optical signal may also have different propagation velocities and this is the origin of "chromatic dispersion", which is one of the major sources of waveform distortion in a single-mode fiber. Another important parameter in an optical fiber is the Kerr-effect nonlinearity, which is caused by the power-dependent refractive index. As the amplitude of information-carrying optical signal in an optical fiber is usually not constant, the variation of the instantaneous signal optical power can modulate the
refractive index of the optical fiber which causes the modulation in the propagation
delay and introduces waveform distortion at the optical receiver. In addition to the
Kerr-effect nonlinearity, nonlinear scatterings also exist in an optical fiber mainly
through Stimulated Brillouin scattering (SBS) and Stimulated Raman scattering (SRS).
SBS, originates from the mechanical property of the silica material, is a narrow band
phenomenon, in which the optical signal creates an acoustic vibration of the material
and been scattered by this acoustic wave to produce a new frequency component
approximately 11GHz away from the original signal optical frequency. On the other
hand, SRS is generated by molecular vibrations of the silica material caused by the
optical signal, which is broadband. The frequency shift of the scattered optical signal
cau sed by the SRS process is on the order of 13THz. The last section of this chapter,
introduces a few special fiber types which were designed to provide various different
properties compare to a standard single mode fiber in terms their fabrication
processes, chromatic dispersion parameters, polarization selectivity, and nonlinear
property.

As optical fiber is the basic transmission medium of fiber-optic communication
systems and networks, a good understanding of wave propagation mechanism and
basic properties of optical fiber is fundamental.

2.1 General properties of optical wave

Lightwave is a special format of electromagnetic wave which occupies the spectral
regions from infrared (IR) to ultraviolet (UV). Popular wavelength windows of
lightwave used for optical communication include 850nm, 1310nm and 1550nm,
which are located in the near IR region. The general property of the lightwave signal
has amplitude, frequency, phase and the state of polarization. A linearly polarized
optical signal propagating in a certain direction can be expressed by a complex
representation as,

$$\vec{E}(\vec{x},t) = \vec{e}E_0 \exp\left\{-j(\omega t - \vec{k} \cdot \vec{x})\right\}$$

(2.1.1)

where, $\vec{E}$ is the complex optical field, which is a vector, with the polarization
orientation represented by a unit vector $\vec{e}$ and a position vector represented by $\vec{x}$. In
Equation 2.1.1, $E_0$ is the field magnitude, $\omega = 2\pi f$ is the angular frequency ($f$ is the
circular frequency), $\vec{k}$ is the known as the vectoral propagation constant with its
absolute value defined by \[ |\mathbf{k}| = \frac{2\pi n}{\lambda}. \] \( n \) is the refractive index of the medium, \( \lambda = \frac{c}{f} \) is the vacuum wavelength of the lightwave and \( c \) is the speed of light in vacuum. Both wavelength and frequency are often used to describe an optical signal depending on convenience. For example, optical communication wavelengths of \( \lambda = 850\text{nm}, 1310\text{nm} \) and \( 1550\text{nm} \) correspond to the frequencies of \( 353\text{THz}, 229\text{THz} \) and \( 193.5\text{THz} \), respectively. The following are a few common properties of lightwave.

*Phase front* (also known as wave-front) refers to a surface of equal phase. That is represented by \[ \omega t - \mathbf{k} \cdot \mathbf{x} = \text{constant}. \] With this definition, for a fixed frequency \( \omega \) and at each moment of time, \( \mathbf{k} \cdot \mathbf{x} \) is a constant. The spatial distribution of this equal-phase depends only on the nature of \( \mathbf{k} \). The wave front of a point source can be spherical, while a collimated light beam often has a plane wave front perpendicular to the propagating direction.

*Speed of propagation* is represented by the speed of equal-phase front in the direction determined by \( \mathbf{k} \). Based on Equation 2.1.1, for an incremental time \( \Delta t \) the lightwave travels a distance \( \Delta x \), the equal-phase relation is \( \omega \Delta t - \mathbf{k} \cdot \Delta \mathbf{x} = 0 \) and thus, \[ \frac{\Delta \mathbf{x}}{\Delta t} = \frac{\omega}{\mathbf{k}}. \] Then if \( \Delta \mathbf{x} \) is in the same direction of \( \mathbf{k} \), \[ \left| \frac{\Delta x}{\Delta t} \right| = \frac{|\omega|}{|\mathbf{k}|} = f\lambda / n = c / n \] which is indeed the speed of light in the medium with a refractive index \( n \). If we consider both the positive and the negative frequencies, a lightwave can also be conveniently represented as a real field, \( \tilde{E}(\mathbf{x}, t) = \tilde{e}E_0 \cos(\omega t - \mathbf{k} \cdot \mathbf{x}) \).

*State of polarization* (SOP): Although wave-front can have various spatial shapes depending on the focusing or diverging nature of the light beam, the simplest wave format is the plane wave, also known as *transverse wave*, which represents a collimated light beam with a plane wave-front. In this case the electrical field \( \tilde{E} \) and the magnetic field \( \tilde{H} \) are both perpendicular to the direction of wave propagation. For example in a Cartesian coordination for a plane wave traveling in the \( z \)-direction: \( \tilde{E} = \tilde{e}_x E_x + \tilde{e}_y E_y \) and \( \tilde{k} = \tilde{e}_z k_z \), so that, \( \tilde{k} \cdot \mathbf{x} = k_z x \), where \( \tilde{e}_x, \tilde{e}_y \) and \( \tilde{e}_z \) are unit vectors in the \( x, y, \) and \( z \) directions, respectively.

In fact, a polarized plane wave electrical field has the general expression,
\[ \vec{E}(z,t) = \vec{e}_x E_x(z,t) + \vec{e}_y E_y(z,t) \]  

(2.1.2)

where,

\[ E_x(z,t) = E_{0x} \cos(\omega t - k z) \]  

(2.1.3a)

\[ E_y(z,t) = E_{0y} \cos(\omega t - k z + \delta) \]  

(2.1.3b)

with \( \delta \) the relative phase difference between \( x \) and \( y \) field components. The state of polarization of the lightwave is determined by field magnitudes \( E_{0x}, E_{0y} \) and differential phase \( \delta \). The state of polarization of a polarized light can be categorized linear, circular, and elliptical.

![Figure 2.1.1 Illustration of state of polarization of a plane wave.](image)

Figure 2.1.1 Illustration of state of polarization of a plane wave.

Linear polarization is obtained with no phase difference between the \( E_x \) and the \( E_y \) components (\( \delta = 0 \) or \( \pi \)) as illustrated in Figure 2.1.1 (a). In this case, the \( E_x \) and \( E_y \) components increase or decrease at the same time, so that the tip of the vector \( \vec{E} \) moves along a straight line with an angle \( \phi = \tan^{-1}(E_{0y} / E_{0x}) \) with respect to the \( x \)-axis.

For a circular polarization, \( E_{0x} = E_{0y} \) and the phase difference has to be \( \delta = 2m\pi \pm \pi/2 \) where \( m \) is an integer. In this case, when the magnitude of \( E_x \) is the maximum, \( E_y \) is zero, and vise versa. Thus, the tip of the vector \( \vec{E} \) moves along a circle as shown in Figure 2.1.1 (b). For \( \delta = 2m\pi + \pi/2 \), \( E_y \) advances \( E_x \) by \( \pi/2 \), so that the circle is clockwise. Otherwise for \( \delta = 2m\pi - \pi/2 \), the circle is counter-clockwise.

The most general state of polarization is elliptical as shown in Figure 2.1.1 (c), which has no special requirement for \( E_{0x}, E_{0y} \) and \( \delta \). The trajectory of vector \( \vec{E} \) can be expressed by [Keiser, 2011],
\[
\left( \frac{E_x}{E_{0x}} \right)^2 + \left( \frac{E_y}{E_{0y}} \right)^2 - 2 \frac{E_x E_y}{E_{0x} E_{0y}} \cos \delta = \sin^2 \delta
\]  
(2.1.4)

The orientation angle of the ellipse with respect to the \(x\)-axis is,

\[
\phi = \frac{1}{2} \tan^{-1} \left( \frac{2E_{0x} E_{0y} \cos \delta}{E_{0x}^2 - E_{0y}^2} \right)
\]  
(2.1.5)

Note that, the state of polarization is only valid for the polarized light for which there is a deterministic phase difference \(\delta\) between the \(E_x\) and \(E_y\) components. If the phase difference \(\delta\) is random, the state of polarization will also be random so that the definition of SOP becomes meaningless. Polarized light can be produced by coherent light sources such as lasers (here "coherent" means that \(E_x\) and \(E_y\) components are mutually coherent so that \(\delta\) is deterministic), but many light sources in the nature, such as sun light, are not coherent, and a practical light source is often partially coherent.

Degree of polarization (DOP) is defined as the percentage of the polarized light power \((P_{\text{polarized}})\) in the total power of the light \((P_{\text{polarized}} + P_{\text{unpolarized}})\),

\[
DOP = \frac{P_{\text{polarized}}}{P_{\text{polarized}} + P_{\text{unpolarized}}}
\]  
(2.1.6)

Thus, \(0 \leq DOP \leq 1\), with 0 and 1 representing completely un-polarized and completely polarized lights, respectively. In practice, light generated from a semiconductor laser for optical communication has the DOP value of generally higher than 95%.

2.2 Reflection and refraction

An optical interface is generally defined as a plane across which optical property discontinues. For example, water surface is an optical interface because the refractive indices suddenly change from \(n = 1\) in the air to \(n = 1.3\) in the water. To simplify our discussion, the following assumptions have been made:

1. Plane wave propagation
2. Linear medium
3. Isotropic medium
4. Smooth planar optical interface

As illustrated in Figure 2.2.1, an optical interface is formed between two optical materials with refractive indices of \( n_1 \) and \( n_2 \), respectively. A plane optical wave is projected onto the optical interface at an incident angle \( \theta_i \) (with respect to the surface normal). The optical wave is linearly polarized, and its field amplitude vector can be decomposed into two orthogonal components, \( E_{\parallel}^i \) and \( E_{\perp}^i \) parallel and perpendicular to the incidence plane. At the optical interface, part of the energy is reflected back to the same side of the interface and the other part is refracted across the interface.

\[
n_2 \sin \theta_2 = n_1 \sin \theta_i
\]  

(2.2.1)
is known as the Snell's Law, which tells the propagation direction of the refracted light wave with respect to that of the incident wave. The wave propagation direction change is proportional to the refractive index difference across the interface.

Snell’s Law is derived based on the fact that phase velocity along z-direction should be continuous across the interface. Since the phase velocities in the z-direction are

\[
v_{\parallel 1} = \frac{c}{n_1 \sin \theta_i} \quad \text{and} \quad v_{\parallel 2} = \frac{c}{n_2 \sin \theta_2}
\]
at the two sides of the interface, Equation 2.2.1 can be obtained with \( v_{\parallel 1} = v_{\parallel 2} \).

Figure 2.2.1 Plane wave reflection and refraction at an optical interface.
Because Snell’s Law was obtained without any assumption of light wave polarization state and wavelength, it is independent of these parameters. An important implication of Snell’s Law is that \( \theta_2 > \theta_1 \) with \( n_2 < n_1 \).

2.2.1 Fresnel reflection coefficients

To find out the strength and the phase of the optical field that is reflected back to the same side of the interface, we have to treat optical field components \( E_\parallel \) and \( E_\perp \) separately. An important fact is that optical field components parallel to the interface must be continuous at both sides of the interface.

Let’s first consider the field components \( E_\parallel \), \( E_\parallel \) and \( E_\parallel \) (they are all parallel to the incident plane but not to the interface). They can be decomposed into components parallel with and perpendicular to the interface; the parallel components are, \( E_\parallel \cos \theta_1 \), \( E_\parallel \cos \theta_2 \), and \(-E_\parallel \cos \theta_1\), respectively, which can be derived from Figure 2.2. Because of the field continuity across the interface, we have

\[
(E_\parallel - E_\parallel) \cos \theta_1 = E_\parallel \cos \theta_2 \tag{2.2.2}
\]

At the same time, the magnetic field components associated with \( E_\parallel \), \( E_\parallel \) and \( E_\parallel \) have to be perpendicular to the incident plane, and they are \( H_\perp = \sqrt{\varepsilon_1 / \mu_1} E_\parallel \), \( H_\perp = \sqrt{\varepsilon_2 / \mu_2} E_\parallel \) and \( H_\perp = \sqrt{\varepsilon_1 / \mu_1} E_\parallel \), respectively, where \( \varepsilon_1 \) and \( \varepsilon_2 \) are electrical permittivities and \( \mu_1 \) and \( \mu_2 \) are magnetic permittivities of the optical materials at two sides of the interface. Since \( H_\perp \), \( H_\perp \) and \( H_\perp \) are all parallel to the interface (although perpendicular to the incident plane), magnetic field continuity requires \( H_\perp + H_\perp = H_\perp \). Assume that \( \mu_1 = \mu_2 \), \( \sqrt{\varepsilon_1} = n_1 \) and \( \sqrt{\varepsilon_2} = n_2 \), and we have

\[
n_1 E_\parallel + n_2 E_\parallel = n_2 E_\parallel \tag{2.2.3}
\]

Combine Equations 2.2.2 and 2.2.3 and we can find the field reflectivity:

\[
\rho_\parallel = \frac{E_\parallel}{E_\parallel} = \frac{n_1 \cos \theta_2 - n_2 \cos \theta_1}{n_1 \cos \theta_2 + n_2 \cos \theta_1} \tag{2.2.4}
\]

Using Snell’s Law, Equation 2.2.4 can also be written as
where variable $\theta_2$ is eliminated.

Similar analysis can also find the reflectivity for optical field components perpendicular to the incident plane as

$$
\rho_\perp = \frac{E_\perp^r}{E_\perp} = \frac{n_1 \cos \theta_1 - n_2 \cos \theta_2}{n_1 \cos \theta_1 + n_2 \cos \theta_2} \tag{2.2.6}
$$

Or, equivalently,

$$
\rho_\perp = \frac{n_1 \cos \theta_1 - \sqrt{n_2^2 - n_1^2 \sin^2 \theta_1}}{n_1 \cos \theta_1 + \sqrt{n_2^2 - n_1^2 \sin^2 \theta_1}} \tag{2.2.7}
$$

Power reflectivities for parallel and perpendicular field components are therefore

$$
R_\parallel = |\rho_\parallel|^2 = \left| \frac{E_\parallel^r}{E_\parallel} \right|^2 \tag{2.2.8}
$$

and

$$
R_\perp = |\rho_\perp|^2 = \left| \frac{E_\perp^r}{E_\perp} \right|^2 \tag{2.2.9}
$$

Then, according to energy conservation, the power transmission coefficients can be found as

$$
T_\parallel = \left| \frac{E_\parallel^t}{E_\parallel} \right|^2 = 1 - |\rho_\parallel|^2 \tag{2.2.10}
$$

and

$$
T_\perp = \left| \frac{E_\perp^t}{E_\perp} \right|^2 = 1 - |\rho_\perp|^2 \tag{2.2.11}
$$

In practice, for an arbitrary incidence polarization state, the input field can always be decomposed into $E_\parallel$ and $E_\perp$ components. Each can be treated independently.

### 2.2.2 Special cases of reflection angles

(A) Normal incidence
This is when the light is launched perpendicular to the material interface. In this case, \( \theta_1 = \theta_2 = 0 \) and \( \cos \theta_1 = \cos \theta_2 = 1 \), the field reflectivity can be simplified as

\[
\rho_{\parallel} = \rho_{\perp} = \frac{n_1 - n_2}{n_1 + n_2} \quad (2.2.12)
\]

Note that there is no phase shift between incident and reflected field if \( n_1 > n_2 \) (the phase of both \( \rho_{\parallel} \) and \( \rho_{\perp} \) is zero). On the other hand, if \( n_1 < n_2 \), there is a \( \pi \) phase shift for both \( \rho_{\parallel} \) and \( \rho_{\perp} \) because they both become negative.

With normal incidence, the power reflectivity is

\[
R_{\parallel} = R_{\perp} = \left| \frac{n_1 - n_2}{n_1 + n_2} \right|^2 \quad (2.2.13)
\]

This is a very often used equation to evaluate optical reflection. For example, reflection at an open fiber end is approximately 4 percent. This is because the refractive index in the fiber core is \( n_1 \approx 1.5 \) (silica fiber) and refractive of air is \( n_1 = 1 \). Therefore:

\[
R = \left| \frac{n_1 - n_2}{n_1 + n_2} \right|^2 = \left| \frac{1.5 - 1}{1.5 + 1} \right|^2 = 0.2^2 = 0.04 \approx -14dB
\]

In practical optical measurement setups using optical fibers, if optical connectors are not properly terminated, the reflections from fiber-end surface can potentially cause significant measurement errors.

(B) Critical angle

Critical angle is defined as an incident angle \( \theta_i \) at which total reflection happens at the interface. According to Fresnel Equations 2.2.5 and 2.2.7, the only possibility that \( |\rho_{\parallel}| = |\rho_{\perp}| = 1 \) is to have \( n_2^2 - n_i^2 \sin^2 \theta_i = 0 \) or

\[
\theta_i = \theta_c = \sin^{-1} \left( \frac{n_2}{n_1} \right) \quad (2.2.14)
\]

where \( \theta_c \) is defined as the critical angle. Obviously the necessary condition to have a critical angle depends on the interface condition.
First, if \( n_1 < n_2 \), there is no real solution for \( \sin^2 \theta_1 = \left( \frac{n_2}{n_1} \right)^2 \). This means that when a light beam goes from a low index material to a high index material, total reflection is not possible.

Second, if \( n_1 > n_2 \), a real solution can be found for \( \theta_c = \sin^{-1} \left( \frac{n_2}{n_1} \right) \) and therefore total reflection can only happen when a light beam launches from a high index material to a low index material.

It is important to note that at a larger incidence angle \( \theta_1 > \theta_c \), \( n_2^2 - n_1^2 \sin^2 \theta_1 < 0 \) and \( \sqrt{n_2^2 - n_1^2 \sin^2 \theta_1} \) becomes imaginary. Equations 2.2.5 and 2.2.7 clearly show that if \( \sqrt{n_2^2 - n_1^2 \sin^2 \theta_1} \) is imaginary, both \( |\rho_1| \) and \( |\rho_2| \) are equal to 1.

The important conclusion is that for all incidence angles satisfying \( \theta_1 > \theta_c \), total internal reflection will happen with \( R = 1 \).

**Evanescent field:** Note that if the total reflection condition is satisfied, there is no optical power flow across the interface. However, because of the continuity constrain, the optical field on the other side of the interface does not suddenly reduce to zero, which is known as the evanescent field.

![Figure 2.2.2](image-url) Illustration of evanescent wave for (a) partial reflection with \( \theta_1 < \theta_c \) and (b) total reflection with \( \theta_1 > \theta_c \).

As illustrated in Figure 2.2.2 (a), when the incidence angle is smaller than the critical angle, \( \theta_1 < \theta_c \), the reflection is partial, and the optical field that propagates in the z-direction in the \( n_2 \) medium can be described by \( E(z) = E_0 e^{i\beta_z z} \), where \( \beta_z = \left( \frac{2\pi}{\lambda} \right) n_2 \cos \theta_2 \) which is the propagation constant projected on the z-direction. Based on the Snell's Law, this projected propagation constant can be expressed as, \( \beta_z = \left( \frac{2\pi}{\lambda} \right) \sqrt{n_2^2 - n_1^2 \sin^2 \theta_1} \). The value of \( \beta_z \) reduces with the increase of the incidence angle, and \( \beta_z \) becomes zero when the incidence angle is equal to the critical
Further increasing the incidence angle for $\theta_1 > \theta_c$ results in a
imaginary $\beta_z$ value, $\beta_z = j(2\pi / \lambda)\sqrt{n_1^2 \sin^2 \theta_1 - n_2^2} = j\alpha$, where

$$\alpha = \frac{2\pi}{\lambda} \sqrt{n_1^2 \sin^2 \theta_1 - n_2^2}$$

is defined as the attenuation parameter of the evanescent field on the $n_2$ side of the
medium, and the optical field is then,

$$E(z) = E_0 e^{j\beta_z z} = E_0 e^{-\alpha z}$$

As illustrated in Figure 2.2.2 (b) the penetration depth, $z_e$, of the evanescent field
across the interface is usually defined by the distance at which the field is reduced by
1/e, and thus $z_e = 1/\alpha$. This evanescent field penetration depth not only depends on
the values of $n_1$ and $n_2$, but also depends on the wavelength of the optical signal and
the wave incidence angle. As an example, the Figure 2.2.3 shows the penetration
depth as the function the incidence angle for two different materials: glass ($n_1 = 1.5$)
and silicon ($n_1 = 3.5$) in the air ($n_2 = 1$), for the signal wavelength of 1.5 $\mu$m. This
figure indicates that the evanescent field penetration depth $z_e$ is much shorter than the
wavelength of the optical signal, especially when the index difference ($n_1 - n_2$) is
large, and the incidence angle approaches $\pi/2$. This unique property of tight field
concentration has been utilized to make evanescent field photonic bio-sensors, in
which only the molecules immobilized on the waveguide surface are illuminated by
the optical field [Taitt, 2005].

Figure 2.2.3, Penetration depth of evanescent field as the function of incidence
angle, for air/glass interface and air/silicon interface.
Example 2.1. Assume the refractive indices of water and glass are 1.3 and 1.5, respectively. Please find critical angles on the air/water interface and air/glass interface. Is it possible to have 100% power reflection on the air/water surface when the light beam is launched from the air?

Solution,

Based on Equation 2.2.14, the critical angles on the air/water and air/glass surfaces are

\[ \theta_{c,\text{water}} = \sin^{-1}(1/1.3) = \sin^{-1}(1.5/1.3) = 50.28^\circ, \quad \text{and} \quad \theta_{c,\text{glass}} = \sin^{-1}(1/1.5) = 41.8^\circ. \]

If the light beam is launched from air to the water surface, \( n_2 = 1.3 \) and \( n_1 = 1 \) in Equation 2.2.14, and thus \( \theta_c = \sin^{-1}(1.3) \) which is imaginary, implying that critical angle does not exist in this case and the power reflectivity cannot be 100% for \( 90^\circ > \theta > 0 \). In fact, this is the reason that optical wave can be confined with high index material, such as an optical waveguide or a fiber core which will be discussed in this chapter.

2.2.3 Optical field phase shift between the incident and the reflected beams

(a) When \( \theta_i < \theta_c \) (partial reflection and partial transmission), both \( \rho_\parallel \) and \( \rho_\perp \) are real and therefore there is no phase shift for the reflected wave at the interface.

(b) When total internal reflection happens, \( \theta_i > \theta_c \), \( \sqrt{n_2^2 - n_1^2 \sin^2 \theta_i} \) is imaginary.

Fresnel Equations 2.2.5 and 2.2.7 can be written as

\[
\rho_\parallel = \frac{-n_2^2 \cos \theta_i + jn_1 \sqrt{(n_2^2 \sin^2 \theta_i - n_1^2)}}{n_2^2 \cos \theta_i + jn_1 \sqrt{(n_1^2 \sin^2 \theta_i - n_2^2)}} \tag{2.2.15}
\]

\[
\rho_\perp = \frac{n_1 \cos \theta_i - jn_1 \sqrt{(n_1^2 \sin^2 \theta_i - n_2^2)}}{n_1 \cos \theta_i + jn_1 \sqrt{(n_1^2 \sin^2 \theta_i - n_2^2)}} \tag{2.2.16}
\]

Therefore phase shift for the parallel and the perpendicular electrical field components are, respectively,

\[
\Delta \Phi_\parallel = \arg \left( \frac{E_\parallel'}{E_\parallel} \right) = -2 \tan^{-1} \left( \frac{n_2 \sqrt{n_1^2 - n_1^2 \sin^2 \theta_i}}{n_2^2 \cos \theta_i} \right) \tag{2.2.17}
\]

\[
\Delta \Phi_\perp = \arg \left( \frac{E_\perp'}{E_\perp} \right) = -2 \tan^{-1} \left( \frac{n_1 \sqrt{n_2^2 - n_1^2 \sin^2 \theta_i}}{n_1 \cos \theta_i} \right) \tag{2.2.18}
\]
This optical phase shift happens at the optical interface, which has to be considered in optical waveguide design, as will be discussed later.

### 2.2.4 Brewster angle

Consider a light beam launched onto an optical interface. If the input electrical field is parallel to the incidence plane, there exists a specific incidence angle $\theta_b$ at which the reflection is equal to zero. Therefore the energy is totally transmitted across the interface. This angle $\theta_b$ is defined as the *Brewster angle*.

Consider the Fresnel Equation 2.2.5 for parallel field components. If we solve this equation for $\rho_\parallel = 0$ and use $\theta_1$ as a free parameter, the only solution is $\tan \theta_1 = n_2 / n_1$ and therefore the Brewster angle is defined as

$$\theta_B = \tan^{-1}(n_2 / n_1) \quad (2.2.19)$$

![Diagram of field reflectivities and phase shifts versus incidence angle. Optical interface is formed with $n_1 = 1.5$ and $n_2 = 1.4$.](image)

Two important points we need to note: (1) The Brewster angle is only valid for the polarization component which has the electrical field vector parallel to the incidence
plane. For the perpendicular polarized component, no matter how you choose $\theta_1$, total transmission will never happen. (2) $\rho_\| = 0$ happens only at one angle $\theta_1 = \theta_b$. This is very different from the critical angle where total reflection happens for all incidence angles within the range of $\theta_c < \theta_1 < \pi/2$.

The Brewster angle is often used to minimize the optical reflection and it can also be used to select the polarization. Figure 2.2.4 shows an example of optical field reflectivities $\rho_\|$ and $\rho_\perp$, and their corresponding phase shifts $\Delta \Phi_\|$ and $\Delta \Phi_\perp$ at an optical interface of two materials with $n_1 = 1.5$ and $n_2 = 1.4$. In this example, the critical angle is $\theta_c \approx 69^\circ$ and the Brewster angle is $\theta_b \approx 43^\circ$.

2.3 Propagation modes in optical fibers

The cross-section of a planar optic waveguide on the PLC platform is usually rectangle, and the details of PLC design will be presented in Chapter 13 where integrated photonic circuits will be discussed. This section will focus on the discussion of optical fiber which is a cylindrical glass bar with a core, a cladding, and an external coating, as shown in Figure 2.1 (b). To confine and guide the light wave signal within the fiber core, a total internal reflection is required at the core-cladding interface. According to what we have discussed in Section 2.1, this requires the refractive index of the core to be higher than that of the cladding.

Practical optical fibers can be divided into two categories: step-index fiber and graded-index fiber. The index profiles of these two types of fibers are shown in Figure 2.3.1. In a step-index fiber the refractive index is $n_1$ in the core and $n_2$ in the cladding; there is an index discontinuity at the core-cladding interface. A light wave signal is bounced back and forth at this interface, which forms guided modes propagating in the longitudinal direction. On the other hand, in a graded-index fiber, the refractive index in the core gradually reduces its value along the radius. A generalized Fresnel equation indicates that in a medium with a continual index profile, a light trace would always bend toward high refractive areas. In fact this graded-index profile creates a self-focus effect within the fiber core to form an optical waveguide [E. G. Meumann, 1988]. Although graded-index fibers form a unique category, they are usually made for multimode applications. The popular single-mode fibers are made with step-index
fibers. Because of their popularity and simplicity, we will focus our analysis on step-index fibers.

![Step-index fiber](image1)

![Graded-index fiber](image2)

Figure 2.3.1 Illustration and index profiles of step-index and graded-index fibers.

Rigorous description of wave propagation in optical fibers requires solving Maxwell’s equations and applying appropriate boundary conditions. In this section, we first use geometric ray trace approximation to provide a simplified analysis, which helps us understand the basic physics of wave propagation. Then we present electromagnetic field analysis, which provides precise mode cutoff conditions.

### 2.3.1 Geometric optics analysis

In this geometric optics analysis, different propagation modes in an optical fiber can be seen as rays traveling at different angles. There are two types of light rays that can propagate along the fiber: skew rays and meridional rays. Figure 2.3.2(a) shows an example of skew rays, which are not confined to any particular plane along the fiber. Although skew rays represent a general case of fiber modes, they are difficult to analyze. A simplified special case is the meridional rays shown in Figure 2.3.2(b), which are confined to the meridian plane, which contains the symmetry axis of the fiber. The analysis of meridional rays is relatively easy and provides a general picture of ray propagation along an optical fiber.
Consider meridional rays as shown in Figure 2.3.2(b). This is a two-dimensional problem where the optical field propagates in the longitudinal direction $z$ and its amplitude varies over the transversal direction $r$. We define $\beta_i = n_i \omega / c = 2 \pi m_i / \lambda$ (in $\text{rad./m}$) as the propagation constant in a homogeneous core medium with a refraction index of $n_1$. Each fiber mode can be explained as a light ray that travels at a certain angle, as shown in Figure 2.3.3. Therefore, for $i$\textsuperscript{th} mode propagating in $+z$ direction, the propagation constant can be decomposed into a longitudinal component $\beta_z$ and a transversal component $k_{i1}$ such that

$$\beta_i^2 = \beta_z^2 + k_{i1}^2 \quad (2.3.1)$$

Then the optical field vector of the $i$\textsuperscript{th} mode can be expressed as

$$\tilde{E}_i(r,z) = \tilde{E}_{i0}(r,z) \exp\{-j(\omega t - \beta_z z)\} \quad (2.3.2)$$

where $\tilde{E}_{i0}(r,z)$ is the field amplitude of the mode.
Since the mode is propagating in the fiber core, both $k_1$ and $\beta_z$ must be real. First, for $k_1$ to be real in the fiber core, we must have

$$k_1^2 = \beta_1^2 - \beta_z^2 \geq 0$$  \hspace{1cm} (2.3.3)

The physical meaning of this real propagation constant in the transversal direction is that the light wave propagates in the transverse direction but is bounced back and forth between the core-cladding interfaces. This creates a standing wave pattern in the transverse direction, like a resonant cavity. In addition, $k_1$ can only have discrete values because the standing wave pattern in the transversal direction requires phase matching. This is the reason that propagating optical modes in a fiber have to be discrete.

Now let’s look at what happens in the cladding. Because the optical mode is guided in the fiber core, there should be no energy propagating in the transversal direction in the cladding (otherwise optical energy would be leaked). Therefore, $k_i$ has to be imaginary in the cladding, that is,

$$k_i^2 = \beta_i^2 - \beta_z^2 < 0$$  \hspace{1cm} (2.3.4)

where subscript 2 represents parameters in the cladding and $\beta_2 = n_2 \omega / c = 2 \pi n_2 / \lambda$ is the propagation constant in the homogeneous cladding medium.
Note that since the optical field has to propagate with the same phase velocity in the z-direction both in the core and in cladding, $\beta_{zi}$ has the same value in both Equations 2.3.3 and 2.3.4.

Equations 2.3.3 and 2.3.4 can be simplified as

$$\frac{\beta_{zi}}{\beta_1} \leq 1$$  
(2.3.5)

and

$$\frac{\beta_{zi}}{\beta_2} > 1$$  
(2.3.6)

Bringing equations 2.3.5 and 2.3.6 together with $\beta_2 = \beta n_2 / n_1$, we can find the necessary condition for a propagation mode,

$$1 \geq \frac{\beta_{zi}}{\beta_1} > \frac{n_2}{n_1}$$  
(2.3.7)

It is interesting to note that in Figure 2.3.3, $\theta_i$ is, in fact, the incidence angle of the $i^{th}$ mode at the core-cladding interface. The triangle in Figure 2.3.3 clearly shows that $\beta_{zi} / \beta_1 = \sin \theta_i$. This turns Equation 2.3.7 into $1 \geq \sin \theta_i > n_2 / n_1$, which is the same as the definition of the critical angle as given by Equation 2.2.14.

The concept of discrete propagation modes comes from the fact that the transversal propagation constant $k_{i1}$ in the fiber core can only take discrete values to satisfy standing wave conditions in the transverse direction. Since $\beta_1$ is a constant, the propagation constant in the z-direction $\beta_{zi}^2 = \beta_1^2 - k_{i}^2$ can only take discrete values as well. Or equivalently the ray angle $\theta_i$ can only take discrete values within the range defined by $1 \geq \sin \theta_i > n_2 / n_1$.

The geometric optics description given here is simple and it qualitatively explains the general concept of fiber modes. However, it is not adequate to obtain quantitative mode field profiles and cutoff conditions. Therefore electromagnetic field theory has to be applied by solving Maxwell’s equations and using appropriate boundary conditions, which we discuss next.

2.3.2 Mode analysis using electromagnetic field theory
Mode analysis in optical fibers can be accomplished more rigorously by solving Maxwell’s equations and applying appropriate boundary conditions defined by fiber geometries and parameters. We start with classical Maxwell’s Equations,

\[ \nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad (2.3.8) \]
\[ \nabla \times \vec{H} = \varepsilon \frac{\partial \vec{E}}{\partial t} \quad (2.3.9) \]

The complex electrical and the magnetic fields are represented by their amplitudes and phases,

\[ \vec{E}(t, \vec{r}) = \vec{E}_0 \exp(-j(\omega t - \vec{k} \cdot \vec{r})) \quad (2.3.10) \]
\[ \vec{H}(t, \vec{r}) = \vec{H}_0 \exp(-j(\omega t - \vec{k} \cdot \vec{r})) \quad (2.3.11) \]

Since fiber material is passive and there is no generation source within the fiber,

\[ (\nabla \cdot \vec{E}) = 0 \quad (2.3.12) \]
\[ \nabla \times \nabla \times \vec{E} = \nabla (\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -\nabla^2 \vec{E} \quad (2.3.13) \]

Combining Equations 2.3.8–2.3.13 yields,

\[ \nabla \times \nabla \times \vec{E} = j \omega \mu (\nabla \times \vec{H}) = j \omega \mu (-j \omega \varepsilon \vec{E}) \quad (2.3.14) \]

And the Helmholtz equation,

\[ \nabla^2 \vec{E} + \omega^2 \mu \varepsilon \vec{E} = 0 \quad (2.3.15) \]

Similarly, a Helmholtz equation can also be obtained for the magnetic field \( \vec{H} \):

\[ \nabla^2 \vec{H} + \omega^2 \mu \varepsilon \vec{H} = 0 \quad (2.3.16) \]

The next task is to solve Helmholtz equations for the electrical and the magnetic fields. Because the geometric shape of an optical fiber is cylindrical, we can take advantage of this axial symmetry to simplify the analysis by using cylindrical coordinates. In cylindrical coordinates, the electrical field can be decomposed into radial, azimuthal and longitudinal components:

\[ \vec{E} = \vec{a}_r E_r + \vec{a}_\phi E_\phi + \vec{a}_z E_z \]
\[ \vec{H} = \vec{a}_r H_r + \vec{a}_\phi H_\phi + \vec{a}_z H_z \]

where \( \vec{a}_r, \vec{a}_\phi, \) and \( \vec{a}_z \) are unit vectors. With this separation, the Helmholtz Equations 2.3.15 and 2.3.16 can be decomposed into separate equations for
\( E_r, E_\phi, E_z, H_r, H_\phi, \text{ and } H_z \), respectively. However, these three components are not completely independent. In fact, classic electromagnetic theory indicates that in cylindrical coordinate the transversal field components \( E_r, E_\phi, H_r, \text{ and } H_\phi \) can be expressed as a combination of longitudinal field components \( E_z \text{ and } H_z \) [Iizuka, 2002]. This means that \( E_z \) and \( H_z \) need to be determined first and then we can find all other field components.

In cylindrical coordinates, the Helmholtz equation for \( E_z \) is:

\[
\frac{\partial^2 E_z}{\partial r^2} + \frac{1}{r} \frac{\partial E_z}{\partial r} + \frac{1}{r^2} \frac{\partial^2 E_z}{\partial \phi^2} + \frac{\partial^2 E_z}{\partial z^2} + \omega^2 \mu \epsilon E_z = 0 \tag{2.3.17}
\]

Since \( E_z = E_z(r, \phi, z) \) is a function of both \( r, \phi, \text{ and } z \), Equation 2.3.17 cannot be solved analytically. We assume a standing wave in the azimuthal direction and a propagating wave in the longitudinal direction, then the variables can be separated as

\[
E_z(r, \phi, z) = E_z(r)e^{i\beta_z z}
\tag{2.3.18}
\]

where \( l = 0, \pm 1, \pm 2, \ldots \) is an integer. Substituting 2.3.8 into 2.3.7, we can obtain a one-dimensional wave equation:

\[
\frac{\partial^2 E_z(r)}{\partial r^2} + \frac{1}{r} \frac{\partial E_z(r)}{\partial r} + \left( \frac{n^2 \omega^2}{c^2} - \beta_z^2 \right) E_z(r) = 0 \tag{2.3.19}
\]

This is commonly referred to as a Bessel equation because its solutions can be expressed as Bessel functions.

For a step-index fiber with a core radius \( a \), its index profile can be expressed as,

\[
n = \begin{cases} 
n_1 & (r \leq a) \\
n_2 & (r > a) 
\end{cases} \tag{2.3.20}
\]

We have assumed that the diameter of the cladding is infinite in this expression. The Bessel Equation 2.3.19 has solutions only for discrete \( \beta_z \) values, which correspond to discrete modes. The general solutions of Bessel Equation 2.3.19 can be expressed in Bessel functions as,

\[
E_z(r) = \begin{cases} 
AJ_1(U_{in}r) + A'Y_1(U_{in}r) & (r \leq a) \\
CK_1(W_{in}r) + C'I_1(W_{in}r) & (r > a)
\end{cases} \tag{2.3.21}
\]
where \( U_{lm}^2 = \beta_l^2 - \beta_{z,lm}^2 \) and \( W_{lm}^2 = \beta_{z,lm}^2 - \beta_z^2 \) represent equivalent transversal propagation constants in the core and cladding, respectively, with \( \beta_l = n_l \omega / c \) and \( \beta_z = n_z \omega / c \) as defined before. \( \beta_{z,lm} \) is the propagation constant in the \( z \)-direction. This is similar to the vectorial relation of propagation constants shown in Equation 2.3.1 in geometric optics analysis. However, we have two mode indices here, \( l \) and \( m \). The physical meanings of these two mode indices are the amplitude maximums of the standing wave patterns in the azimuthal and the radial directions, respectively.

In Equation 2.3.21, \( J_l \) and \( Y_l \) are the first and the second kind of Bessel functions of the \( l^{th} \) order, and \( K_l \) and \( I_l \) are the first and the second kind of modified Bessel functions of the \( l^{th} \) order. Their values are shown in Figure 2.3.4. \( A \), \( A' \), \( C \), and \( C' \) in Equation 2.3.21 are constants that need to be defined using boundary appropriate conditions.

The first boundary condition is that the field amplitude of a guided mode should be finite at the center of the core \( (r = 0) \). Since the special function \( Y_l(0) = -\infty \), one must set \( A' = 0 \) to ensure that \( E_z(0) \) has a finite value.

The second boundary condition is that the field amplitude of a guided mode should be zero far away from the core \( (r = \infty) \). Since \( I_l(\infty) \neq 0 \), one must set \( C' = 0 \) to ensure that \( E_z(\infty) = 0 \). Consider \( A' = C' = 0 \); Equation 2.3.21 can be simplified, and for the mode index of \( (l, m) \), it becomes:

\[
E_{z,lm}(r, \phi, z) = \begin{cases} 
AJ_l(U_{lm}, r)e^{i\beta_l r} \cdot e^{i\beta_{z,lm} z} & (r \leq a) \\
CK_l(W_{lm}, r)e^{i\beta_{z,lm} r} \cdot e^{i\beta_z z} & (r > a) 
\end{cases} 
\tag{2.3.22a}
\]

Similarly, we can write the magnetic field as,

\[
H_{z,lm}(r, \phi, z) = \begin{cases} 
BJ_l(U_{lm}, r)e^{i\beta_l r} \cdot e^{i\beta_{z,lm} z} & (r \leq a) \\
DK_l(W_{lm}, r)e^{i\beta_{z,lm} r} \cdot e^{i\beta_z z} & (r > a) 
\end{cases} 
\tag{2.3.22b}
\]

And other field components can be found through the Maxwell's equations as [Iizuka, 2002],

\[
E_{r,lm} = \begin{cases} 
\left( \frac{j}{U_{lm}^2} \beta_{z,lm} \frac{\partial E_{z,lm}}{\partial r} + \mu_0 \frac{\omega}{r} \frac{\partial H_{z,lm}}{\partial \phi} \right) & (r \leq a) \\
-\left( \frac{j}{W_{lm}^2} \beta_{z,lm} \frac{\partial E_{z,lm}}{\partial r} + \mu_0 \frac{\omega}{r} \frac{\partial H_{z,lm}}{\partial \phi} \right) & (r > a) 
\end{cases} 
\tag{2.3.23a}
\]
And

and

and

$E_{\phi,lm} = \left\{ \begin{array}{ll}
\frac{j}{U_{lm}^2} \left( \beta_{z,lm} \frac{\partial E_{z,lm}}{\partial r} + \mu_0 \omega \frac{\partial H_{z,lm}}{\partial r} \right) & r \leq a \\
- \frac{j}{W_{lm}^2} \left( \beta_{z,lm} \frac{\partial E_{z,lm}}{\partial r} + \mu_0 \omega \frac{\partial H_{z,lm}}{\partial r} \right) & r > a
\end{array} \right.$  

\begin{equation}
(2.3.23b)
\end{equation}

$H_{r,lm} = \left\{ \begin{array}{ll}
\frac{j}{U_{lm}^2} \left( \beta_{z,lm} \frac{\partial H_{z,lm}}{\partial r} + \epsilon_0 n^2 \omega \frac{\partial E_{z,lm}}{\partial r} \right) & r \leq a \\
- \frac{j}{W_{lm}^2} \left( \beta_{z,lm} \frac{\partial H_{z,lm}}{\partial r} + \epsilon_0 n^2 \omega \frac{\partial E_{z,lm}}{\partial r} \right) & r > a
\end{array} \right.$  

\begin{equation}
(2.3.23c)
\end{equation}

$H_{\phi,lm} = \left\{ \begin{array}{ll}
\frac{j}{U_{lm}^2} \left( \beta_{z,lm} \frac{\partial H_{z,lm}}{\partial r} + \epsilon_0 n^2 \omega \frac{\partial E_{z,lm}}{\partial r} \right) & r \leq a \\
- \frac{j}{W_{lm}^2} \left( \beta_{z,lm} \frac{\partial H_{z,lm}}{\partial r} + \epsilon_0 n^2 \omega \frac{\partial E_{z,lm}}{\partial r} \right) & r > a
\end{array} \right.$  

\begin{equation}
(2.3.23d)
\end{equation}

$E_{z,lm}, H_{z,lm}, E_{\phi,lm},$ and $H_{\phi,lm}$, have to be continuous at the core-cladding interface ($r = a$), which can be satisfied by only a set of discrete values of $U_{lm}$ and $W_{lm}$. This explains the reason why the fiber modes have to be discrete.

Figure 2.3.4 Bessel function (top) and modified Bessel functions (bottom).

Mathematically, the modified Bessel function fits well to an exponential characteristic $K_l(W_{lm}r) \propto \exp(-W_{lm}r)$, so that $K_l(W_{lm}r)$ represents an exponential decay
of optical field over \( r \) in the fiber cladding. For a propagation mode, \( W_{lm} > 0 \) is required to ensure that energy does not leak through the cladding. In the fiber core, the Bessel function \( J_l(U_m r) \) oscillates as shown in Figure 2.3.4, which represents a standing-wave pattern in the core over the radius direction. For a propagating mode, \( U_m \geq 0 \) is required to ensure this standing-wave pattern in the fiber core.

It is interesting to note that based on the definitions of \( U_m^2 = \beta_i^2 - \beta_{z,lm}^2 \) and \( W_m^2 = \beta_{z,lm}^2 - \beta_z^2 \), the requirement of \( W_{lm} > 0 \) and \( U_m \geq 0 \) is equivalent to \( \beta_z^2 < \beta_{z,lm}^2 \leq \beta_i^2 \) or \( (n_z/n_l) < \beta_{z,lm}^2 / \beta_i \leq 1 \). This is indeed equivalent to the mode condition (2.27) derived by the ray optics.

There are a few often used definitions to categorize the propagation modes in the fiber:

- Transverse electric-field mode (TE mode): \( E_z = 0 \)
- Transverse magnetic-field mode (TM mode): \( H_z = 0 \)
- Hybrid mode (HE mode) \( E_z \neq 0 \) and \( H_z \neq 0 \)

\( V \)-number is an important parameter of a fiber, which is defined as

\[
V = a \sqrt{U_m^2 + W_m^2}
\]  
(2.3.24)

since

\[
U_m^2 = \beta_i^2 - \beta_{z,lm}^2 = \left( \frac{2 \pi n_l}{\lambda} \right)^2 - \beta_{z,lm}^2
\]

and

\[
W_m^2 = \beta_{z,lm}^2 - \beta_z^2 = \beta_{z,lm}^2 - \left( \frac{2 \pi n_z}{\lambda} \right)^2
\]

\( V \)-number can be expressed as

\[
V = a \sqrt{U_m^2 + W_m^2} = \frac{2 \pi a}{\lambda} \sqrt{n_l^2 - n_z^2}
\]  
(2.3.25)

In an optical fiber with large core size and large core-cladding index difference, it will support a large number of propagating modes. Approximately, the total number of guided modes in a fiber is related to the \( V \)-number as [Keiser, 2000]
\[ M \approx \frac{V^2}{2} \quad (2.3.26) \]

In a multimode fiber, the number of guided modes can be on the order of several hundred. Imagine that a short optical pulse is injected into a fiber and the optical energy is carried by many different modes. Because different modes have different propagation constants \( \beta_{z,m} \) in the longitudinal direction and they will arrive at the output of the fiber in different times, the short optical pulse at the input will become a broad pulse at the output. In optical communications systems, this introduces signal waveform distortions and bandwidth limitations. This is the reason single-mode fiber is required in high-speed long distance optical systems.

In a single-mode fiber, only the lowest-order mode is allowed to propagate; all higher-order modes are cut off. In a fiber, the lowest-order propagation mode is \( HE_{11} \), whereas the next lowest modes are \( TE_{01} \) and \( TM_{01} \) \((l = 0 \text{ and } m = 1)\). In fact, \( TE_{01} \) and \( TM_{01} \) have the same cutoff conditions: \((1) \, W_{01} = 0 \) so that these two modes radiate in the cladding and \((2) \, J_0 (U_{01}a) = 0 \) so that the field amplitude at core/cladding interface \((r = a)\) is zero. Under the first condition \((W_{01} = 0)\), we can find the cutoff \( V \)-number \( V = a\sqrt{U_{01}^2 + W_{01}^2} = aU_{01} \), whereas under the second condition \((J_0 (U_{01}a) = 0)\), we can find \( J_0(aU_{01}) = J_0(V) = 0 \), which implies that \( V = 2.405 \) as the first root of \( J_0(V) = 0 \).

Therefore, the single-mode condition is

\[ V = 2\pi a \sqrt{n_1^2 - n_2^2} < 2.405 \quad (2.3.27) \]

Note that the approximation given in (2.45) is valid only for a fiber with large number of modes so that \( V >> 2.45 \) is valid. For a few-modes fiber, the number of modes can be counted from Figure 2.3.5 as calculated through the actual solutions of Maxwell's Equations.

2.3.3 Mode Classification

In general, each mode in the fiber has its unique identification number determined by the indices \( l \) and \( m \). Multiple solutions \((m = 1, 2, 3, \ldots)\) can exist for each given \( l \) value. Meridional modes refer to the modes with \( l = 0 \), and the mode field is independent of azimuthal angle \( \phi \). Meridional modes include \( TE_{0m} \) \((E_z = 0)\) and \( TM_{0m} \) \((H_z = 0)\). Skew modes are commonly referred to as hybrid modes for \( l \neq 0 \), and they are often denoted as \( EH_{lm} \) \((E_z \text{ dominant})\) and \( HE_{lm} \) \((H_z \text{ dominant})\).
Figure 2.3.5 Normalized propagation constant $b$ as the function of the $V$-number [Gloge, 1971]

For a guided mode, the projection of the propagation constant in $z$-direction, $\beta_z$, has to satisfy $\beta_1^z < \beta_z^2 \leq \beta_2^z$ (see Figure 2.3.3), where $\beta_1 = k n_1$, $\beta_2 = k n_2$ and $k = 2\pi / \lambda$. This is equivalent to

$$0 < \frac{(\beta_z/k)^2 - n_2^2}{n_1^2 - n_2^2} < 1$$  \hspace{1cm} (2.3.28)

In Figure 2.3.5, the normalized propagation constant is defined as

$$b = \frac{(\beta_z/k)^2 - n_2^2}{n_1^2 - n_2^2}$$  \hspace{1cm} (2.3.29)

Thus, $0 < b < 1$ is the necessary condition for the existence of a propagation mode, and the relation between $b$ and the $V$-number is shown in Figure 2.3.5. Mode cut-off condition is for $b$ to approach zero. Figure 2.3.5 indicates that the lowest order mode in a fiber is $HE_{11}$, which is a hybrid mode corresponding to $l = 1$ and $m = 1$, and the electrical field is only in the transversal direction ($E_z = 0$). High order modes ($TE_{01}$ and $TM_{01}$) start to exist when the $V$-number reaches to 2.405.

For most practical optical fibers, the index contrast between the core and the cladding is small so that $\Delta = (n_1 - n_2)/n_1 << 1$. Under such a weak guidance condition, certain groups of propagating modes have almost identical propagation constants, so that the linear combination of these modes within a group can form a so called LP
mode ("LP" stands for linearly polarized). For example, the combination of TE$_{01}$, TM$_{01}$ and HE$_{21}$ forms the LP$_{11}$ mode; and EH$_{11}$ and HE$_{31}$ forms LP$_{21}$.

Figure 2.3.6 Illustration of transversal electrical (solid lines) and magnetic (dashed lines) fields on the fiber cross section for different fiber modes. The images represent mode field intensity.

Figure 2.3.6 shows the transversal $E$ and $H$ field distributions of HE$_{11}$, TM$_{01}$, TE$_{01}$ and HE$_{21}$. While HE$_{11}$ constitutes the LP$_{01}$ mode by itself as shown by Figure 2.3.6(a), LP$_{11}$ mode is formed by the superposition of HE$_{21}$ and TM$_{01}$, or HE$_{21}$ and TE$_{01}$ as shown in Figure 2.3.6 (b) and (c).

Because of the central symmetry of the fiber cross section, the mode structure will be identical when the fiber is rotated by 90 degrees (exchange $x$ and $y$ axis). Thus, each mode shown in Figure 2.3.6 has a degenerate mode with the orthogonal polarization orientation. As the result, there are 2 LP$_{01}$ modes, 4 LP$_{11}$ modes, 4 LP$_{21}$ modes, and 2 LP$_{02}$ modes. A detailed mode list can be found at [Buck, 2004].

2.3.4 Numerical Aperture
Numerical aperture is a parameter that is often used to specify the acceptance angle of a fiber. Figure 2.3.7 shows an azimuthal cross-section of a step-index fiber and a light ray that is coupled into the fiber from the left side end surface.

Figure 2.3.7 Illustration of light coupling into a step-index fiber.

For the light to be into the guided mode in the fiber, total internal reflection has to occur inside the core and $\theta_i > \theta_c$ is required, as shown in Figure 2.3.7, where $\theta_c = \sin^{-1}(n_2/n_1)$ is the critical angle of the core-cladding interface. With this requirement on $\theta_i$, there is a corresponding requirement on incident angle $\theta_a$ at the fiber end surface. It is easy to see from the drawing that $\sin \theta_i = \sqrt{1 - \sin^2 \theta_i}$, and by Snell’s Law,

$$n_0 \sin \theta_a = n_1 \sqrt{1 - \sin^2 \theta_i}$$  \hspace{1cm} (2.3.30)

If total reflection happens at the core-cladding interface, which requires $\theta_i \geq \theta_c$, then $\sin \theta_i \geq \sin \theta_c = n_2/n_1$. This requires the incidence angle $\theta_a$ to satisfy the following condition:

$$n_0 \sin \theta_a \leq \sqrt{n_1^2 - n_2^2}$$  \hspace{1cm} (2.3.31)

The definition of numerical aperture is

$$NA = \sqrt{n_1^2 - n_2^2}$$  \hspace{1cm} (2.3.32)

For weak optical waveguide like a single-mode fiber, the difference between $n_1$ and $n_2$ is very small (not more than 1 percent). Use $\Delta = (n_1 - n_2)/n_1$ to define a normalized index difference between core and cladding, then $\Delta$ must also be very small ($\Delta \ll 1$). In this case, the expression of numerical aperture can be simplified as
In most cases fibers are placed in air and \( n_0 = 1 \). \( \sin \theta_a \approx \theta_a \) is valid when \( \sin \theta_a \ll 1 \) (weak waveguide); therefore, Equation 2.3.31 reduces to

\[
\theta_a \leq n_1 \sqrt{2 \Delta} = NA
\]  

(2.3.34)

Figure 2.3.8 Light can be coupled to an optical fiber only when the incidence angle is smaller than the numerical aperture.

From this discussion, the physical meaning of numerical aperture is very clear. Light entering a fiber within a cone of acceptance angle, as shown in Figure 2.3.8, will be converted into guided modes and will be able to propagate along the fiber. Outside this cone, light coupled into fiber will radiate into the cladding. Similarly, light exits a fiber will have a divergence angle also defined by the numerical aperture. This is often used to design focusing optics if a collimated beam is needed at the fiber output.

Typically parameters of a single-mode fiber are \( NA \approx 0.1 \sim 0.2 \) and \( \Delta \approx 0.2\% \sim 1\% \). Therefore, \( \theta_a \approx \sin^{-1}(NA) \approx 5.7^\circ \sim 11.5^\circ \). This is a very small angle and it makes difficult to couple light into a single-mode fiber. Not only that, the source spot size has to be small (~80\( \mu \)m\(^2\)) but also the angle has to be within \( \pm 10 \) degrees.

With the definition of the numerical aperture in Equation 2.3.32, the \( V \)-number of a fiber can be expressed as a function of \( NA \):

\[
V = \frac{2\pi a}{\lambda} NA
\]  

(2.3.35)
Another important fiber parameter is the cutoff wavelength $\lambda_c$. It is defined such that the second lowest mode ceases to exist when the signal wavelength is longer than $\lambda_c$, and therefore when $\lambda < \lambda_c$ a single-mode fiber will become multimode. According to Equation 2.3.27, cutoff wavelength is

$$\lambda_c = \frac{\pi d}{2.405} NA$$  \hspace{1cm} (2.3.36)

where $d$ is the core diameter of the step-index fiber. As an example, for a typical standard single-mode fiber with, $n_1 = 1.47$, $n_2 = 1.467$, and $d = 9$ $\mu$m, the numerical aperture is

$$NA = \sqrt{n_1^2 - n_2^2} = 0.0939$$

The maximum incident angle at the fiber input is

$$\theta_a = \sin^{-1}(0.0939) = 5.38^\circ$$

and the cutoff wavelength is

$$\lambda_c = \frac{\pi d \cdot NA}{2.405} = 1.1\mu m$$

**Example 2.2**

To reduce the Fresnel reflection, the end surface of a fiber connector can be made non-perpendicular to the fiber axis. This is usually referred to as APC (angled physical contact) contactor. If the fiber has the core index $n_1 = 1.47$ and cladding index $n_2 = 1.467$, what is the minimum angle $\phi$ such that the Fresnel reflection by the fiber end facet will not become the guided fiber mode?

**Solution:** To solve this problem, we use ray trace method and consider three extreme light beam angles in the fiber. The surface has to be designed such that after reflection at the fiber end surface, all these three light beams will not be coupled into fiber-guided mode in the backward propagation direction.

As illustrated in Figure 2.3.9(a), first, for the light beam propagating in the fiber axial direction ($z$-direction), the direction of the reflected beam from the end surface has an angle $\theta$ in respect to the surface normal of the fiber sidewall: $\theta = \pi / 2 - 2\phi < \theta_a$. In order for this reflected light beam not to become the guided
mode of the fiber, \( \theta < \theta_c \) is required, where \( \theta_c \) is the critical angle defined by Equation 2.2.14. Therefore, the first requirement for \( \phi \) is

\[
\phi > \pi / 4 - \theta_c / 2
\]

(2.3.37)

Figure 2.3.9 Illustration of an angle-polished fiber surface.

Second, for the light beam propagating at the critical angle of the fiber as shown in Figure 2.3.9(b), the beam has an angle \( \theta_1 \) with respect to the surface normal of the fiber end surface, which is related to \( \phi \) by \( \theta_1 = (\pi / 2 - \theta_c) + \phi \). Then the \( \theta \) angle of the reflected beam from the end surface with respect to the fiber sidewall can be found as \( \theta = \pi - \theta_c - 2\theta_1 = \theta_c - 2\phi \). This angle also has to be smaller than the critical angle, that is, \( \theta_c - 2\phi < \theta_c \), or

\[
\phi > 0
\]

(2.3.38)

In the third case as shown in Figure 2.3.9(c), the light beam propagates at the critical angle of the fiber but at the opposite side as compared to the ray trace shown in Figure 2.3.9(b). This produces the smallest \( \theta_1 \) angle, which is, \( \theta_1 = \phi - (\pi / 2 - \theta_c) \).
This corresponds to the biggest \( \theta \) angle, \( \theta = \pi - \theta_c - 2 \theta_t = 2 \pi - 3 \theta_c - 2 \phi \). Again, this \( \theta \) angle has to be smaller than the critical angle, \( \theta < \theta_c \), that is,

\[
\phi > \pi - 2 \theta_c \tag{2.3.39}
\]

Since in this example

\[
\theta_c = \sin^{-1}\left( \frac{n_2}{n_1} \right) = \sin^{-1}\left( \frac{1.467}{1.47} \right) = 86.34^\circ \tag{2.3.40}
\]

The three constraints given by Equations (2.3.37), (2.3.38), and (2.3.39) become \( \phi > 1.83^\circ \), \( \phi > 0 \), and \( \phi > 7.3^\circ \), respectively. Obviously, in order to satisfy all these three specific conditions, the required surface angle is \( \phi > 7.3^\circ \). In fact, as an industry standard, commercial APC connectors usually have the surface tilt angle of \( \phi = 8^\circ \).

2.3.5 Field distribution profile of single mode fiber

When \( V \leq 2.405 \) is satisfied, the fiber only supports a single mode, which is the LP01 mode illustrated in Figure 2.3.6. A stand single mode fiber (SMF), such as Corning SMF-28, has the core diameter of \( d = 8 \) \( \mu m \) and the normalized core/cladding index difference \( \Delta n = (n_1-n_2)/n_2 \) of approximately 0.35\%. Because of the circular geometry of an optical fiber, the field distribution of the fundamental mode in a single-mode optical fiber is circularly symmetrical, which can be specified by a single parameter known as the mode-field diameter (MFD) and the electrical field distribution can often be assumed as Gaussian:

\[
E(r) = E_0 \exp\left( -\frac{r^2}{W_0^2} \right) \tag{2.3.41}
\]

where \( r \) is the radius, \( E_0 \) is the optical field at \( r = 0 \), and \( W_0 \) is the width of the field distribution. Specifically, the MFD is defined as \( 2W_0 \), which is

\[
2W_0 = 2 \left[ \frac{\int_0^\infty r^2 E^2(r)dr}{\int_0^\infty r E^2(r)dr} \right]^{1/2} \tag{2.3.42}
\]
Figure 2.3.10 illustrates the mode-field distribution of a single mode fiber in which Gaussian approximation is used. The physical meaning of the MFD definition given by Equation 2.3.42 can be explained as follows: The denominator in Equation 2.3.42 is proportional to the integration of the power density across the entire fiber cross-section, which is the total power of the fundamental mode, whereas the numerator is the integration of the square of the radial distance \( r^2 \) weighted by the power density over the fiber cross-section. Therefore MFD defined by Equation 2.3.42 represents a root mean square \((rms)\) value of the mode distribution of the optical field on the fiber cross-section. Mode field radius \( W_0 \) is proportional to the geometric core radius, \( a \), of the fiber, but they are not equal. In fact, within \( 1.2 < V < 2.4 \), mode field radius can be approximated as \([D. Marcuse 1978]\),

\[
W_0 \approx a \left( 0.65 + 1.619V^{-3/2} + 2.879V^{-6} \right)
\]  

(2.3.43)

It is important to point out that mode-field distribution \( E(r) \) in Equation 2.3.41 represents the field distribution inside the fiber; thus it is equivalent to the optical field distribution exactly on the output surface of the fiber. It is commonly referred to as the near-field (NF) distribution. Near-field distribution is a very important parameter of the fiber that determines the effective cross-section area of the fiber as

\[
A_{\text{eff}} = \frac{2\pi \left[ \int_0^2 r |E(r)|^2 rdr \right]^2}{\int_0^2 r |E(r)|^4 rdr}
\]  

(2.3.44)
If the total optical power $P$ carried by the fiber is known, the power density in the fiber core can be determined by using the effective cross-section area as $I_{\text{density}} = P / A_{\text{eff}}$. This effective cross-section area will be used later in this chapter when discussing fiber nonlinearities.

### 2.4 Optical fiber attenuation

Optical fiber is an ideal medium that can be used to carry optical signals over long distances. Attenuation is one of the most important parameters of an optical fiber; it, to a large extent, determines how far an optical signal can be delivered at a detectable power level. There are several sources that contribute to fiber attenuation, such as absorption, scattering, and radiation.

Material absorption is mainly caused by photo-induced molecular vibration, which absorbs signal optical power and turns it into heat. Pure silica molecules absorb optical signals in ultraviolet (UV) and infrared (IR) wavelength bands. At the same time, there are often impurities inside silica material such as OH$^-$ ions, which may be introduced in the fiber perform fabrication process. These impurity molecules create additional absorption in the fiber. Typically, OH$^-$ ions have high absorptions around 700nm, 900nm, and 1400nm, which are commonly referred to as *water absorption peaks*.

Scattering loss arises from microscopic defects and structural inhomogeneities. In general, the optical characteristics of scattering depend on the size of the scatter in comparison to the signal wavelength. However, in optical fibers, the scatters are most likely much smaller than the wavelength of the optical signal, and in this case the scattering is often characterized as *Rayleigh scattering*. A very important spectral property of Rayleigh scattering is that the scattering loss is inversely proportional to the fourth power of the wavelength. Therefore, Rayleigh scattering loss is high in a short wavelength region.

In the last few decades, the loss of optical fiber has been decreased significantly by reducing the OH$^-$ impurity in the material and eliminating defects in the structure. However, absorption by silica molecules in the UV and IR wavelength regions and Rayleigh scattering still constitute fundamental limits to the loss of silica-based optical fiber. Figure 2.4.1 shows the typical absorption spectra of silica fiber. The dotted line shows the attenuation of old fibers that were made before the 1980s. In
addition to strong water absorption peaks, the attenuation is generally higher than new fiber due to material impurity and waveguide scattering. Three wavelength windows have been used since the 1970s for optical communications in 850nm, 1310nm, and 1550nm where optical attenuation has local minimums. In the early days of optical communication, the first wavelength window in 850nm was used partly because of the availability of GaAs-based laser sources, which emit in that wavelength window. The advances in longer wavelength semiconductor lasers based on InGaAs and InGaAsP pushed optical communications toward the second and the third wavelength windows in 1310nm and 1550nm where optical losses are significantly reduced and optical systems can reach longer distances without regeneration.

Another category of optical loss that may occur in optical fiber cables is radiation loss. It is mainly caused by fiber bending. Micro bending, usually caused by irregularities in the pulling process, may introduce coupling between the fundamental optical mode and high-order radiation modes and thus creating losses. On the other hand, macro bending, often introduced by cabling processing and fiber handling, causes the spreading of optical energy from fiber core into the cladding. For example, for a standard single-mode fiber, bending loss starts to be significant when the bending diameter is smaller than approximately 30cm.

Mathematically, the complex representation of an optical wave propagating in *z*-direction is
\[ E(z,t) = E_0 \exp[-j(\omega t - kz)] \]  \hspace{1cm} (2.4.1)

where \( E_0 \) is the complex amplitude, \( \omega \) is the optical frequency and \( k \) is the propagation constant. Considering attenuation in the medium, the propagation constant should be complex:

\[ k = \beta + j \frac{\alpha}{2} \]  \hspace{1cm} (2.4.2)

where \( \beta = \frac{2\pi}{\lambda} \) is the real propagation constant and \( \alpha \) is the power attenuation coefficient. By separating the real and the imaginary parts of the propagation constant, Equation 2.4.1 can be written as

\[ E(z,t) = E_0 \exp[-j(\omega t - \beta z)] \cdot \exp\left(-\frac{\alpha}{2}z\right) \]  \hspace{1cm} (2.4.3)

The average optical power can be simply expressed as

\[ P(z) = P_0 e^{-\alpha z} \]  \hspace{1cm} (2.4.4)

where \( P_0 \) is the input optical power. Note here the unit of \( \alpha \) is in Neper per meter.

This attenuation coefficient \( \alpha \) of an optical fiber can be obtained by measuring the input and the output optical power:

\[ \alpha = \frac{1}{L} \ln \left( \frac{P_0}{P(L)} \right) \]  \hspace{1cm} (2.4.5)

where \( L \) is the fiber length and \( P(L) \) is the optical powers measured at the output of the fiber.

However, engineers like use decibel \((dB)\) to describe fiber attenuation and use \(dB/km\) as the unit of attenuation coefficient. If we define \( \alpha_{dB} \) as the attenuation coefficient which has the unit of \( dB/km \), then the optical power level along the fiber length can be expressed as

\[ P(z) = P_0 \times 10^{\frac{\alpha_{dB} z}{10}} \]  \hspace{1cm} (2.4.6)

Similar to Equation 2.4.5, for a fiber of length \( L \), \( \alpha_{dB} \) can be estimated using

\[ \alpha_{dB} = \frac{1}{L} 10 \log_{10} \left( \frac{P_0}{P(L)} \right) \]  \hspace{1cm} (2.4.7)
Comparing Equations 2.4.5 and 2.4.7, the relationship between $\alpha$ and $\alpha_{dB}$ can be found as

$$\frac{\alpha_{dB}}{\alpha} = \frac{10\log_{10}[P_0 / P(L)]}{\ln[P_0 / P(L)]} = 10\log(e) = 4.343 \quad (2.4.8)$$

or simply, $\alpha_{dB} = 4.343\alpha$.

$\alpha_{dB}$ is a simpler parameter to use for evaluation of fiber loss. For example, for an 80km-long fiber with $\alpha_{dB} = 0.25\text{dB/km}$ attenuation coefficient, the total fiber loss can be easily found as $80 \times 0.25 = 20\text{dB}$. On the other hand, if complex optical field expression is required to solve wave propagation equations, $\alpha$ needs to be used instead. In practice, people always use the symbol $\alpha$ to represent optical-loss coefficient, no matter in [Neper/m] or in [dB/km]. But one should be very clear which unit to use when it comes to finding numerical values.

The following is an interesting example that may help to understand the impact of fiber numerical aperture and attenuation.

**Example 2.3**

A very long step-index, single-mode fiber has a numerical aperture $NA = 0.1$ and a core refractive index $n_1 = 1.45$. Assume that the fiber end surface is ideally antireflection coated and fiber loss is only caused by Rayleigh scattering. Find the

Figure 2.4.2 Illustrations of fiber reflection (a) and scattering in fiber core (b).
reflectivity of the fiber $R_{ref} = P_b / P_i$ where $P_i$ is the optical power injected into the fiber and $P_b$ is the optical power that is reflected back to the input fiber terminal, as illustrated in Figure 2.4.2(a).

**Solution:**

In this problem, Rayleigh scattering is the only source of attenuation in the fiber. Each scattering source scatters the light into all directions uniformly, which fills the entire $4\pi$ solid angle. However, only a small part of the scattered light with the angle within the numerical aperture can be converted into guided mode within the fiber core and travels back to the input. The rest will be radiated into cladding and lost. For simplicity, we assume that scatters are only located along the center of the fiber core, as shown in Figure 2.4.2(b). Therefore the conversion efficiency from the scattered light to that captured by the fiber is

$$
\eta = \frac{2\pi(1-\cos \theta_i)}{4\pi} \quad (2.4.9)
$$

where the numerator is the area of the spherical cap shown as the shaded area in Figure 2.4.2(b), whereas the denominator $4\pi$ is the total area of the unit sphere. $\theta_i$ is the maximum trace angle of the guided mode with respect to the fiber axis. Since the numerical aperture is defined as the maximum acceptance angle $\theta_a$ at the fiber entrance as given in Equation 2.3.34, $\theta_i$ can be expressed as a function of $NA$ as

$$
\theta_i = \frac{n_0}{n_i} NA = \frac{1}{1.45} \times 0.1 = 0.069 \quad (2.4.10)
$$

where $n_0 = 1$ is used for the index of air. Substitute the value of $\theta_i$ into Equation 2.4.9, the ratio between the captured and the total scattered optical power can be found as $\eta = 1.19 \times 10^{-3} = -29 dB$.

Now let’s consider a short fiber section $\Delta z$ located at position $z$ along the fiber. The optical power at this location is $P(z) = P_i e^{-\alpha z}$, where $\alpha$ is the fiber attenuation coefficient. The optical power loss within this section is

$$
\Delta P(z) = \frac{dP(z)}{dz} \Delta z = -\alpha P_i e^{-\alpha z} \Delta z \quad (2.4.11)
$$
Since we assumed that the fiber loss is only caused by Rayleigh scattering, this power loss $\Delta P(z)$ should be equal to the total scattered power within the short section. $\eta \Delta P(z)$ is the amount of scattered power that is turned into the guided mode that travels back to the fiber input.

However, during the traveling from location $z$ back to the fiber input, attenuation also applies, and the total amount of power loss is, again, $e^{-\alpha z}$. Considering that the fiber is composed of many short sections and adding up the contributions from all sections, the total reflected optical power is

$$P_r = \sum \eta |\Delta P(z)| e^{-\alpha z} = \int_0^\infty \eta \alpha P e^{-2\alpha z} dz = \frac{\eta P_0}{2}$$

(2.4.12)

Therefore the reflectivity is

$$R_{ref} = \frac{P_r}{P_i} = \frac{\eta}{2} = 5.95 \times 10^{-4} = -32dB$$

(2.4.13)

This result looks surprisingly simple. The reflectivity, sometimes referred to as return loss, only depends on the fiber numerical aperture and is independent of the fiber loss. The physical explanation is that since Rayleigh scattering is assumed to be the only source of loss, increasing the fiber loss will increase both scattered signal generation and its attenuation. In practical single-mode fibers, this approximation is quite accurate, and the experimentally measured return loss in a standard single-mode fiber is between $-31dB$ and $-34dB$.

2.5 Group velocity and dispersion

When an optical signal propagates along an optical fiber, not only is the signal optical power attenuated but also different frequency components within the optical signal propagate in slightly different speeds. This frequency-dependency of propagation speed is commonly known as the chromatic dispersion.

2.5.1 Phase velocity and group velocity

Neglecting attenuation, the electric field of a single-frequency plane optical wave propagating in $z$-direction is often expressed as

$$E(z,t) = E_0 \exp[-j\Phi(t,z)]$$

(2.5.1)
where $\Phi(t,z) = (\omega_t t - \beta_0 z)$ is the optical phase, $\omega_0$ is the optical angular frequency, and $\beta_0 = 2\pi n/\lambda = n\omega_0/c$ is the propagation constant.

The phase front of this light wave is the plane where the optical phase is constant:

$$ (\omega_t t - \beta_0 z) = \text{constant} \quad (2.5.2) $$

The propagation speed of the phase front is called *phase velocity*, which can be obtained by differentiating both sides of Equation 2.5.2:

$$ v_p = \frac{dz}{dt} = \frac{\omega_0}{\beta_0} \quad (2.5.3) $$

Now consider that this single-frequency optical wave is modulated by a sinusoid signal of frequency $\Delta\omega$. Then the electrical field is

$$ E(z,t) = E_0 \exp\left[- j(\omega_0 t - \beta_0 z)\right] \cos(\Delta\omega t) \quad (2.5.4) $$

This modulation splits the signal frequency light wave into two frequency components. At the input ($z = 0$) of the optical fiber, the optical field is

$$ E(0,t) = E_0 e^{-j\omega_0 t} \cos(\Delta\omega t) = \frac{1}{2} E_0 \left( e^{-j(\omega_0 + \Delta\omega) t} + e^{-j(\omega_0 - \Delta\omega) t} \right) \quad (2.5.5) $$

Since wave propagation constant $\beta = n\omega/c$ is linearly proportional to the frequency of the optical signal, the two frequency components at $\omega_0 \pm \Delta\omega$ will have two different propagation constants $\beta_0 \pm \Delta\beta$. Therefore, the general expression of the optical field is

$$ E(z,t) = \frac{1}{2} E_0 \left\{ e^{-j[(\omega_0 + \Delta\omega) t - (\beta_0 + \Delta\beta) z]} + e^{-j[(\omega_0 - \Delta\omega) t - (\beta_0 - \Delta\beta) z]} \right\} \quad (2.5.6) $$

where $E_0 e^{-j(\omega_0 t - \beta_0 z)}$ represents an optical carrier, which is identical to that given in Equation 2.5.4, whereas $\cos(\Delta\omega t - \Delta\beta z)$ is an envelope that is carried by the optical carrier. In fact, this envelope represents the information that is modulated onto the optical carrier. The propagation speed of this information-carrying envelope is called *group velocity*. Similar to the derivation of phase velocity, one can find group velocity by differentiating both sides of $(\Delta\omega t - \Delta\beta z) = \text{constant}$, which yields

$$ v_g = \frac{dz}{dt} = \Delta\omega / \Delta\beta $$
With infinitesimally low modulation frequency, \( \Delta \omega \rightarrow d\omega \) and \( \Delta \beta \rightarrow d\beta \), so the general expression of group velocity is

\[
v_g = \frac{d\omega}{d\beta}
\]  

(2.5.7)

In a nondispersive medium, the refractive index \( n \) is a constant that is independent of the frequency of the optical signal. In this case, the group velocity is equal to the phase velocity: \( v_g = v_p = n/c \). However, in many practical optical materials, the refractive index \( n(\omega) \) is a function of the optical frequency and therefore \( v_g \neq v_p \) in these materials.

Over a unit length of 1 meter, the propagation phase delay is equal to the inverse of the phase velocity:

\[
\tau_p = \frac{1}{v_p} = \frac{\beta_0}{\omega_0}
\]  

(2.5.8)

And similarly, the propagation group delay over a 1 meter length is defined as the inverse of the group velocity:

\[
\tau_g = \frac{1}{v_g} = \frac{d\beta}{d\omega}
\]  

(2.5.9)

2.5.2 Group velocity dispersion

To understand group velocity dispersion, we consider that two sinusoids with the frequencies \( \Delta \omega \pm \delta\omega/2 \) are modulated onto an optical carrier of frequency \( \omega_0 \). When propagating along a fiber, each modulating frequency will have its own group velocity; then over a unit fiber length, the group delay difference between these two frequency components can be found as

\[
\delta \tau_g = \frac{d\tau_g}{d\omega} \delta\omega = \frac{d}{d\omega} \left( \frac{d\beta}{d\omega} \right) \delta\omega = \frac{d^2\beta}{d\omega^2} \delta\omega
\]  

(2.5.10)
Figure 2.5.1 Spectrum of two-tone modulation on an optical carrier, where $\omega_0$ is the carrier frequency and $\Delta \omega \pm \delta \omega / 2$ are the modulation frequencies.

Obviously, this group delay difference is introduced by the frequency dependency of the propagation constant. In general, the frequency-dependent propagation constant $\beta(\omega)$ can expanded in a Taylor series around a central frequency $\omega_0$:

$$
\beta(\omega) = \beta(\omega_0) + \frac{d\beta}{d\omega}|_{\omega=\omega_0}(\omega-\omega_0) + \frac{1}{2} \frac{d^2\beta}{d\omega^2}|_{\omega=\omega_0}(\omega-\omega_0)^2 + \ldots \tag{2.5.11}
$$

$$
= \beta(\omega_0) + \beta_1(\omega-\omega_0) + \frac{1}{2} \beta_2(\omega-\omega_0)^2 + \ldots
$$

where

$$
\beta_1 = \frac{d\beta}{d\omega} \tag{2.5.12}
$$

represents the group delay and

$$
\beta_2 = \frac{d^2\beta}{d\omega^2} \tag{2.5.13}
$$

is the group delay dispersion parameter.

If the fiber is uniform with length $L$, use Equation 2.5.10, we can find the relative time delay between two frequency components separated by $\delta \omega$ as

$$
\Delta \tau_g = \delta \tau_g L = \beta_2 L \delta \omega \tag{2.5.14}
$$

Sometimes it might be convenient to use wavelength separation $\delta \lambda$ instead of frequency separation $\delta \omega$ between the two frequency (or wavelength) components. In this case, the relative delay over a unit fiber length can be expressed as

$$
\delta \tau_g = \frac{d \tau_g}{d \lambda} \delta \lambda \equiv D \delta \lambda \tag{2.5.15}
$$
where \( D = d\tau_g / d\lambda \) is another group delay dispersion parameter. The relationship between these two dispersion parameters \( D \) and \( \beta_2 \) can be found as

\[
D = \frac{d\tau_g}{d\lambda} = \frac{d\omega}{d\lambda} \cdot \frac{d\tau_g}{d\omega} = -\frac{2\pi c}{\lambda^2} \beta_2
\]

(2.5.16)

For a fiber of length \( L \), we can easily find the relative time delay between two wavelength components separated by \( \delta\lambda \) as

\[
\Delta\tau_g = D \cdot L \cdot \delta\lambda
\]

(2.5.17)

In practical fiber-optic systems, the relative delay between different wavelength components is usually measured in picoseconds; wavelength separation is usually expressed in nanometers and fiber length is usually measured in kilometers. Therefore, the most commonly used units for \( \beta_1, \beta_2, \) and \( D \) are \([\text{ps/nm}], [\text{ps}^2/\text{km}], \) and \([\text{ps/nm-km}], \) respectively.

2.5.3 Sources of chromatic dispersion

The physical reason of chromatic dispersion is the wavelength-dependent propagation constant \( \beta(\lambda) \). Both material property and waveguide structure may contribute to this wavelength dependency of \( \beta(\lambda) \), which are referred to as material dispersion and waveguide dispersion, respectively.

**Material dispersion** is originated by the wavelength dependent material refractive index \( n = n(\lambda) \); thus the wavelength dependent propagation constant is

\[
\beta(\lambda) = 2\pi n(\lambda) / \lambda.
\]

For a unit fiber length, the wavelength-dependent group delay is

\[
\tau_g = \frac{d\beta(\lambda)}{d\omega} = -\left( \frac{\lambda^2}{2\pi} \right) \frac{d\beta(\lambda)}{d\lambda} = -\frac{1}{c} \left[ n(\lambda) - \frac{\lambda}{\lambda} \frac{dn(\lambda)}{d\lambda} \right]
\]

(2.5.18)

The group delay dispersion between two wavelength components separated by \( \delta\lambda \) is then

\[
\delta\tau_g = \frac{d\tau_g}{d\lambda} \delta\lambda = -\frac{1}{c} \left[ \frac{\lambda}{\lambda} \frac{d^2 n(\lambda)}{d\lambda^2} \right] \delta\lambda
\]

(2.5.19)

Therefore material induced dispersion is proportional to the second derivative of the refractive index.
Waveguide dispersion can be explained as the wavelength-dependent angle of the light ray propagating inside the fiber core, as illustrated by Figure 2.3.3. Under weak guidance condition, the normalized propagation constant defined in Equation 2.3.29 can be linearized as,

$$b = \frac{\left(\beta_z / k\right)^2 - n_z^2}{n_1^2 - n_z^2} \approx \frac{\left(\beta_z / k\right) - n_z}{n_1 - n_z}$$

(2.5.20)

The actual propagation constant in the $z$-direction, $\beta_z$ can be expressed as a function of $b$ as

$$\beta_z(\lambda) = kn_z(b \Delta + 1)$$

(2.5.21)

where $\Delta = (n_1 - n_z)/n_2$ is the normalized index difference between the core and the cladding. Then the group delay can be found as

$$\tau_g = \frac{d\beta_z(\lambda)}{d\omega} = \frac{n_z}{c} \left(1 + b \Delta + k \frac{db}{dk} \Delta\right)$$

Figure 2.3.5 shows the variation of $b$ as a function of $V$ for different modes. Since $V$ is inversely proportional to $\lambda$, as defined by Equation 2.3.25, $\tau_g$ varies with $\lambda$ as well.

In general, material dispersion is difficult to modify, because doping other materials into silica might introduce excess attenuation, but waveguide dispersion can be modified by index profile design.

The overall chromatic dispersion in an optical fiber is the combination of material dispersion and waveguide dispersion. In general, different types of fiber have different dispersion characteristics. However, for a standard single-mode fiber, the dispersion parameter $D$ can usually be described by a Sellmeier equation [Meumann, 1988]:

$$D(\lambda) = \frac{S_0}{4} \left(\lambda - \frac{\lambda_0^4}{\lambda^2}\right)$$

(2.5.22)

where $S_0$ is the dispersion slope, which ranges from 0.08 to 0.1 ps/nm$^2$-km and $\lambda_0$ is the zero-dispersion wavelength, which is around 1315nm.

Figure 2.5.2(a) shows the dispersion parameter $D$ versus wavelength for standard single-mode fiber, which has dispersion slope $S_0 = 0.09$ ps/nm$^2$-km and zero-dispersion wavelength $\lambda_0 = 1315$nm. $D(\lambda)$ is generally nonlinear; however, if we are
only interested in a relatively narrow wavelength window, it is often convenient to linearize this parameter. For example, if the central frequency of an optical signal is at \( \lambda = \lambda_a \), then \( D(\lambda) \) can be linearized in the vicinity of \( \lambda_a \) as

\[
D(\lambda) \approx D(\lambda_a) + S(\lambda_a) \cdot (\lambda - \lambda_a)
\]  

(2.5.23)

where

\[
D(\lambda_a) = \frac{S_0}{4} \left( \lambda - \frac{\lambda_a^4}{\lambda_a^2} \right)
\]  

(2.5.24)

and the local dispersion slope at \( \lambda_a \) is

\[
S(\lambda_a) = \frac{S_0}{4} \left( 1 + \frac{3\lambda_a^2}{\lambda_a^4} \right)
\]  

(2.5.25)

Figure 2.5.2 (a) Chromatic dispersion \( D \) versus wavelength and (b) relative group delay versus wavelength. \( S_0 = 0.09 \text{ ps/nm}^2\text{-km}, \lambda_0 = 1315\text{nm}. \)

In general, \( S(\lambda_a) \neq S_0 \), except when the optical signal is near the zero-dispersion wavelength \( \lambda_a = \lambda_0 \).

As a consequence of wavelength-dependent dispersion parameter, the group delay is also wavelength-dependent. Considering the definition of dispersion \( D = d\tau_g / d\lambda \) as given by Equation 2.5.10, the wavelength-dependent group delay \( \tau_g(\lambda) \) can be found by integrating \( D(\lambda) \) over wavelength. Based on Equation 2.5.22, the group delay can be derived as

\[
\tau_g(\lambda) = \int D(\lambda) d\lambda = \tau_0 + \frac{S_0}{8} \left( \lambda - \frac{\lambda_0^2}{\lambda} \right)^2
\]  

(2.5.26)
Figure 2.5.2(b) shows the relative group delay $\Delta \tau_g(\lambda) = \tau_g(\lambda) - \tau_0$ versus wavelength. The group delay is not sensitive to wavelength change around $\lambda = \lambda_0$ because where the dispersion is zero.

2.5.4 Modal dispersion

Chromatic dispersion discussed earlier specifies wavelength-dependent group velocity within one optical mode. If a fiber has more than one mode, different modes will also have different propagation speeds; this is called modal dispersion. In a multimode fiber, the effect of modal dispersion is typically much stronger than the chromatic dispersion within each mode; therefore chromatic dispersion is usually neglected.

Modal dispersion depends on the number of propagation modes that exist in the fiber, which, in turn, is determined by the fiber core size and the index difference between the core and the cladding. By using geometric optics analysis as described in Section 2.3, we can find the delay difference between the fastest propagation mode and the slowest propagation mode. Obviously, the fastest mode is the one that travels along the fiber longitudinal axis, whereas the ray trace of the slowest mode has the largest angle with the fiber longitudinal axis or the smallest $\theta_i$ shown in Figure 2.3.2(b). This smallest angle $\theta_i$ is limited by the condition of total reflection at the core-cladding interface, $\theta_i > \sin^{-1}(n_2/n_1)$. Since the group velocity of the fastest ray trace is $c/n_1$ (here we assume $n_1$ is a constant), the group velocity of the slowest ray trace should be $(c/n_1)\sin \theta_i = (cn_2/n_1^2)$. Therefore, for a fiber of length $L$, the maximum group delay difference is approximately

$$\delta T_{\text{max}} = \frac{n_1 L}{c} \left( \frac{n_1 - n_2}{n_2} \right)$$

(2.5.27)

This expression does not consider the core size of the fiber, and therefore it only provides an absolute maximum of the modal dispersion.

2.5.5 Polarization mode dispersion (PMD)

Polarization mode dispersion is a special type of modal dispersion that exists in single mode fibers. It is worth noting that there are actually two fundamental modes that coexist in a single mode fiber. As shown in Figure 2.5.3, these two modes are orthogonally polarized. In an optical fiber with perfect cylindrical symmetry, these
two modes have the same cutoff condition and they are referred to as degenerate modes.

Figure 2.5.3 Illustration of optical field vector across the core cross-section of a single-mode fiber. Two degenerate modes exist in a single-mode fiber.

However, practical optical fibers might not have perfect cylindrical symmetry due to birefringence; therefore these two fundamental modes may propagate in different speeds. Birefringence in an optical fiber is usually caused by small perturbations of the structure geometry as well as the anisotropy of the refractive index. The sources of the perturbations can be categorized as intrinsic and extrinsic. **Intrinsic perturbation** refers to permanent structural perturbations of fiber geometry, which are often caused by errors in the manufacturing process. The effect of intrinsic perturbation include (1) noncircular fiber core, which is called *geometric birefringence* and (2) nonsymmetric stress originated from the non-ideal perform, which is usually called *stress birefringence*. On the other hand, **extrinsic perturbation** usually refers to perturbations due to random external forces in the cabling and installation process. Extrinsic perturbation also causes both geometric and stress birefringence.

The effect of birefringence is that the two orthogonal polarization modes $HE_{11}^{x}$ and $HE_{11}^{y}$ experience slightly different propagation constants when they travel along the fiber; therefore their group delays become different. Assuming that the effective indices in the core of a birefringence fiber are $n_x$ and $n_y$ for the two polarization modes, their corresponding propagation constants will be $\beta_x = \omega n_x / c$ and $\beta_y = \omega n_y / c$, respectively. Due to birefringence, $\beta_x$ and $\beta_y$ are not equal and their difference is

$$\Delta \beta = (\beta_x - \beta_y) = \frac{\omega}{c} \Delta n_{\text{eff}}$$

(2.5.28)
where $\Delta n_{\text{eff}} = n_\parallel - n_\perp$ is the effective differential refractive index of the two modes.

For a fiber of length $L$, the relative group delay between the two orthogonal polarization modes is

$$\Delta \tau_g = \left(\frac{n_\parallel - n_\perp}{c}\right) L = \frac{L \Delta n_{\text{eff}}}{c} \quad \text{(2.5.29)}$$

This is commonly referred to as differential group delay (DGD).

As a result of fiber birefringence, the state of polarization of the optical signal will rotate while propagating along the fiber because of the accumulated relative phase change $\Delta \Phi$ between the two polarization modes:

$$\Delta \Phi = \frac{\omega \Delta n_{\text{eff}}}{c} L \quad \text{(2.5.30)}$$

According to Equation 2.5.30, when an optical signal is launched into a birefringence fiber, its polarization evolution can be accomplished by the changes of either the fiber length $L$, the differential refractive index $\Delta n_{\text{eff}}$, or the lightwave signal angular frequency $\omega$.

At a certain fiber length $L = L_p$, if the state of polarization of the optical signal completes a full $\Delta \Phi = 2\pi$ rotation, $L_p$ is therefore defined as the birefringence beat-length. On the other hand, at a fixed fiber length $L$, the polarization state of the optical signal can also be varied by changing the frequency. For a complete polarization rotation, the change of optical frequency should be

$$\Delta \omega_{\text{cycle}} = \frac{2\pi}{L \Delta n_{\text{eff}}} = \frac{2\pi}{\Delta \tau_g} \quad \text{(2.5.31)}$$

When the fiber length is short enough, energy coupling between the two orthogonally polarized modes is negligible, and the DGD shown in Equation (2.5.29) is linearly proportional to the fiber length $L$. (Equation (2.5.29) is also valid for polarization-maintaining fiber in which there no coupling between the two modes) However, when the fiber is long enough and the fiber is not polarization maintaining, energy carried by the two polarization modes may exchange between each other, known as mode coupling. This mode coupling is often random due to the random
nature of perturbation such as bending and stressing along the fiber. As a result, the overall DGD scales with the fiber length by \(\sqrt{L}\), so that \(\Delta \tau_g \approx (\Delta n_{\text{eff}} / c) \sqrt{L}\).

In modern high-speed optical communications using single-mode fiber, polarization mode dispersion has become one of the most notorious sources of transmission performance degradation. Due to the random nature of the perturbations that cause birefringence, polarization mode dispersion in an optical fiber is a stochastic process. Standard single mode fiber for telecommunications in 1550nm wavelength usually has the unit-length DGD value less than 0.1 ps/√km.

Example 2.3

A 1550nm optical signal from a multi-longitudinal mode laser diode has two discrete wavelength components separated by 0.8nm. There are two pieces of optical fiber; one of them is a standard single-mode fiber with chromatic dispersion parameter \(D = 17\text{ps/nm/km}\) at 1550nm wavelength, and the other is a step-index multimode fiber with core index \(n_1 = 1.48\), cladding index \(n_2 = 1.46\), and core diameter \(d = 50\mu\text{m}\). Both of these two fibers have the same length of 20km. Find the allowed maximum signal data rate that can be carried by each of these two fibers.

**Solution:** For the single-mode fiber, chromatic dispersion is the major source of pulse broadening of the optical signal. In this example, the chromatic dispersion-induced pulse broadening is

\[\Delta \tau_{\text{SMF}} = D \cdot L \cdot \Delta \lambda = 17 \times 20 \times 0.8 = 272 \text{ps}\]

For the multimode fiber, the major source of pulse broadening is modal dispersion. Using Equation 2.5.27, this pulse broadening is

\[\Delta \tau_{\text{MMF}} \approx \frac{n_1 L}{c} \left( \frac{n_1 - n_2}{n_2} \right) = 1.35 \mu\text{s}\]

Obviously, multimode fiber introduces pulse broadening more than three orders of magnitude higher than the single-mode fiber. The data rate of the optical signal can be on the order of 1Gb/s if the single-mode fiber is used, whereas it is limited to less than 1Mb/s with the multimode fiber.

2.5.6 Mode division multiplexing
Propagation modes in a multi-mode fiber are mutually orthogonal, and ideally there is no energy exchange between different modes during propagation. Under this assumption each mode can provide an independent communication channel for information transmission through the fiber. However, energy exchange between modes, known mode coupling, always exist in practical optical fibers due to intrinsic and extrinsic perturbations. The major difficulty arises from the randomness of the mode coupling which makes the crosstalk between modes unpredictable. Despite this difficulty, polarization multiplexing (PM), the simplest form of mode division multiplexing, has been successfully applied in fiber-optic transmission systems which utilizing two orthogonal polarization states of the fundamental LP_{01} mode to carry independent channels. The practical application of PM was enabled by the availability of high speed digital electronics for signal processing which adaptively tracks and corrects the random crosstalk between the two polarization modes. Mode division multiplexing is an active area of research to further extend the capacity of optical transmission systems. Transmission using few-mode fiber with up to 5 or 6 modes has been shown to be possible [Randel, 2011]. As the number of modes increases, the difficulty of separating these modes at the receiver and removing the crosstalk between them increases exponentially. The complexity of electronic digital signal processing and the power consumption will become the major concern challenging the practicality of mode-division multiplexing.

2.6 Nonlinear effects in an optical fiber

Fiber parameters we have discussed so far, such as attenuation, chromatic dispersion, and modal dispersion, are all linear effects. The values of these parameters do not change with the change in the signal optical power. On the other hand, the effects of fiber nonlinearity depend on the optical power density inside the fiber core. The typical optical power carried by an optical fiber may not seem very high, but since the fiber core cross-section area is very small, the power density can be very high to cause significant nonlinear effects. For example, for a standard single-mode fiber, the cross-section area of the core is about $80 \, \mu m^2$. If the fiber carries $10 mW$ of average optical power, the power density will be as high as $12.5 kW/cm^2$. Stimulated Brillouin scattering (SBS), stimulated Raman scattering (SRS), and the Kerr effect are the three most important nonlinear effects in silica optical fibers.

2.6.1 Stimulated Brillouin scattering
Stimulated Brillouin scattering (SBS) is originated by the interaction between the signal photons and the traveling sound waves, also called *acoustic phonons* [Boyd, 1992]. It is just like that when you blow air into an open-ended tube: A sound wave may be generated. Because of the SBS, the signal light wave is modulated by the traveling sound wave. Stokes photons are generated in this process, and the frequency of the Stokes photons is downshifted from that of the original optical frequency. The amount of this frequency shift can be estimated approximately by

\[
\Delta f = 2f_0 \frac{V}{c/n_1}
\]

where \(n_1\) is the refractive of the fiber core, \(c/n_1\) is the group velocity of the light wave in the fiber, \(V\) is the velocity of the sound wave, and \(f_0\) is the original frequency of the optical signal. In a silica-based optical fiber, sound velocity along the longitudinal direction is \(V = 5760\text{m/s}\). Assume a refractive index of \(n_1 = 1.47\) in the fiber core at 1550nm signal wavelength, this SBS frequency shift is about 11GHz.

SBS is highly directional and narrowband. The generated Stokes photons only propagate in the opposite direction of the original photons, and therefore the scattered energy is always counter-propagating with respect to the signal. In addition, since SBS relies on the resonance of a sound wave, which has very narrow spectral linewidth, the effective SBS bandwidth is as narrow as 20MHz. Therefore, SBS efficiency is high only when the linewidth of the optical signal is narrow.

When the optical power is high enough, SBS turns signal optical photons into frequency-shifted Stokes photons that travel in the opposite direction. If another optical signal travels in the same fiber, in the same direction and at the same frequency of this Stokes wave, it can be amplified by the SBS process. Based on this, the SBS effect has been used to make optical amplifiers; however, the narrow amplification bandwidth nature of SBS limits their applications. On the other hand, in optical fiber communications, the effect of SBS introduces an extra loss for the optical signal and sets an upper limit for the amount of optical power that can be used in the fiber. In commercial fiber-optic systems, an effective way to suppress the effect of SBS is to frequency-modulate the optical carrier and increase the spectral linewidth of the optical source to a level much wider than 20MHz.

2.6.2 Stimulated Raman scattering
Stimulated Raman scattering (SRS) is an inelastic process where a photon of the incident optical signal (pump) stimulates molecular vibration of the material and loses part of its energy. Because of the energy loss, the photon reemits in a lower frequency [Smith, 1972]. The introduced vibrational energy of the molecules is referred to as an optical phonon. Instead of relying on the acoustic vibration as in the case of SBS, SRS in a fiber is caused by the molecular-level vibration of the silica material. Consequently, through the SRS process, pump photons are progressively absorbed by the fiber, whereas new photons, called Stokes photons, are created at a downshifted frequency.

Unlike SBS, where the Stokes wave only propagates in the backward direction, the Stokes waves produced by the SRS process propagate in both forward and backward directions. Therefore SRS can be used to amplify both co- and counterpropagated optical signals if their frequencies are within the SRS bandwidth. Also, the spectral bandwidth of SRS is much wider than that of SBS. As shown in Figure 2.6.1, in silica-based optical fibers, the maximum Raman efficiency happens at a frequency shift of about 13.2THz, and the bandwidth can be as wide as 10THz. Optical amplifiers based on the SRS effect have become popular in recent years because of their unique characteristics compared to other types of optical amplifiers.

![A normalized Raman gain spectrum of a silica fiber.](image)

On the other hand, SRS also may create interchannel crosstalk in wavelength-division multiplexed (WDM) optical systems. In a fiber carrying multiple wavelength
channels, SRS effect may create energy transfer from short wavelength (higher-frequency) channels to long wavelength (lower-frequency) channels.

2.6.3 Kerr effect nonlinearity and nonlinear Schrödinger equation

Kerr effect nonlinearity is introduced by the fact that the refractive index of an optical material is often a weak function of the optical power density:

\[ n = n_0 + n_2 \frac{P}{A_{\text{eff}}} \]  \hspace{1cm} (2.6.2)

where \( n_0 \) is the linear refractive index of the material, \( n_2 \) is the nonlinear index coefficient, \( P \) is the optical power, and \( A_{\text{eff}} \) is the effective cross-section area of the optical waveguide. \( P/A_{\text{eff}} \) represents optical power density.

Considering both linear and nonlinear effects, a nonlinear differential equation is often used to describe the envelope of optical field propagating along an optical fiber [Agrawal, 2001]:

\[
\frac{\partial A(t,z)}{\partial z} + \frac{j \beta_2}{2} \frac{\partial^2 A(t,z)}{\partial t^2} + \frac{\alpha}{2} A(t,z) - j \gamma |A(t,z)|^2 A(t,z) = 0
\]  \hspace{1cm} (2.6.3)

This equation is known as the nonlinear Schrödinger (NLS) equation. \( A(t,z) \) is the complex amplitude of the optical field. The fiber parameters \( \beta_2 \) and \( \alpha \) are group delay dispersion and attenuation, respectively. \( \gamma \) is defined as the nonlinear parameter:

\[
\gamma = \frac{n_2 \omega_0}{c A_{\text{eff}}}
\]  \hspace{1cm} (2.6.4)

On the left side of Equation 2.6.3 the second term represents the effect of chromatic dispersion; the third term is optical attenuation; and the last term represents a nonlinear phase modulation caused by the Kerr effect nonlinearity. To understand the physical meaning of each term in the nonlinear Schrödinger equation, we can consider dispersion and nonlinearity separately.

First, we only consider the dispersion effect and assume

\[
\frac{\partial A}{\partial z} + \frac{j \beta_2}{2} \frac{\partial^2 A}{\partial t^2} = 0
\]  \hspace{1cm} (2.6.5)

This equation can easily be solved in Fourier domain as
\[
\tilde{A}(\omega, L) = \tilde{A}(\omega, 0) \exp \left( -j \frac{\omega^2}{2} \beta_2 L \right) = \tilde{A}(\omega, 0) e^{\Phi(\omega)}
\] (2.6.6)

where \( L \) is the fiber length, \( \tilde{A}(\omega, L) \) is the Fourier transform of \( A(t, L) \), and \( \tilde{A}(\omega, 0) \) is the optical field at the fiber input. The differential phase between frequency components \( \omega \) and \( \omega_0 \) at the end of the fiber is

\[
\Phi = \frac{\omega^2 - \omega_0^2}{2} \beta_2 L \approx \omega_0^2 (\omega - \omega_0) \beta_2 L
\] (2.6.7)

where we have assumed that \( |\omega - \omega_0| \ll \omega_0 \), so that \( \omega + \omega_0 \approx 2\omega_0 \). If we let \( \Phi = \omega_0 \delta \tau \), where \( \delta \tau \) is the arrival time difference at the fiber end between these two frequency components, \( \Delta \tau \) can be found as \( \delta \tau \approx \delta \omega \beta_2 L \) with \( \delta \omega = \omega - \omega_0 \). Then if we convert \( \beta_2 \) into \( D \) using Equation 2.5.16, we find

\[
\delta \tau \approx D \cdot L \cdot \Delta \lambda
\] (2.6.8)

where \( \Delta \lambda = \delta \omega \lambda^2 / (2\pi) \) is the wavelength separation between these two components. In fact, Equation 2.6.8 is identical to Equation 2.5.17.

Now let’s neglect dispersion and only consider fiber attenuation and Kerr effect nonlinearity. Then the nonlinear Schrödinger equation is simplified to

\[
\frac{\partial A(t,z)}{\partial z} + \frac{\alpha}{2} A(t,z) = j \gamma |A(t,z)|^2 A(t,z)
\] (2.6.9)

If we start by considering the optical power, \( P(z,t) = |A(z,t)|^2 \), Equation 2.6.9 gives \( P(z,t) = P(0,t) e^{-\alpha z} \). Then we can use a normalized variable \( E(z,t) \) such that

\[
A(z,t) = \sqrt{P(0,t)} \exp \left( -\frac{\alpha z}{2} \right) E(z,t)
\] (2.6.10)

Equation 2.6.9 becomes

\[
\frac{\partial E(z,t)}{\partial z} = j \gamma P(0,t) e^{-\alpha z} E(z,t)
\] (2.6.11)

And the solution is

\[
E(z,t) = E(0,t) \exp \left[ j \Phi_{NL}(t) \right]
\] (2.6.12)

where
\[ \Phi_{NL}(t) = \gamma P(0, t) \int_0^L e^{-\alpha z} \, dz = \gamma P(0, t) L_{\text{eff}} \]  

(2.6.13)

with

\[ L_{\text{eff}} = 1 - \frac{e^{-\alpha L}}{\alpha} \approx \frac{1}{\alpha} \]  

(2.6.14)

known as the nonlinear length of the fiber, which only depends on the fiber attenuation (where \( e^{-\alpha L} \ll 1 \) is assumed). For a standard single-mode fiber operating in a 1550nm wavelength window, the attenuation is about 0.25dB/km (or \( 5.8 \times 10^{-5} \text{Np/m} \)) and the nonlinear length is approximately 17.4km.

According to Equation 2.6.13, the nonlinear phase shift \( \Phi_{NL}(t) \) follows the time-dependent change of the optical power. The corresponding optical frequency change can be found by

\[ \delta\nu(t) = \frac{1}{2\pi} L_{\text{eff}} \frac{\partial}{\partial t} [P(0,t)] \]  

(2.6.15)

Or the corresponding signal wavelength modulation:

\[ \delta\lambda(t) = -\frac{L_{\text{eff}}}{2\pi c} \frac{\partial}{\partial t} [P(0,t)] \]  

(2.6.16)

Figure 2.6.2 illustrates the waveform of an optical pulse and the corresponding nonlinear phase shift. This phase shift is proportional to the signal waveform, an effect known as self-phase modulation (SPM) [Stolen, 1978].
If the fiber has no chromatic dispersion, this phase modulation alone would not introduce optical signal waveform distortion if optical intensity is detected at the fiber output. However, if the fiber chromatic dispersion is considered, wavelength deviation created by SMP at the leading edge and the falling edge of the optical pulse, as shown in Figure 2.6.2, will introduce group delay mismatch between these two edges of the pulse, therefore creating waveform distortion. For example, if the fiber has anomalous dispersion ($D > 0$), short wavelength components will travel faster than long wavelength components. In this case, the blue-shifted pulse falling edge travels faster than the red-shifted leading edge; therefore the pulse will be squeezed by the SMP process. On the other hand, if the fiber dispersion is normal ($D < 0$), the blue-shifted pulse falling edge travels slower than the red-shifted leading edge and this will result in pulse spreading.

In the discussion of self-phase modulation, we have only considered one wavelength channel in the fiber, and its optical phase is affected by the intensity of the same channel. If there is more than one wavelength channel traveling in the same fiber, the situation becomes more complicated and crosstalk-created channels will be created by Kerr effect nonlinearity.

Now let’s consider a system with only two wavelength channels; the combined optical field is

$$A(z,t) = A_1(z,t)e^{-j\vartheta_1} + A_2(z,t)e^{-j\vartheta_2}$$  \hspace{1cm} (2.6.17)
where $A_1$ and $A_2$ are the optical field amplitude of the two wavelength channels and $\theta_1 = n\omega_1 / c$ and $\theta_2 = n\omega_2 / c$ are optical phases of these two optical carriers. Substituting Equation 2.2.6.17 into the nonlinear Schrödinger Equation 2.6.3 and collecting terms having $e^{-j\theta_1}$ and $e^{-j\theta_2}$, respectively, will result in two separate equations:

$$\frac{\partial A_1}{\partial z} + \frac{j\beta_2 \delta^2 A_1}{2 \partial t^2} + \frac{\alpha}{2} A_1 = j\gamma |A_1|^2 A_1 + j2\gamma |A_2|^2 A_1 + j\gamma A_1^2 A_2^* e^{j(\theta_1 - \theta_2)} \quad (2.6.18)$$

$$\frac{\partial A_2}{\partial z} + \frac{j\beta_2 \delta^2 A_2}{2 \partial t^2} + \frac{\alpha}{2} A_2 = j\gamma |A_2|^2 A_2 + j2\gamma |A_1|^2 A_2 + j\gamma A_2^2 A_1^* e^{j(\theta_2 - \theta_1)} \quad (2.6.19)$$

Each of these two equations describes the propagation characteristics of an individual wavelength channel. On the right side of each equation, the first term represents the effect of self-phase modulation as we have described; the second term represents cross-phase modulation (XPM); and the third term is responsible for another nonlinear phenomenon called four-wave mixing (FWM).

XPM is originated from the nonlinear phase modulation of one wavelength channel by the optical power change of the other channel [Islam, 1987]. Similar to SPM, it requires chromatic dispersion of the fiber to convert this nonlinear phase modulation into waveform distortion. Since signal waveforms carried by these wavelength channels are usually not synchronized with each other, the precise time-dependent characteristic of crosstalk is less predictable. Statistical analysis is normally used to estimate the effect of XPM-induced crosstalk.

FWM can be better understood as two optical carriers copropagating along an optical fiber; the beating between the two carriers modulates the refractive index of the fiber at the frequency difference between them. Meanwhile, a third optical carrier propagating along the same fiber is phase modulated by this index modulation and then creates two extra modulation sidebands [Hill, 1978 and Inoue, 1992]. If the frequencies of three optical carriers are $\omega_j$, $\omega_k$, and $\omega_l$, the new frequency component created by this FWM process is

$$\omega_{jl} = \omega_j + \omega_k - \omega_l = \omega_j - \Delta \omega_{kl} \quad (2.6.20)$$

where $l \neq j$, $l \neq k$ and $\Delta \omega_{kl}$ is the frequency spacing between channel $k$ and channel $l$. If there are only two original optical carriers involved as in our example, the third
carrier is simply one of the two original carriers \(( j \neq k)\), and this situation is known as degenerate FWM. Figure 2.6.3 shows the wavelength relations of degenerate FWM where two original carriers at \(\omega_j\) and \(\omega_k\) beat in the fiber, creating an index modulation at the frequency of \(\Delta\omega_{jk} = \omega_j - \omega_k\). Then the original optical carrier at \(\omega_j\) is phase-modulated at the frequency \(\Delta\omega_{jk}\), creating two modulation sidebands at \(\omega_i = \omega_j - \Delta\omega_{jk}\) and \(\omega_k = \omega_j + \Delta\omega_{jk}\). Similarly, the optical carrier at \(\omega_k\) is also phase-modulated at the frequency \(\Delta\omega_{jk}\) and creates two sidebands at \(\omega_j = \omega_k - \Delta\omega_{jk}\) and \(\omega_i = \omega_k + \Delta\omega_{jk}\). In this process, the original two carriers become four, and this is probably where the name “four-wave mixing” came from.

\[
\begin{align*}
\omega_i & \quad \omega_j & \quad \omega_k & \quad \omega_l \\
\end{align*}
\]

Figure 2.6.3 Degenerate four-wave mixing, where two original carriers at \(\omega_j\) and \(\omega_k\) create four new frequency components at \(\omega_i\), \(\omega_j\), \(\omega_k\), and \(\omega_l\).

FWM is an important nonlinear phenomenon in optical fiber; it introduces interchannel crosstalk when multiple wavelength channels are used. The buildup of new frequency components generated by FWM depends on the phase match between the original carriers and the new frequency component when they travel along the fiber. Therefore, the efficiency of FWM is very sensitive to the dispersion-induced relative walk-off between the participating wavelength components. In general, the optical field amplitude of the FWM component created in a fiber of length \(L\) is

\[
A_{jkl}(L) = j\gamma \sqrt{P_j(0)P_k(0)P_l(0)} \int_0^L e^{-\alpha z} \exp(j\Delta\beta_{jkl} z) dz
\]  
(2.6.21)

where \(P_j(0), P_k(0),\) and \(P_l(0)\) are the input power of the three participating optical carriers and

\[
\Delta\beta_{jkl} = \beta_j + \beta_k - \beta_l - \beta_{jkl}
\]  
(2.6.22)
is a propagation constant mismatch, \( \beta_j = \beta(\omega_j) \), \( \beta_k = \beta(\omega_k) \), \( \beta_l = \beta(\omega_l) \), and \( \beta_{jkl} = \beta(\omega_{jkl}) \). Expanding \( \beta \) as \( \beta(\omega) = \beta_0 + \beta_1(\omega - \omega_0) + \frac{\beta_2}{2}(\omega - \omega_0)^2 \) and using the frequency relation given in Equation (2.6.20), we can find

\[
\Delta \beta_{jkl} = -\beta_2(\omega_j - \omega_k)(\omega_k - \omega_l)
\]

(2.6.23)

Here for simplicity we neglected the dispersion slope and considered that the dispersion value is constant over the entire frequency region. This propagation constant mismatch can also be expressed as the functions of the corresponding wavelengths:

\[
\Delta \beta_{jkl} = \frac{2\pi D}{\lambda^2}(\lambda_j - \lambda_k)(\lambda_k - \lambda_l)
\]

(2.6.24)

where dispersion parameter \( \beta_2 \) is converted to \( D \) using Equation 2.5.16. The integration of Equation 2.6.21 yields

\[
A_{jkl}(L) = j \gamma \sqrt{P_j(0)P_k(0)P_l(0)} \frac{e^{j2\beta_{jkl}L} - 1}{j\Delta \beta_{jkl} - \alpha}
\]

(2.6.25)

The power of the FWM component is then

\[
P_{jkl}(L) = \eta_{FWM} \gamma^2 L_{eff} P_j(0)P_k(0)P_l(0)
\]

(2.6.26)

where \( L_{eff} = (1 - e^{-\alpha L})/\alpha \) is the fiber nonlinear length and

\[
\eta_{FWM} = \frac{\alpha^2}{\Delta \beta_{jkl}^2 + \alpha^2} \left[ 1 + \frac{4e^{-\alpha L} \sin^2(\Delta \beta_{jkl} L/2)}{1 - e^{-\alpha L}} \right]
\]

(2.6.27)

is the FWM efficiency. In most of the practical cases, when the fiber is long enough, \( e^{-\alpha L} \ll 1 \) is true. The FWM efficiency can be simplified as

\[
\eta_{FWM} \approx \frac{\alpha^2}{\Delta \beta_{jkl}^2 + \alpha^2}
\]

(2.6.28)

In this simplified expression, FWM efficiency is no longer dependent on the fiber length. The reason is that as long as \( e^{-\alpha L} \ll 1 \), for the fiber lengths far beyond the nonlinear length the optical power is significantly reduced and thus the nonlinear contribution. Consider the propagation constant mismatch given in Equation 2.6.24, the efficiency of FWM can be reduced either by the increase of fiber dispersion or by
the increase of channel separation. Figure 2.6.4 shows the FWM efficiency for several different fiber dispersion parameters calculated with Equations 2.6.24 and 2.6.28, where the fiber loss is $\alpha = 0.25 dB/km$ and the operating wavelength is 1550nm. Note in these calculations, the unit of attenuation $\alpha$ has to be in Np/m when using Equations (2.6.25) – (2.6.28). As an example, if two wavelength channels with 1nm channel spacing, the FWM efficiency increases for approximately 25dB when the chromatic dispersion is decreased from 17ps/nm/km to 1ps/nm/km. As a consequence, in WDM optical systems, if dispersion-shifted fiber is used, interchannel crosstalk introduced by FWM may become a legitimate concern, especially when the optical power is high.

![Figure 2.6.4 Four-wave mixing efficiency $\eta_{FWM}$, calculated for $\alpha =0.25dB/km$, $\lambda = 1550nm$.](image)

2.7 Different types of optical fibers

As we all know, optical fiber is a cylindrical waveguide that supports low-loss propagation of optical signals. The general properties of optical fibers have been discussed in Chapter 1. In recent years, numerous fiber types have been developed and optimized to meet the demand of various applications. Some popular fiber types that are often used in optical communication systems have been standardized by the International Telecommunication Union (ITU-T). The list includes graded index multimode fiber (G.651), nondispersion-shifted single-mode fiber (G.652), dispersion-shifted fiber (G.653), and nonzero dispersion shifted fiber (G.655). In
addition to fibers designed for optical transmission, there are also various specialty fibers for optical signal processing, such as dispersion compensating fibers (DCF), polarization maintaining (PM) fibers, photonic crystal fibers (PCF), and rare-earth doped active fibers for optical amplification. Unlike transmission fibers, these specialty fibers are less standardized.

2.7.1 Standard optical fibers for transmission

The ITU-T G.651 multimode fiber (MMF) has a 50-µm core diameter and a 125-µm cladding diameter. The attenuation parameter for this fiber is on the order of 0.8 dB/km at 1310 nm wavelength. Because of its large core size, MMF is relatively easy to handle with large misalignment tolerance for optical coupling and connection. However, due to its large modal dispersion, MMF is often used for short-reach and low data-rate optical communication systems. Although this fiber is optimized for use in the 1300-nm band, it can also operate in the 850-nm and 1550-nm wavelength bands [Corning Website; OFS Website].

The ITU-T G.652 fiber, also known as standard single-mode fiber, is the most commonly deployed fiber in optical communication systems. This fiber has a simple step-index structure with a 9-µm core diameter and a 125-µm cladding diameter. It is single-mode with a zero-dispersion wavelength around $\lambda_0 = 1310$ nm. The typical chromatic dispersion value at 1550 nm is about 17 ps/nm-km. The attenuation parameter for G.652 fiber is typically 0.5 dB/km at 1310 nm and 0.2 dB/km at 1550 nm. An example of this type of fiber is Corning SMF-28.

Although standard SMF has low loss in the 1550 nm wavelength window, which makes it suitable for long-distance optical communications, it shows relatively high chromatic dispersion values in this wavelength window. In high-speed optical transmission systems, chromatic dispersion introduces significant waveform distortion, which may significantly degrade system performance. The trend of shifting the transmission wavelength window from 1310 nm to 1550 nm in the early 1990s initiated the development of dispersion-shifted fiber (DSF). Through proper cross-section design, DSF shifts the zero-dispersion wavelength $\lambda_0$ from 1310 nm to approximately 1550 nm, making the 1550 nm wavelength window have both the lowest loss and the lowest dispersion. The core diameter of the DSF is about 7 µm, which is slightly smaller than the standard SMF. DSF is able to significantly extend
the dispersion-limited transmission distance if there is only a single optical channel propagating in the fiber, but people soon realized that DSF is not suitable for multiwavelength WDM systems due to the high-level nonlinear crosstalk between optical channels through four-wave mixing and cross-phase modulation. For this reason, the deployment of DSF did not last very long.

To reduce chromatic dispersion while maintaining reasonably low nonlinear crosstalk in WDM systems, nonzero dispersion-shifted fibers (NZDSF) were developed. NZDSF moves the zero-dispersion wavelength outside the 1550 nm window so that the chromatic dispersion for optical signals in the 1550 nm wavelength is less than that in standard SMF but higher than that of DSF. The basic idea of this approach is to keep a reasonable level of chromatic dispersion at 1550 nm, which prevents high nonlinear crosstalk from happening but without the need of dispersion compensation in the system. There are several types of NZDSF depending on the selected value of zero-dispersion wavelength $\lambda_0$. In addition, since $\lambda_0$ can be either longer or shorter than 1550 nm, the dispersion at 1550 nm can be either negative (normal) or positive (anomalous). The typical chromatic dispersion for G.655 fiber at 1550 nm is 4.5 ps/nm-km. Although NZDSF usually has core sizes smaller than standard SMF, which increases the nonlinear effect, some designs such as Corning LEAF (large effective area fiber) have the same effective core area as standard SMF, which is approximately 80 $\mu$m$^2$. 

![Graph showing dispersion vs. wavelength for different types of fibers](image_url)
Figure 2.7.1, Chromatic dispersions of non-dispersion-shifted fiber (NDSF) and various different non-zero-dispersion-shifted fibers (NZDSF)

Figure 2.7.1 shows typical dispersion vs. wavelength characteristics for several major fiber types that have been offered for long-distance links. The detailed specifications of these fibers are listed in Table 4.1.1. In optical communication system applications, the debate on which fiber has the best performance can never settle down, because data rate, optical modulation formats, channel spacing, the number of WDM channels, and the optical powers used in each channel may affect the conclusion. In general, standard SMF has high chromatic dispersion so that the effect of nonlinear crosstalk between channels is generally small; however, a large amount of dispersion compensation must be used, introducing excess loss and requiring higher optical gain in the amplifiers. This in turn will degrade OSNR at the receiver. On the other hand, low dispersion fibers may reduce the requirement of dispersion compensation but at the risk of increasing nonlinear crosstalk.

<table>
<thead>
<tr>
<th>Fiber Type</th>
<th>ITU</th>
<th>Dispersion @ 1550nm (ps/nm/km)</th>
<th>Dispersion Slope (ps/km/nm²)</th>
<th>Effective Area (Aeff) (μm²)</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>NDSF(SMF-28)</td>
<td>NDSF/G.652</td>
<td>16.70</td>
<td>0.06</td>
<td>86.6</td>
<td>1446</td>
</tr>
<tr>
<td>LS</td>
<td>NZNDSF/G.655</td>
<td>-1.60</td>
<td>0.075</td>
<td>50</td>
<td>80</td>
</tr>
<tr>
<td>Truewave (TW)</td>
<td>NZNDSF/G.655</td>
<td>2.90</td>
<td>0.07</td>
<td>55.42</td>
<td>161</td>
</tr>
<tr>
<td>TW – RS</td>
<td>NZNDSF/G.655</td>
<td>4.40</td>
<td>0.042</td>
<td>55.42</td>
<td>244</td>
</tr>
<tr>
<td>LEAF</td>
<td>NZNDSF/G.655</td>
<td>3.67</td>
<td>0.105</td>
<td>72.36</td>
<td>266</td>
</tr>
<tr>
<td>TERALIGHT</td>
<td>NZNDSF/G.655</td>
<td>8.0</td>
<td>0.058</td>
<td>63</td>
<td>504</td>
</tr>
</tbody>
</table>

Table 2.1 Important Parameters for Standard SMF (NDSF), Long-Span (LS) Fiber, Truewave (TW) Fiber, Truewave Reduced-Slope (TW-RS) Fiber, Large Effective Area Fiber (LEAF), and Teralight Fiber

2.7.2 Specialty optical fibers

In addition to the fibers designed for optical transmission, a number of specialty fibers have also been developed for various purposes ranging from linear and nonlinear optical signal processing to interconnection between equipment. The following are a few examples of specialty optical fibers that have been widely used.

Dispersion compensating fiber (DCF) is a widely used specialty fiber that usually provides a large value of negative (normal) dispersion in a 1550 nm wavelength window. It is developed to compensate for chromatic dispersion in optical
transmission systems that are based primarily on standard SMF. The dispersion coefficient of DCF is typically on the order of \( D = -95 \text{ ps/nm-km} \) at a 1550 nm wavelength window. Therefore approximately 14 km DCF is required to compensate for the chromatic dispersion of 80 km standard SMF in an amplified optical span. For practical system applications, DCFs can be packaged into modules, which are commonly referred to as dispersion compensating module (DCM).

Compared to other types of dispersion compensation techniques such as fiber Bragg gratings, the distinct advantage of DCF is its wide wavelength window, which is critical for WDM applications, and its high reliability and negligibly small dispersion ripple over the operating wavelength. In addition, DCF can be designed to compensate the slope of chromatic dispersion, thus making it an ideal candidate for WDM applications involving wide wavelength windows. However, due to the limited dispersion value per unit length, DSF usually has relatively higher attenuation compared to fiber gratings, especially when the required total dispersion is high. In addition, because a large value of waveguide dispersion is needed to achieve normal dispersion in 1550 nm wavelength window, the effective core area of DCF can be as small as \( A_{\text{eff}} \approx 15 \mu m^2 \), which is less than 1/5th that of a standard SMF. Therefore the nonlinear effect in DCF is expected to be significant, which has to be taken into account in designing a measurement setup involving DCF.

_Polarization maintaining (PM) fiber_ is another important category of specialty fibers. It is well known that in an ideal single-mode fiber with circular cross-section geometry, two degenerate modes coexist, with mutually orthogonal polarization states and identical propagation constants. The effect of external stress may cause the fiber to become birefringent, and the propagation constants of these two degenerate modes will become different. The partitioning of the propagating optical signal into the two polarization modes not only depends on the coupling condition from the source to the fiber but also on the energy coupling between the two modes while propagating in the fiber, which is usually random. As a consequence, the polarization state of the output optical signal is usually random, even after only a few meters of propagation length in the fiber; the mode coupling and the output polarization state are very sensitive to external perturbations such as temperature variation, mechanical stress change, and micro and macro bending.
It is also known that energy coupling between the two orthogonal polarization modes can be minimized if the difference between the propagation constants of these two modes is large enough. This can be accomplished by incorporating extra elements in the fiber cladding that apply asymmetric stress to the fiber core. Due to the difference in the thermal expansion coefficients of various materials, the unidirectional stress in the fiber core can be achieved from the manufacturing process when the fiber is drawn from a preform. Depending on the shape of the stress-applying parts (SAP), PM fibers can be classified as “Panda” and “Bowtie,” as illustrated in Fig.2.7.2. The bowtie structure is depicted in which the SAPs are arranged in an arced manner around the fiber core; the Panda structure is named based on its similarity to the face of a panda bear. In the direction of the SAPs, which is parallel to the field of tension (horizontal in Figure 2.22), the fiber core has a slightly higher refractive index so that this axis is also referred to as the slow axis because the horizontally polarized mode propagates slower than the vertically polarized mode. The principle axis perpendicular to the slow axis is thus called the fast axis.

Figure 2.7.2 Cross-sections of polarization fibers: (a) Panda fiber and (b) Bowtie fiber. SAP: stress-applying parts.

It is important to note that a PM fiber is simply a highly birefringent fiber, in which the coupling between the two orthogonally polarized propagation modes is minimized. However, for a PM fiber to maintain the polarization state of an optical signal, the polarization state of input optical signal has to be aligned to either the slow or the fast axis of the PM fiber. Otherwise, both of the two degenerate modes will be excited, although there is minimum energy coupling between them; their relative optical phases will still be affected by the fiber perturbation, and the output polarization state will not be maintained because of the vectorial summation of these two mode fields. To further explain, assume that the input optical field is linearly
polarized with an orientation angle $\theta$ with respect to the birefringence axis of the PM fiber. The optical field $E_0$ will be split into the two polarization modes such that $E_x = E_0 \cos \theta$ and $E_y = E_0 \sin \theta$. At the output of the fiber, the composite optical field vector is $\tilde{E} = E_0 e^{-\alpha \ell} \left( \hat{a}_x \cos \theta + \hat{a}_y \sin \theta \cdot e^{i \Delta \phi} \right)$, where $\hat{a}_x$ and $\hat{a}_y$ are unit vectors, $L$ is the fiber length, and $\alpha$ is the attenuation parameter. $\Delta \phi = (\beta_x - \beta_y) L$ with $\beta_x$ and $\beta_y$ the propagation constants of the two modes. It is important to note that the differential phase $\Delta \phi$ is very sensitive to external perturbations of the fiber. When $\Delta \phi$ varies, the polarization state of the output optical field $\tilde{E}$ also varies if $\theta \neq 0$ (or 90°), as illustrated in Figure 2.7.3. In this case, since the phase difference $\Delta \phi$ between the two mode fields $E_x$ and $E_y$ are considered random, the variation of the output polarization orientation can be as high as $\Delta \Phi$, as shown in Figure 4.1.3.

Figure 2.7.3 Illustration of the output field vector of a PM fiber when the input field polarization state is not aligned to the principle axis of the fiber.

Therefore, in an optical system, if a PM fiber is used, one has to be very careful in the alignment of the signal polarization state at the fiber input. Otherwise the PM fiber could be even worse than a standard single-mode fiber in terms of the output polarization stability. Another issue regarding the use of PM fibers is the difficulty of connecting and splicing. When connecting between two PM fibers, we must make sure that their birefringence axes are perfectly aligned. Misalignment between the axes would cause the same problem as the misalignment of input polarization state, as previously discussed. To provide the functionality of precisely controlled axes rotation and alignment, a PM fiber splicer can be five times more expensive than a conventional fiber splicer due to its complexity.

Photonic crystal fiber (PCF), also known as photonic bandgap fiber, is an entirely new category of optical fibers because of its different wave-guiding mechanisms. As
shown in Figure 2.7.4, a PCS fiber usually has large number of air holes periodically distributed in its cross-section; for that reason it is also known as the “holey” fiber. The guiding mechanism of PCF is based on the Bragg resonance effect in the transversal direction of the fiber; therefore the low-loss transmission window heavily depends on the bandgap structure design.

Figure 2.7.4(a) shows the cross-section view of a hollow core PCF in which the optical signal is guided by the air core. In contrast to the conventional optical wave-guiding mechanism where high refractive index solid dielectric material is required for the core, the photonic bandgap structure in the cladding of PCF acts as a virtual mirror that confines the propagating lightwave to the hollow core. In general, the periodical photonic bandgaps structure of PCF is formed by the index contrast between silica and air by incorporating the holes into a silica matrix. In most of the hollow core PCFs, more than 95 percent of the optical power exists in the air; therefore the interaction between signal optical power and the glass material is very small. Because the nonlinearity of the air is approximately three orders or magnitude lower than in the silica, hollow core PCF can have extremely low nonlinearity and can be used to deliver optical signals with very high optical power. In addition, because of the design flexibility of the photonic bandgap structures, by varying the size and the pitch of the holes, zero dispersion can be achieved at virtually any wavelength. One drawback, though, for the hollow core PCF is its relatively narrow transmission window, which is typically on the order of 200 nm. This is the consequence of the very strong resonance effect of the periodic structure that confines the signal energy in the hollow core.

Figure 2.7.4 Cross-sectional view of photonic crystal fibers: (a) hollow-core PCF, (b) large core area PCF, and (c) highly nonlinear PCF [Thorlabs: crystal fiber].
Another category of PCF as shown in Figure 2.7.4(b) is referred to as large core area PCF. This type of fiber has a solid silica core to guide the optical signal, whereas the periodic holes in the cladding are used to facilitate optical wave guiding and help confine signal optical power in the core area. Generally, standard single-mode fiber with a 10 µm core diameter has a cutoff wavelength at approximately 1100 nm, below which the fiber will have multiple propagation modes. Large core PCF, on the other hand, allows for single-mode operation in a very wide wavelength window—for example, from 750 to 1700 nm—while maintaining a very large core area. Compared to hollow core PCF, large core area PCF has much wider low-loss window. Though a large core area PCF has a lower nonlinear parameter than standard single-mode fiber, its nonlinear parameter is typically much higher than a hollow core PCF.

Highly nonlinear PCF, as shown in Figure 2.7.4(c), is a very useful fiber for optical signal processing and has very small solid core cross-section; therefore the power density in the core is extremely high. For example, for a highly nonlinear PCF with zero-dispersion wavelength at $\lambda_0 = 710$ nm, the core diameter is as small as 1.8 µm and the nonlinear parameter is $\gamma > 100W^{-1} km^{-1}$, which is 40 times higher than that of a standard single-mode fiber. Highly nonlinear PCFs are usually used in nonlinear optical signal processing, such as parametric amplification and supercontinuum generation.

Photonic crystal fibers are considered high-end fiber types that are usually expensive due to their complex fabrication processes. They are generally sold in meters instead of kilometers because relatively short PCF fibers are enough for most of the applications for which PCFs are designed. They are also delicate and difficult to handle; for example, surface treatment, termination, connection, and fusion splicing are not straightforward because of the existence of air holes in the fiber.

Plastic optical fiber (POF) is a low-cost type of optical fiber that is also easy to handle. The core material of POF is typically made of PMMA (polymethyl methacrylate), which is a general-purpose resin, whereas the cladding is usually made of fluorinated polymer, which has a lower refractive index than the core. The cross-section design of POF is more flexible than silica fibers and various core sizes and core/cladding ratio can be easily obtained. For example, in a large-diameter POF, 95 percent of the cross-section is the core that allows the transmission of light. The fabrication of POF does not need an expensive MOCVD process, which is usually
required for the fabrication of silica-based optical fibers; this is one of the reasons for the low cost of the POFs. Although silica-based fiber is widely used for telecommunications, POF has found more and more applications due to its low cost and high flexibility. The cost associated with POF connection and installation costs are also low, which are especially suitable for fiber to the home application. On the other hand, POFs have transmission losses on the order of 0.25 dB/m, which is almost three orders of magnitude higher than silica fibers. This excluded POFs from being used in long-distance optical transmission. In addition, the majority of POFs are multi-mode, and therefore they are often used in low-speed, short-distance applications such as fiber-to-the-home networks, optical interconnections, networks inside automobile, and flexible illumination and instrumentation.
Summary:

In this chapter, we have discussed basic concept of wave propagation in an optical waveguide. Based on free-space Fresnel refraction and refraction on a dielectric interface, we have introduced the concepts of total internal reflection, optical phase shift upon reflection, as well as evanescent wave penetrating into the waveguide cladding. Then, based on the roundtrip phase matching conditions in the transversal direction of the waveguide and the standing wave principle, we have explained the physical meaning and the definition of propagation modes in an optical waveguide. Although geometric ray-tracing in free space is simple and intuitive in explaining the basic concept, precise mode field analysis in an optical fiber has to be accomplished based on the solutions of Maxwell Equations in a cylindrical coordinate with appropriate boundary conditions. In addition to mode-field distributions, several simple, but very important, parameters are resulted from this electromagnetic analysis include mode cut-off conditions, $V$-number, single-mode condition, cut-off wavelength, numerical aperture, and propagation constant of each mode.

Silica-based optical fiber has low loss windows around 1320nm and 1550nm. The typical fiber losses in these two wavelength windows are approximately 0.5dB/km and 0.25dB/km, respectively. With advances in material science and fabrication techniques, the ultimate fiber loss is eventually limited by the Rayleigh scatter. As a common practice of engineering, propagation loss in an optical fiber is defined in [dB/km], which allows straightforward estimation of signal optical power along the fiber with the knowledge of the signal power in [dBm] at the fiber input. On the other hand, when complex optical field expression, for example in Equation (2.4.3), is needed in the calculation, the attenuation parameter $\alpha$ needs to be expressed in [Neper/m]. A conversion factor between [dB/km] and [Neper/m] is $4.343 \times 10^3$.

Dispersion is an effect of differential delay. For a multi-mode fiber, if a signal optical pulse is carried by multiple propagation modes with each mode propagating in a different speed, the narrow pulse at the fiber input would be converted into a broader pulse, or even multiple pulses, at the fiber output. This is the effect of modal dispersion. In addition, even within a single mode, different frequency components of the optical signal may propagate in different speeds, and thus a short optical pulse at the fiber input can also be converted into a broader pulse at the fiber output due to chromatic dispersion. Usually for a multi-mode fiber, the differential delay caused...
model dispersion can be several orders of magnitude bigger than that of chromatic dispersion within each mode. For a single mode fiber, chromatic dispersion is the dominate mechanism and the differential delay is directly proportional to the spectral width of the optical signal. Note that even for a single mode fiber two degenerate modes exist with mutually orthogonal states of polarizations. Differential propagation delay between these two polarization modes is known as the polarization mode dispersion (PMD). PMD, originated from the birefringence and the non-circular core of the fiber, is usually random and easily perturbed by environmental conditions, and often specified by a statistical Maxwellian distribution.

We have also discussed various nonlinear effects in an optical fiber, their origins, and their impacts in optical transmission systems. Kerr-effect nonlinearity, caused by the power-dependent refractive index in the fiber, is responsible for various nonlinear effects such as self-phase modulation (SPM), cross-phase modulation (XPM) and four-wave mixing (FWM), which have been seen to cause performance degradation in fiber-optic systems. Especially in multi-wavelength optical networks, crosstalk caused by XPM and FWM can be quite significant when the signal optical power is high. Nonlinear scattering, including SBS and SRS, can cause optical signal energy shift from short wavelength to longer wavelength components. For a transmission system with only a single wavelength, nonlinear scattering causes a nonlinear loss of optical signal power, while for a fiber system carrying multiple wavelengths, nonlinear crosstalk can introduce a nonlinear crosstalk. Although nonlinear effects are limiting factors for the maximum optical power level than can be used in the fiber system, they can also be utilized to perform nonlinear optic signal processing. One such example is nonlinear pulse compression through SPM which is the basic mechanism to create optical soliton. Another example is to introduce stimulated Raman amplification, in which a strong optical pump at a short wavelength transfers its energy to an optical signal at a longer wavelength through the SRS process, which will be further discussed in Chapter 5.

The last section of this chapter we have presented different types of optical fibers. In addition to standard single-mode fiber (SSMF) and standard multi-mode fiber (SMMF), which are most commonly used in optical communication systems, there are various other fiber types which are often referred to as specialty fibers. Dispersion shifted fiber (DSF) was an early effort to reduce the chromatic dispersion in a single
mode fiber by shifting the zero-dispersion wavelength to the communication window around 1550nm. DSF was then found to be susceptible to nonlinear crosstalk in multi-wavelength systems due to FWM because of the slow walk-off between different wavelength channels. Then other types of DSFs were designed by changing the zero-dispersion wavelength away from the communications wavelength window to make tradeoffs between chromatic dispersion and nonlinear crosstalk. Dispersion compensation fiber with large normal dispersion value in the 1550nm wavelength window was developed to compensate the accumulated anomalous dispersion in optical systems with SSMF. Large effective area fiber (LEAF) is another transmission fiber developed to reduce the power density in the fiber core so that the nonlinear effect can be reduced. Other specialty fibers such as polarization maintaining (PM) fiber and photonic crystal fiber are usually used in special circumstances with relatively short lengths.

With the advances in material science, the maturity of fiber fabrication techniques, and the emergence of new applications, new materials and fiber types will be developed in the future. Good understanding of fundamental physics, wave guiding mechanics, and key parameters will be essential in the design of optical fibers, as well as the design and performance estimation of fiber-optic systems and networks.
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