Chapter 8, Optical transmission system design

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Introduction

Optical communication is one of the most important applications of fiber-optic technology. The introduction of optical fiber into communications revolutionized the entire telecommunications industry. The wide transmission bandwidth and low propagation loss make optical fiber an ideal media for transmission. Nowadays, more than 99 percent of long-distance communication traffic is carried by optical fibers all over the world. Fiber-optic technology is the backbone of the modern internet carried by high speed communication and data networks including wide area, metro area and access networks. With the knowledge of optical components discussed in the previous chapters, we discuss how to construct optical communication systems in this chapter based on these basic building blocks, and characterize system performance.

The physical layer of an optical fiber transmission system comprises a transmitter, a line system, and a receiver. The transmitter provides a means of uploading the electrical signal to be transmitted onto an optical carrier, known as electrical to optical (E/O) conversion. The line system delivers the modulated optical carrier to the receiver, which can be as simple as a length of optical fiber as the transmission medium or as complex as a multi-span, optically amplified and switched wavelength-division multiplexed (WDM) optical network. The receiver detects the optical carrier and down-converts the information carried by the optical carrier back to the electrical domain, known as optical to electric (O/E) conversion. Information on the optical carrier can either be analog or digital. While analog modulation is used for a number of applications including cable TV, digital modulation has clear advantages for high speed and long distance transmission. We discuss the fundamentals of binary modulation in this chapter which is the simplest yet the most often used modulation format for digital transmission systems and communication networks.

The fidelity of digital transmission is quantified as a bit error rate (BER). The BER is defined as the fraction of transmitted data that is mistakenly decoded by the receiver. BER is a function of the system quality factor, $Q$. The quality factor $Q$ is an electrical domain measure of the ratio of the separation of digital states to the noise associated with the states. Both the numerator and the denominator of $Q$ can be partitioned into
contributions whose sources are objects of system design. Examples include accumulated optical noise generated by optical amplifiers, signal optical power, polarization-dependent loss (PDL) and polarization mode dispersion (PMD), receiver and transmitter transfer function, net dispersion, and nonlinear propagation noise and distortion.

Commercial optical systems are designed to operate with a BER lower than a specified maximum value over their lifetime. For example, a maximum BER of $10^{-15}$ is commonly allowed for fiber links spanning city and continental distances. Such links are often designed with forward error correction (FEC) wherein overhead bits encoded with the data payload in such a way as to allow limited correction of errors upon decoding at the receiver [P. V. Kumar, 2002]. There is a wide range of FEC implementations offering a variety of correction capabilities and efficiencies. These FEC algorithms can deliver corrected error rates as low as $10^{-15}$ based on received data with uncorrected (raw) error rates as high as $10^{-3}$.

Although fundamental communication protocols, modulation formats, and performance evaluation criteria are applicable, optical fiber communication has unique characteristics due to its high data rate and the special system properties due to the use of optical fibers. Understanding basic properties of optical systems and the underline physical mechanisms is very important in the design, development, and installation of fiber-optic transmission systems, subsystems and networks.

In this chapter we describe basic parameters defining the performance of optical transmission systems. Although the description is focused on intensity-modulated direct detection (IMDD) systems with binary coding, most of the techniques and their fundamental principles are applicable to other types of modulation formats.

Section 8.1 provides an overview of digital optical transmission systems and their performance specifications, such as bit error rate (BER), the quality factor ($Q$) and their relation for binary modulated systems. Section 5.2 introduces the definitions of receiver sensitivity and the required optical signal-to-noise ratio (R-OSNR). In practical applications, receiver sensitivity is an evaluation criterion useful for optical systems whose performance are limited by noise generated by the receiver, whereas R-OSNR is a criterion often used for systems with inline optical amplifiers in which performance is
mainly determined by the OSNR of the optical signal. Section 8.3 discusses the impact of noise and waveform distortion in the performance of an optical system. While noises, such as thermal noise, shot noise and signal ASE beat noise, are random, waveform distortion which may be caused by limited bandwidth of the transmitter and receiver, as well as fiber chromatic dispersion, is usually deterministic. Thus, their impacts on the eye diagram closure and $Q$-factor are different. Section 8.4 discusses how to make optical system link budget considering the strength of optical signal and all factors of degradation. A design example will be used to help explain the design principles and procedures.

8.1 BER versus Q-value for binary modulated systems

Bit error rate (BER) and quality factor (Q-value) are most important parameters describing the quality of digital signals at a telecommunication receiver, and these two parameters are related. In this section, we discuss the meaning, the implication and the limitation of these two parameters.

8.1.1 Overview of IMDD optical systems

The majority of digital optical transmission systems are based on binary modulation. In the intensity modulation and direct detection (IMDD) mode, data is encoded on the optical power emitted from the transmitter, and the transmitter output has two digital states that are usually chosen to be light-pass (mark) and light-block (space). At the receiver, the optical power is converted into a photocurrent by means of a photodiode, in which the photocurrent is linearly proportional to the optical power received. This conversion eliminates wavelength information of the optical carrier as well as the phase noise of the optical carrier at the receiver. The digital data can also be encoded as frequency or phase of the optical carrier, such as frequency shift key (FSK) and phase shift key (PSK). As the photocurrent of a photodiode is only proportional to the signal optical power, frequency or phase decoders have to be included in the optical receivers before the photodiodes in these systems, so that the data embedded in the phase or frequency of the optical carrier can be recovered.

A block diagram of data flow through the components of an IMDD link is shown in Figure 5.1.1 (a). A data sequence to be transmitted is first encoded with FEC algorithm,
and the encoded binary data stream, is electrically amplified by a driver and applied on to
an O/E converter. Figure 5.1.1 (b) shows an example of ideal binary data sequence with
10 Gb/s data rate so that the bit length is 100 ps. The O/E converter can be a direct-modulated laser diode whose output optical power is linearly proportional to the applied
electric current as described in Chapter 3, or an external electro-optic modulator whose
transmission loss is related to the applied electric voltage as described in Chapter 6. The
modulated optical signal is then launched into an optical fiber system for transmission.
The fiber system can be as simple as a length of optical fiber, or multiple spans of fibers
with optical amplifiers to compensate the transmission loss of the fiber. An O/E converter
at the receiver detects the received optical signal and converts it into an electric current.
The O/E converter can be a simple photodiode for direct detection of the intensity
modulated optical signal. For frequency or phase modulated optical signals, appropriate
frequency or phase-sensitive optical components have to be employed before the
photodiode to extract the information. Because of the impairments throughout the
modulation transmission and photodetection, the electric current signal from the O/E
converter will be distorted and noisy compared to the transmitted waveform, as illustrated
in Figure 5.1.1(c). In the digital receiver, the clock has to be recovered from the corrupted
waveform through narrowband filtering and phase-locking, and this recovered clock is
used to determine the moment within each bit period when the decision has to be made.
Figure 5.1.1 (d) shows the eye diagram obtained by folding the waveform shown in
Figure 5.1.1(c) into a time window of 2 bits, where $T_D$ is the decision time, and $v_{th}$ is the
decision threshold.

Within a bit period, if the instantaneous amplitude of the received waveform is higher (or
lower) than the threshold $v_{th}$ at the decision time $T_D$, that bit is recognized as "1" (or "0").
This decision process converts the analog waveform back into a digital data sequence as
shown in Figure 5.1.1(e), and ideally it should be identical to the original binary data
sequence shown in Figure 5.1.1(b). But in practice, decision errors may happen by
misreading "0" as "1" or vise versa. A proper system design is to minimize these errors
under various application scenarios and sources of performance degradation.
In short-distance and low-speed optical systems without inline optical amplifiers, the system performance is often limited by the signal optical power level that reaches the receiver. Receiver sensitivity is defined as the minimum signal optical power required at the receiver to achieve the targeted bit error rate. If the signal optical power is too low at the receiver, the noise generated by the receiver would make the signal-to-noise ratio (SNR) unacceptable. For high-speed long-distance optical systems employing multiple inline optical amplifiers, signal waveform distortion and accumulated ASE noise throughout the transmission system may become major limitations in the transmission distance. In this case, receiver sensitivity is no longer a relevant parameter to specify an optical receiver. Instead, the receiver should be qualified by its ability to resist the influences of waveform distortion and optical noise.

One of the most important sources of linear performance impairments in high-speed optical transmission is chromatic dispersion of optical fiber. For a standard single-mode
fiber, the chromatic dispersion at a 1550 nm wavelength is on the order of 17 ps/nm-km. This limits the transmission distance of a 10 Gb/s IMDD optical system to about 100 km. Beyond which the intersymbol interference (ISI) due to chromatic dispersion will introduce significant waveform distortion shown as the closure of the signal eye diagrams. Dispersion shifted fibers have been developed to minimize this problem, but they were later found unsuitable for WDM systems due to the increased nonlinear crosstalk between different wavelength channels.

Dispersion compensation has emerged as an effective way to overcome the dispersion-induced waveform distortion and extend the maximum transmission distance. This is usually accomplished using dispersion-compensating modules (DCM) that have the opposite sign of dispersion to the transmission fiber. DCM can be made by dispersion-compensating fibers (DCF) or by passive optical devices such as fiber Bragg gratings. The overall accumulated dispersion in a transmission system can be reduced to an acceptable level with proper system design of dispersion compensation. In WDM systems with large numbers of channels, different wavelengths may experience different levels of dispersion due to the dispersion slope in optical fibers. In this case, slope compensation has also to be applied in high-speed optical systems to equalize the performance of all WDM channels. Adding a dispersion compensator in each fiber span has become a standard industrial practice for long-distance optical systems. In fact, a dispersion compensator can often be packaged into an inline optical amplifier module to simplify optical system implementation. The disadvantage of dispersion compensation in optical domains is the increased optical attenuation due to DCM, which requires a higher level of optical amplification in the system to compensate for this additional loss. This increased gain requirement of optical amplifiers will in turn generate excess ASE noise, which tends to degrade optical SNR in the receiver.

In an amplified multispans WDM optical system with a large number of wavelength channels, interchannel crosstalk is another important concern, especially when the system has a large number of spans and the signal optical power level at the beginning of each span is high enough. In addition to linear crosstalk that might be caused by leakage from optical filters and switches, nonlinear crosstalk is especially notorious because it cannot be eliminated by improving the qualities of optical components. The major sources of
nonlinear crosstalk in high-speed optical transmission systems include cross-phase modulation (XPM), four-wave mixing (FWM), and Raman crosstalk. Understanding the mechanisms of various system performance degradation, is essential for the system design, optimization and performance specification.

An important way to reduce performance degradation due to linear and nonlinear impairments in fiber-optic systems is to use advanced modulation formats. In general, an optical signal with longer pulse duration and (or) narrower spectral width would suffer less from chromatic dispersion. Multilevel modulation [Waklin 1999], phase-shaped binary (PSB) modulation [Penninckx 1997], and digital subcarrier multiplexing [Hui 2002] have been used to reduce the impact of chromatic dispersion because of their reduced spectral width. More recently, electrical domain digital signal processing (DPS) was applied to reduce transmission impairments, which has the potential to completely eliminate the requirement of optical domain dispersion compensation [McNicol 2005]. Without going into great detail on these techniques, it is useful to discuss the fundamental parameters that specify the quality of an optical transmission system.

In an optical network scenario, data is usually encapsulated in a digital wrapper that can be used to record content partitioning, source and destination, enable synchronization, time-domain partitioning performance monitoring, fault isolation, internodal communication, and the algorithm of FEC, to name a few. Additional overhead may be added to simplify clock recovery. These often essential network functionalities increase the line transition rate (equivalently bandwidth) for a given data rate. Depending on transmission standard and FEC algorithm used, such overhead can possibly add up to 25% to the line rate, but it is typically 3% percent to 7% in practice. Even though the data and overhead are often scrambled to regulate pattern length, in some cases the framing structure (which is not scrambled) can contain long patterns, which place demands on the receiver low-frequency response and clock recovery circuits. For example, long pseudorandom bit sequences (PRBS), such as $2^{31}-1$, have to be used in transmission experiments to properly exercise the pattern dependence of a link.
8.1.2 Receiver \textit{BER} and \textit{Q}

\textit{Bit error rate} (BER) is a fundamental measure of digital communication system quality. BER is essentially an error probability of digital bits in the received signal; it is also known as \textit{bit error probability}. By definition, BER is,

\begin{equation}
BER = \frac{Bit_{Error}}{Bit_{Total}}
\end{equation}

where $Bit_{Error}$ is the number of misinterpreted bits by the receiver and $Bit_{Total}$ is the total number of received bits. Both the misinterpreted bits and the total received bits are measured within a certain time window $\Delta T$, which is referred to as \textit{gating time}.

A useful alternative to the estimation of BER is the system \textit{Q} function. It is a quality factor determined by the ratio of separation between implemented digital states and the approximate Gaussian noise associated with those states at the receiver. In an optical receiver, after photodetection and a transimpedance preamplifier, the time-dependent voltage signal is presented to a decision circuit. This latter is typically a gated threshold device synchronized to the recovered clock. The decision circuit reports a logical one for signal voltage above a reference, threshold, value and a logical zero otherwise. A decision is made at each clock cycle. This scheme is shown in Figure 5.1.2. An eye diagram, formed by overlapping consecutive segments of the received electrical waveform, shows the site in phase (horizontal axis) and in voltage (vertical axis) of the decision instant and threshold, respectively. Depending on receiver design, the decision instant and threshold might be optimized once at start of life or in a continuous and automatic manner dictated by a performance cost function.

In the eye diagram shown in Figure 8.1.2, the spread of the voltage values above and below threshold at the sampling instant is attributable to both the waveform distortion caused by intersymbol interference (ISI) and random noises. Sources of ISI include channel memory stemming from receiver and transmitter transfer functions, linear and nonlinear propagation effects such as residual chromatic dispersion, PMD, SPM, XPM, FWM discussed in Chapter 2, and optical filter transfer functions. Random noises can be caused by photodiode thermal noise and shot noise, as well as signal ASE beat noise and
ASA-ASE beat noise as described in Chapter 5. Phase delay variations within the information bandwidth contribute to spreading at eye crossings, usually located \( \frac{1}{2} \) a clock cycle from the decision instant. This spreading is a constituent of timing jitter.

![Eye diagram](image)

**Figure 8.1.2** Illustration of bit decision in a binary receiver.

Figure 8.1.3 shows the probability distribution function (PDF) of the eye diagram so that we can derive the fundamentals of BER and Q-value calculations. In fact, BER is a conditional probability of receiving signal \( y \) while the transmitted signal is \( x \), \( P(y/x) \), where \( x \) and \( y \) can each be digital 0 or 1. Since the transmitted signal digital states can be either 0 or 1, we can define \( P(y/0) \) and \( P(y/1) \) as the PDFs of the received signal at state \( y \) while the transmitted signals are 0 and 1, respectively. Suppose that the probability of sending digital 0 and 1 are \( P(0) \) and \( P(1) \) and the decision threshold is \( v_{th} \); the BER of the receiver should be

\[
BER = P(0)P(v > v_{th}/0) + P(1)P(v < v_{th}/1)
\]  

(8.1.2)
where \( v \) is the received signal level. In most of the binary transmitters, the probabilities of sending 0 and 1 are the same, \( P(0) = P(1) = 0.5 \). Also, Gaussian statistics can be applied to most of the noise sources in the receiver as a first-order approximation,

\[
P_{\text{Gaussian}}(v) = \frac{1}{\sigma \sqrt{2\pi}} \exp\left(-\frac{(v - v_m)^2}{2\sigma^2}\right) \tag{8.1.3}
\]

where \( \sigma \) is the standard deviation and the \( v_m \) is the mean value of the Gaussian probability distribution. Then the probability for the receiver to declare 1 while the transmitter actually sends a 0 is

\[
P(v > v_{th} / 0) = \frac{1}{\sigma_0 \sqrt{2\pi}} v_{th} \exp\left(-\frac{(v - v_0)^2}{2\sigma_0^2}\right) dv = \frac{1}{\sqrt{2\pi}} \frac{v_{th}}{\sigma_0} \exp\left(-\frac{\xi^2}{2}\right) d\xi \tag{8.1.4}
\]

where \( \sigma_0 \) and the \( v_0 \) are the standard deviation and the mean value of the received signal photocurrent at digital 0, \( \xi = (v - v_0) / \sigma_0 \) and

\[
Q_0 = \frac{v_{th} - v_0}{\sigma_0} \tag{8.1.5}
\]

Figure 8.1.3 Probability distribution function (PDF) of the eye diagram.

Similarly, the probability for the receiver to declare 0 while the transmitter actually sends 1 is
\[ P(v < v_{th}/1) = \frac{1}{\sigma_1 \sqrt{2\pi}} \int_{-\infty}^{v_0} \exp \left( -\frac{(v_1 - v)^2}{2\sigma_1^2} \right) dv = \frac{1}{\sqrt{2\pi} \sqrt{v_{th}}} \int_{-\infty}^{v_{th}} \exp \left( -\frac{\xi^2}{2} \right) d\xi \quad (8.1.6) \]

where \( \sigma_1 \) and the \( v_1 \) are the standard deviation and the mean value of the received signal photocurrent at digital 1, \( \xi = (v_1 - v)/\sigma_1 \) and

\[ Q_1 = \frac{v_1 - v_{th}}{\sigma_1} \quad (8.1.7) \]

According to Equation 8.1.2, the overall error probability is

\[ BER = \frac{1}{2} P(v > v_{th}/0) + \frac{1}{2} P(v < v_{th}/1) = \frac{1}{2 \sqrt{2\pi}} \left\{ \int_{-\infty}^{v_{th}} \exp \left( -\frac{\xi^2}{2} \right) d\xi + \int_{v_{th}}^{\infty} \exp \left( -\frac{\xi^2}{2} \right) d\xi \right\} \quad (8.1.8) \]

where \( P(0) = P(1) = 0.5 \) is assumed.

Mathematically, a widely used special function, the error function, is defined as

\[ erf(x) = \frac{2}{\sqrt{\pi}} \int_{0}^{x} \exp(-y^2) dy \quad (8.1.9) \]

and a complementary error function is defined as

\[ erfc(x) = 1 - erf(x) = \frac{2}{\sqrt{\pi}} \int_{x}^{\infty} \exp(-y^2) dy \quad (8.1.10) \]

Therefore Equation 8.1.8 can be expressed as complementary error functions:

\[ BER = \frac{1}{4} \left\{ erfc\left( \frac{Q_0}{\sqrt{2}} \right) + erfc\left( \frac{Q_1}{\sqrt{2}} \right) \right\} \quad (8.1.11) \]

Since both \( Q_0 \) and \( Q_1 \) in Equation 8.1.11 are functions of the decision threshold \( v_{th} \), and usually the lowest BER can be obtained when \( P(0)P(v > v_{th}/0) = P(1)P(v < v_{th}/1) \), we can simply set \( Q_0 = Q_1 \), which is

\[ \frac{v_{th} - v_0}{\sigma_0} = \frac{v_1 - v_{th}}{\sigma_1} \]

Or equivalently
\[ v_{th} = \frac{v_0 \sigma_1 + v_i \sigma_0}{\sigma_0 + \sigma_1} \quad (8.1.12) \]

Under this “optimum” decision threshold, Equations 8.1.5 and 8.1.7 are equal and

\[ Q = Q_1 + Q_2 = \frac{v_i - v_0}{\sigma_1 + \sigma_0} \quad (8.1.13) \]

The BER function in Equation 8.1.11 becomes

\[ BER = \frac{1}{2} \text{erfc}\left( \frac{Q}{\sqrt{2}} \right) \quad (8.1.14) \]

This is a simple yet important equation that establishes the relationship between BER and the receiver \( Q \)-value, as shown in Figure 8.1.4. As a rule of thumb, \( Q = 6, 7, \) and 8 correspond to the BER of approximately \( 10^{-9}, 10^{-12}, \) and \( 10^{-15}, \) respectively.

![Figure 8.1.4 BER as a function of receiver Q-value.](image)

Figure 8.1.4 BER as a function of receiver Q-value.

Note that the relationship between the BER and the \( Q \)-value shown in Equation 8.1.14 is based on a Gaussian noise assumption. In practical systems, the statistics of noise sources are not always Gaussian. For example, shot noise is a Poisson process whose PDF follows a Poisson distribution \([\text{Personick 1977}]\). Another example is that the received photocurrent at digital 0 should never be negative because the received optical power is always positive. Therefore the tail of the PDF should be limited to the positive territory and a Rayleigh distribution may be more appropriate to describe the noise.
statistics. Nevertheless, Gaussian approximation is widely adopted because of its simplicity.

In the Gaussian approximation discussed so far, we have assumed that the eye diagram only has a single line at the digital 1 level and another single line at the digital 0 level. In practice, the eye diagram may have many lines at each digital level due to pattern-dependent waveform distortion. A normalized eye diagram is recast in Figure 8.1.5. This diagram plots the signal voltage referred (as relative equivalent optical power) to the input connector of the receiver. Associated with each of the noise-free lines of the eye diagram is an approximately Gaussian noise distribution that is generated from a handful of independent processes. The aggregate noise power distributions at the sampling instant (within the jitter window) are drawn on the right side of the figure. The jitter window is determined by the quality of clock recovery, which is an uncertainty on the decision phase. The noise distribution on transmitted 1s is typically wider than the distribution on transmitted 0s due to the signal dependence of some of the noise processes.

![Figure 8.1.5 Statistic distribution of noise when the eye diagram is distorted. Smooth lines represent noise-free eye diagram in which eye closure is caused by waveform distortion.](image)

Noise processes also depend on receiver optical-to-electrical conversion technology. For example, a receiver based on a PIN photodiode has different noise properties than an avalanche photodiode (APD)-based optical receiver. In optically amplified systems, the
presence of ASE noise generated from optical amplifiers makes the receiver performance specification and analysis quite different from unamplified optical systems.

8.2 Impacts of noise and waveform distortion on system Q-value

Equation 8.1.13 is a general definition of receiver $Q$ value under Gaussian noise approximation, and it is the ratio between the signal eye opening and the total noise at the decision time. Although both the waveform distortion and the random noise directly affect the system performance, the ways of their impacts on the receiver $Q$-value are different.

8.2.1 $Q$-calculation for optical signals without waveform distortion

Let us first consider a system without waveform distortion so that the $Q$-value is only determined by the accumulated noise at the receiver. In such an ideal case, the eye diagram is wide open with $P_1$ is the average optical power corresponding to the signal "1" level and $P_0 = 0$ is the signal "0" level at the decision time of each bit. Thus, the $Q$-value is,

$$Q = \frac{v_1 - v_0}{\sigma_1 + \sigma_0} = \frac{M^2P_1}{\sqrt{(\sigma_{Ih}^2 + \sigma_{sh}^2 + \sigma_{dk}^2 + \sigma_{S-ASE}^2 + \sigma_{ASE-ASE}^2 + \sigma_{RIN}^2)}B_e + \sqrt{(\sigma_{Ih}^2 + \sigma_{dk}^2 + \sigma_{ASE-ASE}^2)}B_e}$$

(8.2.1)

where, $\Re$ is the responsivity of photodiode, $M$ is the APD gain if an APD is used ($M = 1$ for a PIN photodiode), and $B_e$ is the receiver electric bandwidth.

$$\sigma_{Ih}^2 = 4kT / R_L$$

(8.2.2)

$$\sigma_{sh}^2 = M^2F_M \cdot 2q\Re P_1$$

(8.2.3)

$$\sigma_{dk}^2 = M^2F_M \cdot 2qI_{dk}$$

(8.2.4)

$$\sigma_{S-ASE}^2 = M^2F_M \cdot 2\Re^2 P_1 \rho_{ASE}$$

(8.2.5)

$$\sigma_{ASE-ASE}^2 = M^2F_M \cdot \Re^2 \rho_{ASE}^2 B_0 / 2$$

(8.2.6)

$$\sigma_{RIN}^2 = M^2F_M \cdot 2\Re P_1 \cdot RIN$$

(8.2.7)
represent single-sided power spectral densities of thermal noise, shot noise, dark current noise, signal-ASE beat noise, ASE-ASE beat noise and relative intensity noise, respectively. $R_L$ is the load resistance, $k$ is the Boltzmann’s constant, $T$ is the absolute temperature, $q$ is the electron charge, $I_{dk}$ is the photodiode dark current, $RIN$ is the relative intensity noise of the laser source defined by Equation (3.3.42), $B_0$ is the optical filter bandwidth, and $\rho_{ASE}$ is the accumulated ASE noise optical power spectral density at the input of the photodiode. $F_M$ is the noise figure of APD which is dependent on the APD gain $M$. If a photodiode (instead of an APD) is used, $M = 1$ and $F_M = 1$. Among these noise sources, short noise, signal-ASE beat noise and laser relative intensity noise are proportional to the signal optical power, so that they are categorized as signal-dependent noise. Whereas, thermal noise, dark current noise, and ASE-ASE beat noise are independent of the signal optical power so that they are referred to as signal-independent noise. Note that signal-dependent noise terms do not exist in the $\sigma_0$ term in this ideal case without waveform distortion because the associated signal optical power is $P_0 = 0$.

In the characterization of an optical system, it is always more convenient to measure the average power than measuring the instantaneous power at signal "1"s. For the majority of binary modulated optical signals, the probabilities of "0"s and"1"s in the signal are approximately equal, and they are both 50%. If $P_0 = 0$, and $P_{ave}$ is the average signal optical power, the "1" level of the optical signal should be $P_1 = 2P_{ave}$, which can be used in Equations (8.2.1) to (8.2.7).

### 8.2.2 Q-estimation based on eye diagram parameterization

In practical optical transmission systems, waveform distortion cannot be avoided which contributes to the degradation of optical system performance. As waveform distortion is generally pattern-dependent, the contributions to eye closure caused by isolated single "1"s and by continuous "1"s are not the same. For example, if the system transfer function has a low-pass characteristic, isolated "1"s (supported by high frequency components in the spectrum) will be penalized more than continuous "1"s which are predominately supported by low frequency components of the spectrum. An accurate calculation of $Q$-value has to consider all possible bit patterns of the received signal.
waveform and the probability of their occurrence, as well as the noise associated with each specific bit pattern. This generally pattern-specific nature prohibits analytic estimation of the $Q$-value even when the signal waveform is pseudorandom, and numerical simulations have to be applied in system performance calculation. Here we discuss a simplified technique for the estimation of system $Q$-value based on eye-mask parameterization. This technique is based on the separation of waveform distortion (eye closure) and random noise, which is suitable for systems designed to operate at high $Q$ values ($Q > 7$ or BER < $10^{-12}$). An optical eye distortion mask parameterization can be made at any reference interface of the system to define the distortion-related link performance contributions, independent of the particular noise characteristic [Hui, 1999].

Figure 8.2.1 (a) is an example of a waveform measured at the output of a dispersive fiber link which is affected by both waveform distortion and random noise. This is a section of a pseudorandom bit sequence (PRBS). As a PRBS waveform repeats itself for each pattern length, an average can be made to remove the random noise so that a much smoother (or deterministic) waveform can be obtained as shown in Figure 8.2.1(b). This waveforms can be converted into eye diagrams with and without the noise contribution as shown in Figure 8.2.2(a) and (b), respectively.

![Figure 8.2.1](image)

Figure 8.2.1 An example of photocurrent waveform with (a) and without random noise.
In Figure 8.2.2(b), the noise-free eye distortion mask is defined by a four-level feature \((P_1, P_0, A, B)\) over a timing window, \(W\), which represents the worst-case phase uncertainty in the sampling instant. \(W\) comprises the sum of all bounded uncertainties plus seven times the standard deviation of statistical uncertainty of the decision phase. \(P_1\) and \(P_0\) are the power levels associated with signal long "1"s and long "0"s in the pseudo-random non-return-to-zero (NRZ) bit pattern. The dimensionless parameters \(A\) and \(B\) are the lowest inner upper eye and the highest inner lower eye measured within the phase window \(W\), and they are independent of the noise. According to definitions in Figure 8.2.2, the average signal optical power is \(P_{\text{ave}} = (P_1 + P_0)/2\), given that signal "1"s and "0"s have the same probability.

Figure 8.2.2 Schematic of an optical eye distortion mask mapped onto a measured eye diagram.
In practical systems, waveform distortion, signal-dependent noise, and signal-independent noise are all mixed together at the receiver. According to Equation 8.1.13, the receiver $Q$ factor can be written as

$$Q = \frac{(A-B)2\Re P_{\text{ave}}}{\sqrt{(\sigma_{\text{ind}}^2 + \zeta 2P_{\text{ave}})B_e + \sqrt{\sigma_{\text{ind}}^2 + \eta B^2 P_{\text{ave}}}}B_e}$$  (8.2.8)

where, $\sigma_{\text{ind}}^2 = \sigma_{\text{th}}^2 + \sigma_{\text{dk}}^2 + \sigma_{\text{ASE-ASE}}^2$ is the signal independent noise power spectral density which includes thermal noise, dark current noise and ASE-ASE beat noise. $\zeta$ is a system-dependent multiplication factor which represents the impact of signal-dependent noise,

$$\zeta = 2\Re M^2 F_M (q + \Re \rho_{\text{ASE}} + \text{RIN})$$  (8.2.9)

In the absence of distortion, $B = 0$ and $A = 1$, the system $Q$ is determined only by the noise contribution. In this case,

$$Q = Q_0 = \frac{2\Re P_{\text{ave}}}{\sqrt{(\sigma_{\text{ind}}^2)B_e + \sigma_{\text{ind}}^2 B_e}}$$  (8.2.10)

By this definition of $Q_0$, the system $Q$ degradation caused only by eye distortion can be written in a general form as

$$D(A,B,x) = \frac{Q}{Q_0} = \frac{A-B}{Y_e}$$  (8.2.11)

In this expression, $Y_e$ is an important factor that shows the effect of interaction between distortion and noise:

$$Y_e(A,B,x) = \frac{\sqrt{1+xA} + \sqrt{1+xB}}{1 + \sqrt{1+x}}$$  (8.2.12)

where $x = 2\zeta P_{\text{ave}} / \sigma_{\text{ind}}^2$ is the ratio of signal-dependent noise to signal-independent noise.

In a case where signal-independent noise dominates, $x = 0$ and $Y_e = 1$, so that $D = A-B$. On the other hand, if signal-dependent noise dominates, $x = \infty$ and $Y_e = \sqrt{A} + \sqrt{B}$, therefore $D = \sqrt{A} - \sqrt{B}$. In general, with $x \in (0, \infty)$, the maximum value of $Y_e$ that corresponds to the worst-case distortion can be expressed as
\[ Y_0 = \begin{cases} \sqrt{A} + \sqrt{B} & (\sqrt{A} + \sqrt{B}) \geq 1 \\ 1 & (\sqrt{A} + \sqrt{B}) < 1 \end{cases} \quad (8.2.13) \]

The two possible maxima \( Y_{\text{max}} = \sqrt{A} + \sqrt{B} \) and \( Y_{\text{max}} = 1 \) correspond to \( x = \infty \) and \( x = 0 \), respectively. Using Equation 8.2.11, the worst-case distortion factor, defined as \( D_{\text{wc}} \), can be written as a function of \( Y_{\text{max}} \):

\[ D_{\text{wc}} = (A - B) / Y_{\text{max}} \quad (8.2.14) \]

Obviously, \( D_{\text{wc}} \) is a global worst-case distortion effect, which is independent of the nature of the noise. To demonstrate the impact of noise characteristic on the system distortion penalty, \( D \), defined by Equation 8.2.11, is plotted in Figure 8.2.3 as a function of \( x \). In this plot, two sets of eye-closure parameters were used, corresponding to the conditions for the two solutions of Equation 8.2.13. In one case, \( A = 0.7, B = 0.15 \), and \( \sqrt{A} + \sqrt{B} > 1 \) so that the worst-case distortion happens at \( x = \infty \). In the other case, \( A = 0.4, B = 0.05 \), and \( \sqrt{A} + \sqrt{B} < 1 \), and the worst-case distortion happens at \( x = 0 \). The dashed lines in Figure 8.2.3 are \( 10\log(\sqrt{A} - \sqrt{B}) \), which represents the case where signal-dependent noise dominates \( (x = \infty) \), whereas the dash-dotted lines are \( 10\log(A - B) \), which represents the case of \( x = 0 \). Shown as the solid lines in Figure 8.2.3, \( D(x) \) vs. \( x \) characteristics are not monotonic; however, \( D \) symptomatically approaches its worst-case \( D_0 \) with either \( x = 0 \) or \( x \to \infty \), depending on the value of \( \sqrt{A} + \sqrt{B} \). It is worthwhile to note that generally, Equations 8.2.13 and 8.2.14 overestimate the distortion penalty because \( x \in (0, \infty) \) was used to search for the worst case, but in real systems \( x \) value can never be infinity. The existence of a worst-case distortion factor \( D_{\text{wc}} \) implies a possibility to separate distortion from noise in the system link budgeting. Equation 8.2.14 clearly demonstrates a simple linear relationship between system \( Q \) and the worst-case distortion factor \( D_{\text{wc}} \). Regardless of the fundamental difference in the origins of noise and distortion, isolation of these two gives a clear picture of system budget allocations. Experimental verification of the linear relationship between \( Q \) and \( D_{\text{wc}} \), can be found in [Hui, 1999].
8.3 Receiver sensitivity and required OSNR

In a fiber-optic transmission system, the receiver has to correctly recover the data carried by the optical carrier. Because the optical signal that reaches the receiver is usually very weak after experiencing attenuation of the optical fiber and other optical components, a high-quality receiver has to be sensitive enough. In addition, high-speed and long-distance optical transmission systems suffer from waveform distortion, linear and nonlinear crosstalk during transmission along the fiber, and the accumulated ASE noise due to the use of inline optical amplifiers. A high-quality receiver also has to tolerate these additional quality degradations of the optical signal. The criterion of optical receiver qualification depends on the configuration of the optical system and the dominant degradation sources. In a relatively short distance optical fiber system without in-line optical amplifiers, optical noise is not a big concern at the receiver. In this case, transmission quality can be guaranteed as long as the signal optical power is high enough, and thus receiver sensitivity is the most relevant measure of the system performance. On the other hand, in a long distance fiber-optic transmission system employing multiple in-line optical amplifiers, accumulated optical noise arriving at the receiver can be overwhelming. In such a case, transmission performance cannot be ensured by simply

Figure 8.2.3 $Q$-degradation parameter $D$ as a function of $x$ (solid line). Dashed line: $10\log(A^{1/2} + B^{1/2})$; dashed-dotted line: $10\log(A-B)$.
increasing the signal optical power, and therefore a minimum optical signal to noise ratio (OSNR) has to be met. In the following we discuss receiver sensitivity and receiver OSNR tolerance, two useful receiver specifications.

8.3.1 Receiver sensitivity

Receiver sensitivity is one of the most widely used specifications of optical receivers in fiber-optic systems. It is defined as the minimum signal optical power level required at the receiver to achieve a certain BER performance. For example, in an optical system, for the BER to be less than $10^{-12}$ without forward error correction, the minimum signal optical power reaching the receiver has to be no less than $-35$dBm; the receiver sensitivity is thus $-35$dBm. Obviously the definition of receiver sensitivity depends on the targeted BER level and the signal data rate. However, signal waveform distortion and optical SNR are, in general, not clearly specified in the receiver sensitivity definition, but it assumes that the noise generated in the receiver is the major limiting factor of receiver performance.

Now we consider two types of optical receiver shown in Figure 8.3.1, in which configuration (a) is a simple PIN photodiode followed by a transimpedance amplifier (TIA), whereas, configuration (b) has an optical pre-amplifier added in front of the PIN to boost the optical signal power level.

![Figure 8.3.1 Direct detection receivers with (a) and without (b) optical preamplifier.](image)

We first consider the simplest receiver configuration shown in Figure 8.3.1(a). Neglecting waveform distortion and the impact of crosstalk in the optical signal for a moment, the $Q$-value will only depend on the electrical SNR after photodetection. In an intensity-modulated system with direct detection, the receiver sensitivity is affected by thermal noise, shot noise, and photodiode dark current noise. Equation 8.2.1 can be written as
Figure 8.3.1 shows the calculated receiver $Q$-value as a function of the received average signal optical power $P_{\text{ave}}$. This is a 10Gb/s binary system with direct detection, and the electrical bandwidth of the receiver is $B_e = 7.5$ GHz. Other parameters used are $\mathcal{R} = 0.85mA/mW$, $R_L = 50\Omega$, $I_d = 5nA$, and $T = 300K$. Figure 8.3.1 indicates that to achieve a targeted BER of $10^{-12}$, or equivalently $Q = 7$ ($10\log_{10}(Q) = 8.45$ dB), the received average signal optical power has to be no less than $–19$ dBm. Therefore the sensitivity of this 10Gb/s receiver is $–19$dBm. Every dB decrease in signal optical power will result in a dB decrease of the $Q$-value, as indicated in Figure 8.3.1.

Comparing the impacts of thermal noise, shot noise and dark current noise on the $Q$ value, it is evident that thermal noise dominates in this type of direct detection receiver in the vicinity of the targeted BER level. Other noise terms can be safely neglected without introducing noticeable errors.
In the thermal noise dominated receiver, if we further consider waveform distortion with $A < 1$ and $B > 0$ (referring to the eye mask defined by Figure 8.2.2(b)), the $Q$-value becomes,

$$Q = \sqrt[4]{\frac{R_L}{4kTB_e} \mathcal{R}(A - B) P_{\text{ave}}} \quad (8.3.2)$$

For a targeted performance of $Q = 7$, the receiver sensitivity can then be found as;

$$P_{\text{sen}} = \frac{7}{\mathcal{R}(A - B)} \sqrt[4]{\frac{4kTB_e}{R_L}} \quad (8.3.3)$$

which is inversely proportional to the eye closure penalty ($A-B$).

On the other extreme, if the shot noise is the only noise source while other noises are negligible, the $Q$-value is, $Q = \sqrt[4]{\frac{\mathcal{R}P_{\text{ave}}}{\sqrt{qB_e}}} \left(\sqrt{A - \sqrt{B}}\right)$. Since $\mathcal{R} = \eta \phi / (h\nu)$ with $\eta$ the quantum efficiency and $h\nu$ the photo energy, the shot-noise limited $Q$-value can be expressed as $Q = \sqrt[4]{\eta P_{\text{ave}} / (h\nu B_e)} \left(\sqrt{A - \sqrt{B}}\right)$. In the ideal case with 100% quantum efficiency and without waveform distortion, $Q = \sqrt[4]{P_{\text{ave}} / (h\nu B_e)}$. This corresponds to a receiver sensitivity of $P_{\text{sen}} = 49h\nu B_e$ for $Q = 7$. Assuming a bandwidth efficiency of 1 bit/Hz, this receiver sensitivity is commonly referred to as quantum-limited detection efficiency, which is of 49 photons per bit in this case. In other words, for each bit of information, 49 photons are required to achieve a $Q$-value of 7.

In order to improve receiver sensitivity, an optical preamplifier can be added in front the PIN photodiode as illustrated in Figure 8.3.1(b). As the optical preamplifier is part of the optical receiver, the receiver sensitivity is defined as the minimum optical power that reaches the preamplifier to achieve a targeted $Q$-value. The optical preamplifier increases the signal optical power before it reaches the photodiode, but it also introduces optical noise in the amplification process. In this case, the level of the ASE noise power spectral density is proportional to the gain of the optical preamplifier. Nevertheless, the receiver $Q$-value still increases with the increase of the input average signal optical power $P_{\text{ave}}$, but not linearly. If we neglect the waveform distortion so that average signal power is
equal to \(\frac{1}{2}\) of the instantaneous power at signal digital 1, the \(Q\)-value can be calculated by,

\[
Q = \frac{2R_P P_r}{\sqrt{\left(2q(2RP_r + I_d) + \frac{4kT}{R_L} + 4\rho_{ASE} \gamma^2 P_r \rho_{ASE} \gamma^2 (2B_o - B_e)\right)B_r + \left(2qI_d - \frac{4kT}{R_L} + \rho_{ASE} \gamma^2 (2B_o - B_e)\right)B_r}}
\]

(8.3.4)

where \(P_r = GP_{ave}\) is the amplified signal average optical power that reaches the PIN photodiode. In pre-amplified optical receiver, \(P_r\) is usually fixed to a reasonably high level around 0dBm so that thermal noise and dark current contributions become negligible compared to signal dependent noises. In such a case, the gain of the optical preamplifier becomes a function of the input signal optical power, as does the ASE noise level. For an optical preamplifier with a noise figure of \(F = 5\) dB that corresponds to \(n_{sp} = 1.58\), and with the signal wavelength of 1550nm, the ASE noise optical power spectral density is

\[
\rho_{ASE} = 2n_{sp} \frac{hc}{\lambda} (G - 1) = 4 \times 10^{-19} \left(\frac{P_r}{P_{ave}} - 1\right)
\]

(8.3.5)

in the unit of Watt per Hertz. Figure 5.2.3 (curve marked with "total") shows the calculated receiver \(Q\)-value as a function of the received average signal optical power \(P_{ave}\) at the input of the EDFA preamplifier. The impact of contribution due to each noise term is separately plotted in Figure 8.3.2. The parameters used in the calculation are \(P_r = 0\) dBm, \(R_L = 50\Omega\), \(I_d = 5\) nA, \(T = 300\) K, \(B_0 = 25\) GHz, \(B_e = 7.5\) GHz, and \(\lambda = 1550\) nm. In this case the receiver sensitivity is \(P_{sen} = -41.8\) dBm, which is approximately 23 dB better than the direct detection without the EDFA preamplifier.

In the optically pre-amplified PIN receiver, since the signal optical power \(P_r\) at the preamplifier output is kept constant, thermal noise, shot noise and dark current noise are constants and independent of the input optical power \(P_{ave}\). In this example shown in Figure 8.3.2, \(Q\) values corresponding to thermal, shot and dark current noises are 27.5dB, 29.2dB and 54dB, respectively, and their contributions to the overall \(Q\)-value are practically negligible. The dominant noise term that limits the receiver \(Q\)-value near the targeted BER level is the signal-ASE beat noise as can be easily seen from Figure 8.3.2.
If we only consider the impact of signal-ASE beat noise, Equation 8.3.4 can be greatly simplified as,

\[ Q = \frac{P_r}{\sqrt{\rho_{ASE} B_e}} = \frac{P_r}{\sqrt{\left(2n_{sp}hc/\lambda\right)(G-1)B_e}} \approx \frac{P_{ave}}{\sqrt{\left(2n_{sp}hc/\lambda\right)B_e}} \]  \hspace{1cm} (8.3.6)

Here G \gg 1 is assumed so that \( G - 1 \approx G \). In this receiver, the Q value is proportional to \( \sqrt{P_{ave}} \), instead of \( P_{ave} \) itself as in non-preamplified PIN receiver. This is beneficial because \( P_{ave} \) is usually very small so that \( \sqrt{P_{ave}} >> P_{ave} \) is always true. In a preamplified PIN receiver for every dB signal optical power decrease, there is only half a dB decrease in 10\log(Q).

With the approximation of only considering signal-ASE beat noise in the Q calculation of pre-amplified PIN optical receiver, we can consider the impact of waveform distortion assuming \( A < 1 \) and \( B > 0 \) (referring to the eye mask defined by Figure 8.2.2(b)). In this case, the numerator of Equation (8.1.13) is \( 2(A-B)R_P \) and terms in the denominator are \( \sigma_i = 4\rho_{ASE}R^2AP_r \) and \( \sigma_o = 4\rho_{ASE}R^2BP_r \), so that the Q-value becomes,

\[ Q = \frac{P_r}{\rho_{ASE}B_e} \left(\sqrt{A} - \sqrt{B}\right) = \frac{P_{ave}}{\sqrt{2n_{sp}(hc/\lambda)B_e}} \left(\sqrt{A} - \sqrt{B}\right) \]  \hspace{1cm} (8.3.7)
In comparison to the simple PIN receiver where $Q \propto (A - B)$, optically pre-amplified PIN receiver has $Q \propto \sqrt{A - \sqrt{B}}$. For $A > 0.5$ and $B < 0.5$, so that $\sqrt{A} - \sqrt{B} < A - B$. This implies that optically pre-amplified PIN receiver is more susceptible to waveform distortion than the simple PIN receiver.

8.3.2 Required OSNR (R-OSNR)

In the systems discussed above, optical noise is not present in the optical signal that reaches the optical receiver, and therefore, BER can be reduced by increase the level of signal optical power. On the other hand, in a long distance optical transmission system employing multiple inline optical amplifiers, the level of optical power at the receiver can always be increased by increasing the gain of optical amplifiers. But the accumulated ASE noise generated by these optical amplifiers may become significant when the number of inline optical amplifiers is large. When the optical signal to noise ratio (OSNR) at the input of the receiver is too low, increasing optical power at the receiver may not result in the improvement of BER performance. In this type of systems, the performance is no longer limited by the signal optical power that reaches the receiver; rather, it is limited by the OSNR.

![Fiber optic transmission system with $N$ optically amplified fiber spans](image)

Figure 8.3.3 illustrates a fiber optic transmission system with $N$ optically amplified fiber spans. In this system, the transmission loss of each fiber span is compensated by the gain of each inline optical amplifier so that the average signal optical power that reaches the receiver is equal to that emitted from the transmitter. At the same time, each EDFA generates an optical noise power spectral density $\rho_{\text{ASE},i} = 2n_{\text{sp}} (hc/\lambda) (G_i - 1)$, with $i = 1, 2, ... N$. As the ASE noise generated by each EDFA is also attenuated by the fiber and amplified by the EDFA along the following fiber spans, the accumulated ASE noise power spectral density at the input of the receiver will be simply the addition of
contributions from all inline EDFAs: \( \rho_{ASE} = \sum_{i=1}^{N} P_{ASE,i} \). Then, OSNR at the input of the optical receiver is defined as the ratio between the optical signal power, \( P_{ave} \), and the power spectral density of the accumulated noise \( \rho_{ASE} \), that is

\[
OSNR = \frac{P_{ave}}{\rho_{ASE}} \quad (8.3.8)
\]

Since the unit of the signal average power is \([W]\) and the unit of optical noise power spectral density is \([W/Hz]\), the unit of OSNR should be \([Hz]\), or \([dB \cdot Hz]\). In practice, the optical noise power spectral density is measured by an optical spectrum analyzer (OSA) with a certain resolution bandwidth, \( R_B \), and the OSA reports the measured noise power within a resolution bandwidth, which is \([W/R_B]\). Whereas assume the spectral linewidth of the optical signal itself is narrower than the OSA resolution bandwidth, the OSA actually measures the total power of the optical signal.

In the system with multiple optical amplifiers, since the level of optical power arriving at the receiver is usually high enough, signal independent noises such as thermal noise and dark current noise can be neglected in comparison with signal dependent noises including shot noise, signal-ASE beat noise and ASE-ASE beat noise. If we only consider shot noise, signal-ASE beat noise and ASE-ASE beat noise, and neglect waveform distortion, \( Q \) value can be calculated by,

\[
Q = \frac{2\Re P_{ave}}{\sqrt{\Re(q + \rho_{ASE} \Re P_{ave} + \rho_{ASE}^2 \Re^2 (2B_o - B_e)B_e) + \sqrt{\rho_{ASE}^2 \Re^2 (2B_o - B_e)B_e}}} \quad (8.3.9)
\]

This can be expressed as the function of OSNR as,

\[
Q = \frac{2\Re \cdot OSNR}{\sqrt{\left(4\Re q \cdot \frac{OSNR^2}{P_{ave}} + 4\Re^2 \cdot OSNR + \Re^2 (2B_o - B_e)B_e \right)B_e + \sqrt{\Re^2 (2B_o - B_e)B_e}}} \quad (8.3.10)
\]
Figure 8.3.4, Q-value as the function of signal OSNR (curve marked with Total) considering contributions from shot noise, ASE-ASE beat noise and signal-ASE beat noise.

Figure 8.3.4 shows the calculated receiver $Q$-value as the function of the signal OSNR for a 10Gb/s binary system based on Equation (8.3.10), where waveform distortion is not considered ($A = 1$ and $B = 0$). The OSNR, based on a resolution bandwidth of 0.1 nm, is the signal optical power divided by the noise power within 0.1 nm optical bandwidth. Other system parameters are $P_{ave} = 0$ dBm, $\mathcal{R} = 0.85mA/mW$, $B_o = 25$ GHz, $B_e = 7.5$ GHz, and $\lambda = 1550$ nm. Contributions due to shot noise, ASE-ASE beat noise and signal-ASE beat noise are shown in the same figure. In the vicinity of BER = $10^{-12}$ ($Q = 8.45$ dB), the $Q$-value is mainly determined by the contribution from signal-ASE beat noise, and the required OSNR to achieve the targeted BER of $10^{-12}$ is approximately 16 dB.

Since SAE-ASE beat noise can be reduced by further reducing the bandwidth of the optical filter, the dominant noise in most of the long distance optical systems employing multiple inline optical amplifiers are signal-ASE beat noise. If we only consider signal-ASE beat noise and also take into account the effect of waveform distortion, the $Q$ value will be,

$$Q = \sqrt{\frac{P_{ave}}{P_{ASE}B_e}} (\sqrt{A} - \sqrt{B}) = \frac{(\sqrt{A} - \sqrt{B})}{\sqrt{B_e}} \sqrt{OSNR}$$

(8.3.10)
which is similar to that described in Equation (8.3.7), but here \( P_{\text{ave}} \) and \( \rho_{\text{ASE}} \) are input optical signal and noise, respectively, at the receiver, and their ratio represents the OSNR. Equation (8.3.10) indicates that the \( Q \) value of the receiver is not determined by the received signal optical power level, but rather it is linearly proportional to the square root of the OSNR. If we set a target value of \( Q = 7 \) (for \( \text{BER} = 10^{-12} \)), the required OSNR, or ROSNR is simply,

\[
\text{ROSNR} = \frac{49B_e}{(\sqrt{A} - \sqrt{B})^2}
\]

(8.3.11)

8.4 Concept of wavelength division multiplexing

Standard single mode fiber has two low loss wavelength windows. The first window is in the 1310nm region with about 45 nm width from 1285 nm to 1330 nm, and the attenuation is on the order of 0.35±0.05 dB/km within this window. The second window is in the 1550nm region with about 50nm width from 1525nm to 1575nm, and the attenuation is on the order of 0.22±0.05 dB/km. All together this provides an approximately 14THz optical bandwidth suitable for optical communication. New fibers developed in recent years can also minimize the water absorption peak between the 1310nm and 1550nm wavelength window so that one giant low loss window extending from 1285nm all the way to 1575nm has the potential to provide more than 40THz bandwidth. However, the bandwidth of signal that can be modulated onto an optical carrier is limited by the bandwidth of electronic circuits and the speed of optoelectronic devices. Although wideband RF amplifiers and high speed electro-optic modulators can provide up to > 50GHz bandwidth, it is still a very small fraction of the available bandwidth of the optical fiber.

Wavelength division multiplexing (WDM) is a technique developed to make efficient use of the vast bandwidth resource provided by the optical fiber. Figure 8.4.1 shows the block diagram of a point-to-point fiber-optic WDM transmission system which uses multiple transmitters and receivers. Laser diode used in each transmitter is assigned with a unique wavelength so that there is no spectral overlap between different transmitters. Optical signals emitted from all transmitters are combined into a composite multi-
wavelength signal through a WDM multiplexer, and launched into a single transmission fiber. At the receiver, the multi-wavelength optical signal is first split into individual wavelength channels through a WDM demultiplexer, and each wavelength channel is detected by an optical receiver performing optical to electric conversion. In-line optical amplifiers can be used to compensate the attenuation of the transmission fiber. EDFAs are usually used as in-line amplifiers in WDM systems which require wide optical bandwidth and low crosstalk between different wavelength channels.

![Figure 8.4.1 Block diagram of point-to-point WDM transmission system.](image)

Because of the available gain bandwidth of EDFAs, the most used wavelengths window is between 1531nm and 1570nm, known as the C-band. Other wavelength bands include O-band: 1270nm -1370nm, E-band: 1371nm - 1470nm, S-band:1471nm - 1530nm, and L-band: 1571nm - 1611nm. In order to ensure the interoperability of optical communication equipment produced by different companies, a precise wavelength grid is standardized by International Telecommunication Union (ITU) for WDM systems and networks. Use C-band as an example, with 100GHz grid size the standard WDM optical frequencies are \( f_m = (190,000 + 100 \cdot m) \text{GHz} \), corresponding to the wavelength of the \( m^{th} \) WDM channel of \( \lambda_m = c / f_m \), where \( m \) is an integer representing channel index. Thus, nearly 50 wavelength channels can be put into the WDM C-band. The number of WDM channels can be increased by reducing channel spacing from 100GHz to 50GHz or 25GHz, which are known as dense wavelength multiplexing (DWDM). Similarly, ITU grid for 25GHz channel spacing is specified by \( f_m = (190.100 + 25 \cdot m) \text{GHz} \), and the entire C-band can host about 200 wavelength channels. With binary intensity modulation and direct detection, 10Gb/s data rate can be loaded on each wavelength channel in a 25GHz grid DWDM system, so that up to 2Tb/s total traffic capacity can be carried by a single
fiber in the C-band. Nowadays, with complex modulation and coherent detection with polarization multiplexing, 100Gb/s data rate can be put on each wavelength channel in a 25GHz DWDM grid, the total traffic capacity in the C-band alone can potentially reach 20Tb/s.

Basic components enabling DWDM include high quality WDM multiplexers (MUX) and de-multiplexers (DEMUX), single wavelength semiconductor lasers, and wideband inline optical amplifiers. Characteristics and specifications MUXs and DEMUXs have been discussed in Chapter 7. As far as laser sources are concerned, a DWDM system requires wavelength stabilization of semiconductor lasers so that they can be on the frequency grid defined by ITU. For binary intensity modulated systems with direct detection, DFB lasers with both temperature and current stabilization are often used with spectral linewidths on the order of Megahertz. Narrower spectral linewidths are required for phase modulated systems using coherent detection which will be discussed in the next chapter.

As discussed in Chapter 3, the wavelength of a simple DFB laser diode is primarily determined by the period of the Bragg grating used inside the laser cavity. Although the emitting wavelength can be adjusted to some extent by changing the injection current and the operation temperature, the tuning range is realistically not more than 1nm. Thus, a WDM transmitter has to be specifically designed for each wavelength on the grid. For a WDM system with $N$ wavelength channels, $N$ different transmitters have to be specifically designed each with a different wavelength. For practical system operation, each transmitter also needs to have a backup to avoid the interruption of service when the transmitter is malfunctioning. This requires to maintaining a large inventory which can often be very costly. A good solution to this problem is to use a tunable laser in the transmitter so that it can be provisioned to the required wavelength at the time of system installation, or replacing any transmitter when needed. This requires a wide wavelength tuning range of the tunable laser diode to cover the entire C-band. Although the cost and the complexity of a tunable laser are typically higher than a simple wavelength stabilized DFB laser, it still makes economic sense for developing wavelength tunable WDM transmitters. For the application as the tunable laser source inside a WDM transmitter, the speed of wavelength provisioning does not have to be fast, and thus tunable lasers based
on Micro-Electro-Mechanical Systems (MEMS) can be used, as illustrated in Figure 8.4.2.

Figure 8.4.2 configurations of tunable lasers based on wavelength tuning through MEMS. (a): use MEMS tilt mirror to select the desired wavelength from an array of DFB laser diodes. (b) use MEMS tilt grating to control optical feedback condition in an external cavity to select desired wavelength and minimize spectral linewidth.

Figure 8.4.2(a) shows a tunable laser configuration based on a DFB laser diode array which has \( N \) components. Each laser diode in the array emits a unique wavelength, and they are selected by a MEMS tilt mirror which decides which wavelength is coupled in the output optical fiber. This array-and-selection configuration does not change the spectral structure of each laser diode in the array, so that the spectral linewidth is on the Megahertz level. For the external cavity configuration shown in Figure 8.4.2(b), a reflective grating is used for wavelength selection in the external cavity, and the angle tilt of the grating is controlled by the MEMS actuating mechanism. In this external cavity configuration, both the angle of grating and the length of the external cavity are important to determine the emission wavelength and the spectral linewidth of the laser. More stringent precision and stability is required in this configuration, and feedback control is also required to optimize the optical feedback condition and minimize the spectral linewidth. 100KHz spectral linewidth is usually specified for tunable lasers based on external cavity.

8.5 Sources of optical system performance degradation

WDM is clearly an enabling technology that makes efficient use of the fiber bandwidth and makes transmission of Terabit data traffic possible over a single fiber. But at the same time, it introduces linear and nonlinear crosstalks among different wavelength channels. Understand the nature of these impairments is essential in system design and
optimization, as well as find ways to mitigate and minimize the impact of these crosstalks.

8.5.1 Performance degradation due to linear sources

Linear sources of degradation to the performance of a fiber-optic transmission system include chromatic dispersion, polarization mode dispersion (PMD), accumulated ASE noise from inline optical amplifiers, multi-pass interference due to optical reflections in the system, and cross talk between adjacent wavelength channels due to the extinction ratio of optical filters and WDM MUX and DEMUX. These degradations are in the linear category because they are independent of the signal optical power that is traveling inside the fiber.

8.5.1.1 Eye closure penalty due to chromatic dispersion

Chromatic dispersion, as discussed in Chapter 2, is a frequency-dependent group velocity, which causes different frequency components within a modulated optical signal spectrum to travel in different speeds resulting in an arrival time difference between them. Figure 8.5.1 shows normalized eye diagrams of a 10 Gb/s binary intensity modulated signal propagating through a single mode fiber with 0 km, 40 km, 80 km and 120 km lengths, respectively. Only chromatic dispersion is considered in the fiber. At the signal wavelength of 1553nm, the dispersion parameter of the fiber is 15.8 ps/nm/km, so that the accumulated dispersion values along the system are 0 ps/nm, 632 ps/nm, 1264 ps/nm and 1896 ps/nm, respectively for the fiber lengths considered. A 7.5GHz low-pass filter (5th order Bessel) was used in the receiver to eliminate high frequency components, while maintaining a complete eye opening at the decision phase. After 80 km of propagation through the fiber, the eye opening is reduced to approximately 60%.
Figure 8.5.1 Eye diagrams of received 10Gb/s binary optical signal propagating through single mode fiber with lengths of 0km (a), 40km (b), 80km (c) and 120km (d). Fiber dispersion parameter is $D = 15.8 \text{ps/nm/km}$ at the signal wavelength.

Figure 8.5.1 Eye opening (a) and eye closure penalty (b) as the function of fiber length for a 10Gb/s intensity modulated signal without chirp.

Eye diagrams shown in Figure 8.5.1 is obtained through numerical simulations, in which a non-return to zero (NRZ) pseudo random bit sequence (PRBS) is used. No phase modulation is associated with the intensity modulation, or equivalently the modulation chirp is zero in the optical signal used in the simulation. Figure 8.5.1(a) shows the normalized eye opening as the function of the fiber length, and Figure 8.5.1(b) shows the eye closure penalty, $E_{\text{penalty}}$, which is related to the eye opening, $E_{\text{eye}}$, by
\( E_{\text{penalty}} = -10 \log_{10}(E_{\text{eye}}) \). In this case without modulation chirp, normalized eye opening reduces monotonically with the increase of the fiber length, and reduces to half at around 100km fiber transmission distance. When modulation chirp is considered, the eye closure penalty as the function of fiber length may change. Figure 8.5.2(a) shows the calculated eye closure penalty for the 10Gb/s binary modulated optical signal, similar to that shown in Figure 8.5.1, but with optical phase modulation associated with the intensity modulation. In this case, we assume optical phase shift \( \varphi(t) \) is linearly proportional to the normalized instantaneous optical power, \( p(t) \), as \( \varphi(t) = \alpha_c \cdot p(t) \) with \( 0 < p(t) < 1 \), where \( \alpha_c \) is the chirp parameter, similar to that defined in Equation 3.3.40, for the linewidth enhancement factor. The dashed, solid and dash-dotted lines in Figure 8.5.2(a) show eye closure penalties for the chirp parameters of \(-0.2\pi\), 0, and \(0.2\pi\), respectively.

![Figure 8.5.2(a) calculated eye closure penalty as the function of fiber length using modulated optical signal with chirp parameters of \( \alpha_c = -0.2\pi \) (dashed line), 0 (solid line), and \( 0.2\pi \) (dash-dotted line), respectively. (b) Illustration of instantaneous frequency deviation \( \delta f(t) = d\varphi(t)/dt \) for a single pulse with modulation chirp.]

When the chirp parameter is positive \((\alpha_c > 0)\), the leading edge of a pulse has a positive frequency deviation while the trailing edge of the pulse has a negative frequency deviation as illustrated in Figure 8.5.2(b). For a standard single mode fiber with anomalous dispersion, \( D = d\tau/d\lambda > 0 \) and \( \beta_2 = d\tau/d\omega < 0 \), where \( \tau \) is the propagation delay, higher frequency components travel faster (less delay) than the lower frequency components. Therefore, positive chirp associated with an intensity modulated pulse...
introduces additional pulse broadening in this system and accelerated eye closure versus fiber length. On the other hand, a negative chirp can result in pulse squeezing along an anomalous dispersion fiber, which has the effect of increasing eye opening and compensates for the pulse broadening to some extent as shown in Figure 8.5.2 (a) (dashed line).

Because chromatic dispersion can be seen as a low pass filter applied on the modulated optical signal. For a NRZ PRBS pattern, the most vulnerable bit is the isolated "1"s which contains the highest frequency components. As an approximation, if we only consider an isolated digital "1" bit, optical field waveform can be represented by a normalized Gaussian pulse,

\[ E(t) = \frac{1}{\sigma\sqrt{2\pi}} \exp \left( -\frac{t^2}{2\sigma^2} \right) e^{-j\phi(t)} \]  \hspace{1cm} (8.5.1)

where, \( \phi(t) = \alpha_c t^2 / (2\sigma^2) \) is the optical phase introduced from modulation chirp. The FWHM pulse width of the optical power is \( T_0 = \sigma 2\sqrt{\ln 2} \), and the pulse energy is unity.

For a Gaussian pulse in the time domain, its spectrum in the frequency domain, which can be obtained through Fourier transform of \( E(t) \), is still Gaussian,

\[ \tilde{E}(\omega) = \frac{1}{\sqrt{(1 + j\alpha_c)}} \exp \left[ -\frac{\omega^2 \sigma^2}{2(1 + j\alpha_c)} \right] \]  \hspace{1cm} (8.5.2)

When this optical pulse propagates through an optical fiber, the impact of chromatic dispersion introduces a frequency-dependent phase on the spectrum. So that at the fiber output, the optical field becomes,

\[ \tilde{E}(\omega) = \frac{1}{\sqrt{(1 + j\alpha_c)}} \exp \left[ -\frac{\omega^2 \sigma^2}{2(1 + j\alpha_c)} + j \frac{\beta_2 \omega^2}{2} L \right] \]  \hspace{1cm} (8.5.3)

An inverse Fourier transform converts this optical spectrum back to time domain is,

\[ E(t, L) = \frac{1}{\sqrt{2\pi(\sigma^2 + (\alpha_c - j)\beta_2 L^2)}} \exp \left[ -\frac{(1 + j\alpha_c) t^2}{2\sigma^2 + 2(\alpha_c - j)\beta_2 L^2} \right] \]  \hspace{1cm} (8.5.4)
where $L$ is the fiber length and $\beta_2$ is the dispersion parameter of the fiber. At the fiber input, the peak power of the normalized Gaussian pulse, defined by Equation 8.5.1, is $p(0) = 1/(2\pi\sigma^2)$. While at the fiber output, the peak power becomes,

$$p(L) = \frac{1}{2\pi \sqrt{\left(\sigma^2 + \alpha \beta_2 L \right)^2 + \left(\beta_2 L \right)^2}}$$  \hspace{1cm} (8.5.5)

Thus, the normalized eye opening can be found as,

$$E_{\text{eye}} = \frac{\sigma^2}{\sqrt{\left(\sigma^2 + \alpha \beta_2 L \right)^2 + \left(\beta_2 L \right)^2}}$$  \hspace{1cm} (8.5.6)

For a Gaussian pulse without modulation chirp ($\alpha_c=0$), the eye opening at the receiver is $E_{\text{eye}} = \sqrt{1 + \left(\beta_2 L / \sigma^2 \right)^2}$. If we further define a dispersion length as,

$$L_D = \frac{\sigma^2}{\beta_2}$$  \hspace{1cm} (8.5.7)

eye opening can have a very simple form,

$$E_{\text{eye}} = \frac{1}{\sqrt{1 + \left(L / L_D \right)^2}} \approx 1 - \frac{L^2}{2L_D^2}$$  \hspace{1cm} (8.5.8)

where linearization can be applied if the closure penalty is small enough. The reason that the dispersion length is proportional to the square of the pulse width is that increasing pulse width linearly reduces the spectral width and reducing dispersion induced differential delay, and at the same time increased pulse width will increase the tolerance to the impact of differential delay. For a standard single mode fiber with a dispersion parameter $D = 15.8 \text{ ps/nm/km}$, or $\beta_2 \approx 2 \times 10^{-26} \text{ s}^2 / \text{m}$. The dispersion length is approximately 170km for 100ps FWHM pulse width ($\sigma = 60\text{ps}$). Although Gaussian pulse approximation is simple to analyze for the general rule of dispersion impact, it underestimates the eye closure penalty compared to simulation using PRBS patterns as shown in Figure 8.5.2(a). PRBS is a combination of pulses of different widths which may cause different rise/fall times after experiencing chromatic dispersion, so that eye closure penalty is higher than using a single Gaussian pulse. In fact, if we consider an isolated
digital "0", the 0 level will increase because of the spreading of adjacent "1"s. This can also be seen as the broadening of an inverse Gaussian pulse so that the minimum level increases the same way as the decrease of the amplitude of the "1" pulse. Considering the decrease of the 1 level and the increase of the 0 level, Equation (8.5.8) should be modified to 

$$E_{\text{eye}} \approx 1 - L^2 / L_D^2.$$ 

### 8.5.1.2 Eye closure penalty due to polarization mode dispersion

Polarization mode dispersion (PMD) is a differential delay between the two orthogonally polarized modes in a single mode fiber, which introduces time jitter in a fiber-optic system, resulting in an eye closure penalty. Unlike chromatic dispersion in the fiber system which is deterministic, the differential delay introduced by PMD is random and largely time-dependent, which makes the modeling of PMD impact in a fiber-optic transmission system challenging. Large number of simulations has to be done to have statistical relevance. As the most vulnerable bit of an NRZ modulated waveform subject to time jitter is the isolated "1", a Gaussian pulse approximation can be used to evaluate the impact of PMD on the eye closure penalty. Assume a normalized Gaussian pulse of the signal optical power with a width of \(\sigma_in\) at the input of an optical fiber, corresponding to a FWHM pulse width of \(\Delta t_{\text{FWHM}} = 2\sigma_in \sqrt{2\ln(2)}\),

$$P_{in}(t) = \frac{1}{\sigma_in \sqrt{2\pi}} \exp\left(-\frac{t^2}{2\sigma_{in}^2}\right)$$  \hspace{1cm} (8.5.9)

Only considering the impact of PMD, and neglecting the polarization-independent propagation delay, the optical pulse at the fiber output is the weighted combination of the powers carried by the two polarization modes, which is,

$$P_{out}(t) = \frac{1}{\sigma_{in} \sqrt{2\pi}} \left[ \gamma \exp\left(-\frac{t^2}{2\sigma_{in}^2}\right) + (1-\gamma)\exp\left(-\frac{(t - \tau)^2}{2\sigma_{in}^2}\right) \right]$$  \hspace{1cm} (8.5.10)

where \(0 < \gamma < 1\) is the ratio of the powers carried by the two principal states of polarization, and \(\tau\) is the differential group delay (DGD) between them [Poole, PTL 1991]. The average position of the temporal output pulse can be found as
\[ t_a = \int_{-\infty}^{\infty} t P_{out}(t) \, dt = (1 - \gamma) \tau. \] The width \( \sigma_{out} \) of the output pulse can be found through the calculation of the variance,

\[ \sigma_{out}^2 = \int_{-\infty}^{\infty} (t - t_a)^2 P_{out}(t) \, dt = \sigma_{in}^2 + \tau^2 (1 - \gamma) \]  \hspace{1cm} (8.5.11)

As the peak amplitude of a constant-energy Gaussian pulse is inversely proportional to the pulse width, the normalized eye opening can be found as,

\[ E_{eye} = \frac{\sigma_{in}}{\sigma_{out}} = \frac{1}{\sqrt{1 + \frac{3 \gamma (1 - \gamma) \tau^2}{\sigma_{in}^2}}} \]  \hspace{1cm} (8.5.12)

For a NRZ modulated pulse train at a data rate \( B \), assume the FWHM pulse width of an isolated "1" bit is \( \Delta t_{FWHM} = 1 / B \), so that \( \sigma_{in} = 1 / \left( 2B \sqrt{2 \ln(2)} \right) = 0.425 / B \). Thus, normalized eye opening can be expressed as the function of the data rate \( B \) as,

\[ E_{eye} = \frac{1}{\sqrt{1 + 5.6 \gamma (1 - \gamma) \tau^2 B^2}} \approx 1 - 2.8 \gamma (1 - \gamma) \tau^2 B^2 \]  \hspace{1cm} (8.5.13)

where we consider eye closure penalty is small enough so that linearization can be used. Figure 8.5.3 (a) shows a Gaussian pulse with \( \sigma_{in} = 1 / \sqrt{2} \) (\( \Delta t_{FWHM} \approx 1.67 \)) is split into two parts (shown as dashed and dash-dotted lines) with a peak power ratio of \( \gamma = 0.6 \), and a relatively delay \( \tau = 1 \) between them. The combined pulse shown as solid line in Figure 8.5.3(a) has the average position \( t_a = 0.4 \) and a normalized width of \( \sigma_{out} = 0.8602 \) (corresponding to \( \Delta t_{FWHM} \approx 2.03 \)). Obviously the worst-case pulse broadening due to PMD happens when the signal power is equally split into the two polarization modes \( (\gamma = 1/2) \).

If we further consider an isolated digital "0", the effect of DGD will cause the increase of the 0 level in the similar way as the decrease of the 1 level of digital "1". Thus, the impact on the overall eye closure will be doubled to approximately,

\[ E_{eye} \approx 1 - 5.6 \gamma (1 - \gamma) \tau^2 B^2 \]  \hspace{1cm} (8.5.14)
Figure 8.5.3(b) shows the eye closure penalty as the function of \( \tau B \), which is the DGD normalized by the FWHM pulse width as \( \Delta t_{FWHM} = 1 / B \).

\[
\text{Figure 8.5.3} (a) \text{ Illustration of a Gaussian pulse which is split into two (dashed and dash-dotted lines) with 0.6/0.4 ratio, and relatively delayed. The recombined pulse (solid line) has a shifted average position and broader width. (b) eye closure penalty calculated with Equation (8.5.14).}
\]

Since PMD is a random process, both the DGD value \( \tau \) and the power splitting ratio \( \gamma \) vary with time. The statistics of DGD in long fiber systems has been well studied and the probability distribution function (PDF) of \( \tau \) is Maxwellian,

\[
PDF_{DGD}(\tau) = \sqrt{\frac{6}{\pi \tau_{rms}^2}} \exp\left(-\frac{3\tau^2}{2\tau_{rms}^2}\right) \quad (8.5.15)
\]

where \( \tau_{rms} \) is the root-mean-square (RMS) value of DGD. Figure 8.5.4 shows the statistic distribution of \( \tau \), in which the RMS value of DGD is \( \tau_{rms} = 25 \text{ps}. \) The average DGD is \( \tau_{mean} = 27 \text{ps}, \) which is calculated from the PDF, shown in Equation 8.5.15, as

\[
\tau_{mean} = \int_{0}^{\infty} \tau \cdot PDF_{DGD}(\tau) d\tau = \tau_{rms} \sqrt{\frac{3\pi}{8}} \quad (8.5.16)
\]

On the other hand, the ratio \( \gamma \) of power splitting into the two polarized modes has a uniform distribution, so that the average value of \( \gamma(1-\gamma) \) is \( \gamma_{mean} = \int_{0}^{1} \gamma(1-\gamma) d\gamma = 1/6 \), which is less than the worst case of 0.25. Thus, the average eye opening of the received optical signal is,

\[
E_{eye,mean} \approx 1 - 0.933\tau_{mean}^2 B^2 \quad (8.5.17)
\]
According to Equation 8.5.17, a mean DGD of \( \tau_{\text{mean}} = 27 \text{ps} \) corresponds to an average eye closure penalty of approximately 0.3 dB for a 10 Gb/s NRZ data sequence. However, the Maxwellian distribution has a relatively long tail as shown in Figure 8.5.4(b). For example, the instantaneous DGD can reach 100 ps though at a low probability of \( 10^{-8} \). This may cause short-bursts of system outage over a long period of time. A statistic analysis of system outage probability has to be performed in the system design to guarantee the system performance.

![Figure 8.5.4 Probability of fiber DGD, \( \tau \), which has a Maxwellian distribution with \( \tau_{\text{rms}} = 25 \text{ ps} \) and a mean DGD of \( \tau_{\text{mean}} = 27 \text{ ps} \).

The concept of birefringence is relatively straightforward in a short fiber where refractive indices are slightly different in the two orthogonal axes of the transversal cross-section so that DGD is linearly proportional to the length of the fiber. However, if the fiber is long enough, the birefringence axes may rotate along the fiber due to banding, twisting, and non-uniformity. Meanwhile, there is also energy coupling between the two orthogonally polarized propagation modes in the fiber. In general, both the birefringence axis rotation and the mode coupling are random and unpredictable, which make polarization mode dispersion a complex problem to understand and to solve [Poole 1997].

Despite the random rotation of birefringence axis along a fiber, a concept of Principal state of polarization (PSP) is very useful in the analysis of PMD, which indicates two orthogonal polarization states corresponding to the fast and slow axes of the fiber. Under this definition, if the polarization state of the input optical signal is aligned with one of the two PSPs of the fiber, the output optical signal will keep the same SOP. In this case,
the PMD has no impact in the optical signal, and the fiber only provides a single propagation delay. It is important to note that PSPs exist not only in “short” fibers but also in “long” fibers. In a long fiber, although the birefringence along the fiber is random and there is energy coupling between the two polarization modes, an equivalent set of PSPs can always be found. Again, PMD has no impact on the optical signal if its polarization state is aligned to one of the two PSPs of the fiber. However, in practical fibers the orientation of the PSPs usually change with time, especially when the fiber is long. The change of PSP orientation over time is originated from the random changes in temperature and mechanical perturbations along the fiber.

An important parameter of single mode fiber is its PMD parameter, which is defined as the mean DGD over a unit length of fiber. For the reasons we’ve discussed, for short fibers, mean DGD is $\tau_{mean} \propto L$, where $L$ is the fiber length, the unit of PMD parameter is [ps/km]. Whereas for long fibers, mean DGD is $\tau_{mean} \propto \sqrt{L}$, the unit of PMD parameter is [ps/$\sqrt{km}$]. For example, for a standard single mode fiber, if the PMD parameter is $PMD = 0.06 \text{ps/} \sqrt{\text{km}}$, the mean DGD value for a 100km long fiber will be $\tau_{mean} = 0.6 \text{ ps}$.

### 8.5.1.3 ASE noise accumulation in fiber systems with inline optical amplifiers

In a long distance fiber-optic transmission system, optical amplifiers are used to compensate for the loss of the transmission fiber as illustrated in Figure 8.5.5. The optical amplifier at the optical transmitter, commonly referred to as the post amplifier, boosts the signal optical power before it is sent to the transmission fiber. Post amplifier can be packaged inside an optical transmitter to compensate the loss introduced by the electro-optic modulator and other optical components of the transmitter. The optical amplifier in front of the photodiode is commonly referred to as the pre-amplifier which enhances the signal optical power before photo-detection. The optical pre-amplifier can be treated as part of the optical receiver to form a pre-amplified optical receiver. Other optical amplifiers are called inline optical amplifiers. In a WDM optical system, inline optical amplifiers need to have wide gain bandwidth with flat gain spectrum to equally support all wavelength channels in the system.
In the optical system shown in Figure 8.5.5, there are \( N \) fiber spans and \( N+1 \) optical amplifiers including the post and inline amplifiers. Assume the optical gains of the amplifiers are \( G_1, G_2, \ldots, G_N \) and the losses of fiber spans are \( L_1, L_2, \ldots, L_{N-1} \) and \( L_N \), respectively. If the gain of each optical amplifier compensates the loss of each fiber span, the optical signal power \( P_s \) that reaches the pre-amplified optical receiver is the same as that at the output of the post amplifier which is \( P_t \). The ASE noise generated by each optical amplifier is linearly proportional to its optical gain, so that the accumulated ASE noise at the input of the amplified optical receiver is,

\[
\rho = 2h\sum_{i=1}^{N} \left[ n_{sp,i} (G_i - 1) \left( \prod_{m=2}^{N} G_{m+1} L_m \right) \right] + 2h n_{sp,N+1} (G_{N+1} - 1) \tag{8.5.17}
\]

where \( L_i = e^{-\alpha_i l_i} \) is the fiber loss with \( l_i \) and \( \alpha_i \) the fiber length and attenuation parameter of the \( i^{th} \) span. \( n_{sp,i} \) is the noise parameter of the \( i^{th} \) optical amplifier, and \( h \) is the photon energy as defined in Equation 5.2.1. If we further assume that each fiber span has the same loss which is compensated by the gain of the optical amplifier in that span, assume \((G_i - 1) \approx G\) and all amplifiers have the same noise figure \( n_{sp,i} = n_{sp}\), the ASE noise power spectral density shown in Equation 8.5.17 can be simplified as, \( \rho \approx h n_{sp} NG \). The optical signal-to-noise ratio is

\[
OSNR = \frac{P_t}{\rho} \approx \frac{P_t}{2h n_{sp}(N + 1)G} \tag{8.5.18}
\]

which is inversely proportional to the number of fiber spans, \( N \). Since the unit of the signal optical power is dBm, and the unit of optical noise power spectral density is dBm/Hz, the unit of OSNR calculated from Equation 8.5.18 should be [dB·Hz]. In practice, 0.1nm is often used as the resolution bandwidth of optical spectrum analyzer in the process of evaluating OSNR, which results in a unit of [dB·0.1nm] for OSNR. In the
1550nm wavelength window, 0.1nm is equal to 12.5GHz, so that a factor of $12.5 \times 10^9$ has to be used to translate [dB·Hz] into [dB·0.1nm]. As an example, Figure 8.5.6 shows the calculated OSNR in [dB·0.1nm] as the function of the number of fiber spans. In this example, the noise figure of optical amplifiers is 5dB, equivalent to $n_{sp} = 1.58$. The fiber loss is 0.25dB/km, and signal optical power is $P_s = 1$mW. The total fiber lengths of the system are 1000km, 3000km and 5000km, corresponding to the total fiber loss, $L_{total}$, of 250dB, 750dB and 1250dB, respectively. To compensate the fiber loss, the gain of each optical amplifier has to be $G^{N+1} = 10^{L_{total}/10}$, where $L_{total}$ is in dB. Thus $G^{N+1} = 10^{L_{total}/10(N+1)}$ and Equation 8.5.18 can be written as the function of total fiber loss as

$$OSNR = \frac{P_i}{2hn_{sp} (N + 1) \times 10^{L_{total}/10(N+1)}}$$

(8.5.18)

Figure 8.5.6 OSNR as the function of the number of fiber spans for the total system length of 1000km (solid line), 3000km (dashed line) and 5000km (dash-dotted line). The solid dot on each curve indicates where the length of each fiber span is 80km.

As shown in Figure 8.5.6, for a certain total length of the fiber system, OSNR generally increases with the increasing number of optical amplifiers so that the gain of each optical amplifier is reduced. However, the rate of increase saturates when the length of each fiber span is small enough. From an engineering point of view, small number of optical amplifiers in the system would simply system implementation, maintenance, and inventory, but would reduce the OSNR, and thus a tradeoff has to be made in the determination of the fiber span length. The solid dot on each curve in Figure 8.5.6 indicates the place where the length of each fiber span is 80km. This example
clearly tells that when the length of each fiber span is shorter than 80km, further
decreasing the span length would not significantly improve the system OSNR. Thus, in
practical fiber optic systems, amplified fiber span lengths are typically between 60km and
100km.

8.5.1.4 Multipath interference

In practical optical systems, an optical signal may go through different paths before
recombining at the receiver. This can be caused by beam splitting and combining, as well
as reflections along a fiber link with multiple optical connectors and interfaces, as
illustrated in Figure 8.5.7.

\[
E(t) + \eta E(t - \tau) = 0
\]

Figure 8.5.7 Linear crosstalk caused by (a) beam splitting and combining, and (b) optical
reflections

In Figure 8.5.7, \( \eta \) is a crosstalk coefficient, and \( R_1 \) and \( R_2 \) are reflectivities of optical
interfaces along the fiber. \( \tau = n\Delta L/c \) is a relative time delay between the main and the
delayed passes with \( n \) the refractive index, \( \Delta L \) the difference of path length, and \( c \) the
speed of light. In most cases, the optical signal going through the delayed path has much
lower amplitude than that through the main path, that is \( \eta \ll 1 \) or \( R_1 \) \( R_2 \ll 1 \), thus
multipath interference can be treated as a perturbation. Consider an optical signal
\( E(t) = A(t)e^{j\varphi(t)} \) with amplitude \( A(t) \) and phase \( \varphi(t) \), the photocurrent of this optical signal
mixes with its delayed and attenuated replica is,

\[
I(t) = \Re[A(t)e^{j\varphi(t)} + \zeta A(t - \tau)e^{j\varphi(t-\tau)}] = \Re[P(t) + \Re \zeta^2 P(t - \tau) + 2\Re \zeta \sqrt{P(t)P(t-\tau)} \cos \delta \varphi(t)]
\]

(8.5.19)

where \( \Re \) is the responsivity of the photodiode, \( P(t) = |E(t)|^2 \) is the signal optical
power, and \( \zeta = \eta \) for beam splitting/combining coefficient and \( \zeta = \sqrt{R_1R_2} \) for multiple
reflections as shown in Figure 8.5.7 (a) and (b), respectively. The optical phase is
\( \varphi(t) = \omega t + \varphi_0(t) \), where \( \omega \) is the angular frequency and \( \varphi_0 \) represents a reference phase, the difference phase is

\[
\delta \varphi(t) = \varphi(t) - \varphi(t - \tau) = \omega \tau + \varphi_0(t) - \varphi_0(t - \tau)
\]  

(8.5.20)

The impact of multipath interference can be regarded as a conversion from phase noise of the optical source into an intensity noise. The multipath configuration shown in Figure 8.5.7(a) and the multiple-reflection configuration shown in Figure 8.5.7(b) have the transfer functions of a Mach-zehnder interferometer and a Febry-Perot interferometer, respectively. For a constant wave (CW) optical source of constant intensity \( P(t) = P_a \), the photocurrent at the detector is

\[
I(t) \approx |R_P_a[1 + 2 \zeta \cos \delta \varphi(t)]|
\]

where interference has been assumed so that the \( \zeta^2 \) term is neglected for simplicity. When the relative delay \( \tau \) is much shorter than the coherence length, linear approximation can be used for the differential phase, \( \delta \varphi(t) \approx \tau [\omega + \delta \omega(t)] \) where \( \delta \omega(t) = d \varphi_0(t) / dt \) is the optical frequency deviation caused by the phase noise. Maximum conversion efficiency happens when \( \omega \tau = m \pi + \pi / 2 \), with \( m \) an integer, and the maximum intensity noise peak-to-peak amplitude on the photocurrent is \( 4 \zeta I_a \), as shown in Figure 8.5.8, where \( I_a \approx |R_P_a| \) is the average photocurrent.

Figure 8.5.8 illustration of phase noise to intensity noise conversion through phase-dependent transfer function of multipath interference.

Now, let us look at the impact of coherent multipath interference on a binary intensity modulated waveform as illustrated in Figure 8.5.9 (a). For simplicity, we assume that the waveform is ideal before the multipath interference, so that \( P_1 = 2P_{ave} \) and \( P_0 = 0 \) for optical power levels at digital "0" and "1", respectively, with \( 2P_{ave} \) being the average optical power. Based on Equation 8.5.19, the maximum photocurrent of digital "0" (corresponds to \( P(t) = 0 \) and \( P(t-\tau) = P_1 \)) is \( \Re \zeta^2 P_1 \), while the minimum photocurrent of
digital "1" (corresponds to \( P(t) = P_1 \) and \( P(t-\tau) = P_1 \)) is \( \Re P_1(1 + \zeta^2 - 2\zeta) \). The worst case peak-to-peak variation caused by multipath interference at signal digital "1" is \( \Delta I = 4\zeta \), and the normalized eye closure penalty is \( A - B = (1 - 2\zeta) \). This value has been normalized by the nominal digital "1" photocurrent which is \( I_1 = \Re P_1 \).

Figure 8.5.9 (a) illustration of normalized binary waveform with multipath interference, and (b) probability distribution function of the waveform.

Figure 8.5.9(b) shows the probability density function (PDF) of the normalized photocurrent assume \( P(t-\tau) = P(t) \). For a uniformly distributed differential phase \( \delta\phi \), the PDF of \( i(\delta\phi) = 1 + \zeta^2 + 2\zeta \cos(\delta\phi) \) is,

\[
PDF = \frac{1}{2\pi\zeta} \sqrt{1 - \left(\frac{i - 1 - \zeta^2}{2\zeta}\right)^2}
\]

(8.5.21)

This PDF is infinity when \( i = 1 + \zeta^2 \pm 2\zeta \).

The coherence interference happens when the relative delay time \( \tau \) is much shorter than the coherence time of the laser source defined by Equation 3.3.55, that is \( \tau << 1/(\pi\Delta\nu\tau) \), where \( \Delta\nu \) is the spectral linewidth of the laser source. On the other hand, if the relative delay is very long such that \( \tau >> 1/(\pi\Delta\nu\tau) \), statistically, \( \delta\phi(t) = \phi(t) \) is valid and multipath interference will result in an incoherent addition. In such a case, multipath interference can be seen as a mechanism which converts the phase noise into a relative intensity noise (RIN), which is [Gimlett, JLT, 1989],

\[
RIN(f) \approx \frac{4\zeta^2}{\pi} \left[ \frac{\Delta\nu}{f^2 + (\Delta\nu)^2} \right]
\]

(8.5.22)
which is dependent on the spectral linewidth, $\Delta \nu$, of the laser source, but independent of the relative delay $\tau$. In this case, the impact of multipath interference can be treated simply as an additional noise term in the system performance evaluation as discussed in section 8.3.

### 8.5.2 Performance degradation due to fiber nonlinearities

While sources of linear degradation in a fiber-optic system discussed above are independent of the signal optical power level, performance of a fiber-optic system can also be degraded by nonlinear effects in the optical fiber which is sensitive to the level of optical power used. Because of the small diameter of the fiber core, on the order of 9$\mu$m for a standard single mode fiber, even for a moderate optical power level of 1mW, the power density is more than 1kW/cm$^2$. Kerr effect nonlinearity originates from the power density dependent refractive index, which is responsible to a number of well known effects including self-phase modulation, cross-phase modulation, four-wave mixing, and modulation instability. Nonlinear scattering effects such as Stimulated Brillouin scattering (SBS) and Stimulated Raman Scattering (SRS) caused by interaction between the signal photons and the traveling acoustic phonons and photonic phonons, respectively, can cause energy translation from high energy photons to low energy photons and generating new wavelength components. Mechanisms of both the Kerr effect and the nonlinear scattering have been introduced in Chapter 2. In the following we will discuss the impact of these fiber nonlinearities in performance of fiber-optical communication systems.

As discussed in Chapter 2, the analysis of Kerr effect fiber nonlinearity usually starts from a nonlinear differential equation which describes the envelope of optical field propagating along an optical fiber [Agrawal, 2001]:

$$\frac{\partial A(t, z)}{\partial z} + \frac{i\beta_2}{2} \frac{\partial^2 A(t, z)}{\partial t^2} + \frac{\alpha}{2} A(t, z) - i\gamma |A(t, z)|^2 A(t, z) = 0$$

(8.5.23)

where $A(t, z)$ is a time and position dependent complex optical field along the fiber, $z$ is the longitudinal coordinate along the fiber, $\beta_2$ is the chromatic dispersion parameter, $\alpha$ is the fiber loss parameter, and $\gamma = n_2 \omega / (c A_{eff})$ is the nonlinear parameter with $n2$ the
nonlinear refractive index, \( w \) the optical frequency, \( c \) the speed of light and \( A_{\text{eff}} \) the effective cross section area of the fiber core. We have neglected high order dispersion terms for simplicity.

Because the complex optical field \( A(t, z) \) is both time and position dependent, there is no standard analytical solution to the nonlinear differential equation 8.5.23. Numerical simulation is commonly used for transmission performance estimation. In this section, we first introduce split-step Fourier method of numerical simulation, and then discuss analytical and semi-analytical solutions for the impact of specific nonlinear degradation mechanisms.

Split-step Fourier method (SSFM) is a popular technique commonly used in numerical simulations for fiber-optic system performance evaluation. It models the dispersion effect in the frequency domain and nonlinear Kerr-Effect in the time domain. Because signal optical power reduces gradually along the fiber due to attenuation, SSFM divides the transmission fiber into many short sections so that an average power can be used for each section.

Based on the nonlinear differential equation 8.5.23, if only linear effects including dispersion and attenuation of the fiber are considered, Equation 8.5.23 reduces to,

\[
\frac{\partial A(t, z)}{\partial z} = \frac{-i\beta_2}{2} \frac{\partial^2 A(t, z)}{\partial t^2} - \frac{\alpha}{2} A(t, z) \tag{8.5.24}
\]

which has an analytic solution in the frequency domain as,

\[
\tilde{A}(-\omega, L) = \exp\left(\frac{i\beta_2\omega^2}{2} - \frac{\alpha}{2} L\right) \tilde{A}(-\omega, 0) \tag{8.5.25}
\]

Where, \( L \) is the fiber length. \( \tilde{A}(-\omega, L) \) and \( \tilde{A}(-\omega, 0) \) are the Fourier transforms of \( A(t, L) \) and \( A(t, 0) \) at the output (\( z = L \)) and the input (\( z = 0 \)) of the fiber, respectively.

On the other hand, if only the nonlinear Kerr-Effect is considered, and assuming the optical power is constant along the fiber, Equation 8.5.23 reduces to,

\[
\frac{\partial A(t, z)}{\partial z} = i\gamma |A(t, z)|^2 A(t, z) \tag{8.5.26}
\]
it can be solved analytically in the time domain, and the solution relates the optical field at the fiber input $A(t,0)$ and that at the fiber output $A(t,L)$ by,

$$A(t,L) = A(t,0) \exp(i \gamma |A|^2 L)$$  

(8.5.27)

To generalize, we can separate the frequency domain and the time domain solutions into two operators for the nonlinear differential equation:

$$\frac{\partial A(t,z)}{\partial z} = (\hat{D} + \hat{N}) A(t,z)$$  

(8.5.28)

Where,

$$\hat{D} = - \frac{i \beta_2}{2} \frac{\partial^2}{\partial t^2} - \frac{\alpha}{2}$$  

(8.5.29)

is a linear differential operator for dispersion and loss, and

$$\hat{N} = i \gamma |A(t,z)|^2$$  

(8.5.30)

is a nonlinear operator representing the nonlinear phase shift.

In practical a fiber system, signal optical power $|A(t,z)|^2$ is a function of $z$, and the dispersion parameter $\beta_2$ may also be $z$-dependent, which makes the solution of Equation (8.5.28) difficult. In the numerical solution process, one can divide the optical fiber into $k$ sections as illustrated in Figure 8.5.10, where $h_n$ is the length of the $n^{th}$ section. The purpose of dividing the fiber into many short sections is to make sure that within each section both $|A(t,z)|^2$ and $\beta_2$ are independent of $z$, so that an average value can be used for each of them within a short section.

![Figure 8.5.10 Divide a long fiber into k short sections for split-step Fourier simulation](image)

Figure 8.5.10 Divide a long fiber into $k$ short sections for split-step Fourier simulation
Assume that over a short fiber section \( n \) with length \( h_n \), as shown in Figure 8.5.10, dispersion and nonlinear effects act independently, the solution of Equation 8.5.28 can be written as,

\[
A(t, z + h_n) = \exp(\hat{D} h_n) \cdot \exp(\hat{N} h_n) A(t, z)
\]  
(8.5.31)

The linear operator representing dispersion and attenuation can be calculated in the Fourier domain as,

\[
\exp(\hat{D} h_n) A(t, z) = F^{-1} \left\{ \exp[h_n \hat{D}(j \omega)] \cdot F[A(t, z)] \right\}
\]  
(8.5.32)

where, \( F(x) \) and \( F^{-1}(x) \) represent Fourier transform and inverse Fourier transform, respectively, and the operator in the frequency domain is,

\[
\hat{D}(j \omega) = -i \left( \frac{\lambda_0^2}{4 \pi} \right) \omega^2 D - \frac{\alpha}{2}
\]  
(8.5.33)

where, \( \lambda_0 \) is the central wavelength of the optical signal, and \( D = -2 \pi c \beta_2 / \lambda_0^2 \).

The nonlinear effect can be calculated directly in the time domain as,

\[
\exp(\hat{N} h_n) A(t, z) = \exp \left( i \gamma |A(z, t)|^2 h_n \right) A(t, z)
\]  
(8.5.34)

This allows us to find the relationship between the input field \( A(t, z) \) and the output field \( A(t, z + h_n) \) of section \( n \),

\[
A(t, z + h_n) = \exp \left( i \gamma |A(z, t)|^2 h_n \right) F^{-1} \left\{ \exp \left( \frac{i \beta_2 \omega^2}{2} h_n - \frac{\alpha}{2} h_n \right) F[A(z, t)] \right\}
\]  
(8.5.35)

The overall transfer function of the fiber can then be obtained by repeating the process of Equation 8.5.35 section by section from the beginning to the end of the fiber. A Fourier transform and an inverse Fourier transform has to be used for each short fiber section.

As a numerical algorithm, the major advantage of SSFM is that the results automatically include both linear chromatic dispersion effect and the nonlinear Kerr effect. Given an input complex optical field in time-domain \( A(0, t) \) one can find the output complex optical field in time-domain \( A(L, t) \) through a section-by-section calculation. The impacts of self-phase modulation, cross-phase modulation and four-wave mixing are all included.
Figure 8.5.11 shows examples of optical spectra and eye diagrams of a 3 channel WDM system with per channel average optical power of -10dBm, 0dBm and 5dBm, respectively. The system has 80km standard single mode fiber with 16ps/nm/km chromatic dispersion at 1550nm signal wavelength. Fiber cross section is 80 μm² with a nonlinear refractive index $n_2 = 2.35 \times 10^{-20} m^2/W$ and a loss coefficient of 0.25dB/km. NRZ modulation format is used at 10Gb/s, and WDM channel spacing is 100GHz. A dispersion compensator is used at the receiver to compensate the impact of chromatic dispersion. Only the middle channel is selected at the receiver to measure the eye diagram.

Figure 8.5.11 optical spectra (top row) and eye diagrams (bottom row) of 3-channel WDM system with per-channel average powers of -10dBm ((a) and (d)), 0dBm ((b) and (e)), and 5dBm ((c) and (f)) at the input of 80km standard single mode fiber.

Because chromatic dispersion is completely compensated in front of the receiver, the eye diagram is widely open when the average signal optical power is low (-10dBm per channel). With the increase of the launched signal optical power to 0dBm per channel, Kerr effect nonlinearity starts to show its impact in the eye diagram. Further increasing the per channel signal power to 5dBm, the effect of nonlinearity becomes significant, primarily due to nonlinear crosstalk introduced through four-wave mixing (FWM). A number of new frequency components are generated in the optical spectrum as shown in Figure 8.5.11(c). As has been discussed in section 2.6, the newly generated mixing
products coherently interfere with the original optical channel at the same frequency which creates an eye closure penalty.

Examples shown in Figure 8.5.11 are for a system with only a single fiber span of 80km. The split step simulation can also be used for WDM fiber systems with multiple fiber spans with in-line optical amplifiers. This provides a powerful tool for system design and performance evaluation. Notice that the in the simulated results shown in Figure 8.5.11, we have assumed that all signal optical channels are co-polarized along the fiber so that the effect of FWM is the maximum. In a practical system, the state of polarizations (SOP) of WDM channels at different wavelengths may walk-off from each other along the fiber due to polarization mode dispersion (PMD), and thus the actual effect of FWM may be lower especially in long distance fiber-optical systems with multiple fiber spans.

The numerical method of SSFM is a powerful tool for system design, but on the flip side it is usually time consuming in the computation, which can often be prohibitive especially for systems with wide optical bandwidth and large numbers of WDM channels. In such cases the sizes of Fourier transform and inverse Fourier transform have to be excessively large and the speed of computation can be painfully slow. Optical spectra and eye diagrams obtained through numerical simulation shown in Figure 8.5.11 automatically include all Kerr effect nonlinearities including SPM, XPM and FWM. It lacks direct explanation of the physical mechanism behind each individual effect. Therefore, analytic or semi-analytic methods can help understand the impact of each individual nonlinear effect and avoid lengthy numerical processes.

8.5.3 Semi-analytical approaches to evaluate nonlinear crosstalks in fiber-optic systems

In an amplified multispans WDM optical system, interchannel crosstalk is an important concern, especially when the system has a large number of spans and the signal optical power level is high. Although split-step Fourier method discussed in the previous section can be used to evaluate the nonlinear crosstalks through numerical solutions to the nonlinear wave propagation equation, analytical or semi-analytical solutions based on first principles can be very helpful in understanding physical mechanisms.
In this section we discuss analytic and semi-analytic solutions to evaluate the impact of nonlinear crosstalks including XPM, FWM and modulation instability in multispans WDM fiber-optic systems.

### 8.5.3.1 XPM-induced intensity modulation in IMDD optical systems

XPM originates from the Kerr effect in optical fibers, in which intensity modulation of one optical carrier can modulate the phases of other co-propagating optical signals in the same fiber [Chraplyvy 1990; Marcuse 1994]. Unlike phase encoded optical systems, intensity-modulation direct-detection (IMDD) optical systems are not particularly sensitive to signal phase fluctuations. Therefore XPM-induced phase modulation alone is not a direct source of performance degradation in IMDD systems. However, due to the chromatic dispersion of optical fibers, phase modulation can be converted into intensity modulation [J. Wang 1992] and thus can degrade the IMDD system performance. On one hand, nonlinear phase modulation created by XPM is inversely proportional to the signal baseband modulation frequency [Chiang 1996]; on the other hand, the efficiency of phase noise to intensity noise conversion through chromatic dispersion increases with the modulation frequency [J. Wang 1992]. Therefore XPM-induced overall intensity modulation is a complicated function of the signal modulation frequency.

The theoretical analysis of XPM begins with the nonlinear wave propagation equation 8.5.23. As illustrated in Figure 8.5.12, assume that there are only two wavelength channels copropagating along the fiber. They are defined as the probe and the pump, and their optical fields are denoted as \( A_j(t, z) \) and \( A_k(t, z) \), respectively. The evolution of the probe wave (a similar equation can be written for the pump wave) is described by

\[
\frac{\partial A_j(t, z)}{\partial z} = -\frac{\alpha}{2} A_j(t, z) - \frac{1}{v_j} \frac{\partial A_j(t, z)}{\partial t} - i\beta_2 \frac{\partial^2 A_j(t, z)}{\partial t^2} + i\gamma_j p_j(t, z) A_j(t, z) + i\gamma_j 2 p_k(t - z / v_k, z) A_j(t, z)
\]

(8.5.36)

where \( \alpha \) is the fiber attenuation coefficient, \( \beta_2 \) is the chromatic dispersion parameter, \( \gamma_j = 2 n_2 / (\lambda_j A_{\text{eff}}) \) is the nonlinear coefficient, \( n_2 \) is the nonlinear refractive index, \( \lambda_j \) and \( \lambda_k \) are the probe and the pump wavelengths, \( A_{\text{eff}} \) is the fiber effective core area, and \( p_k = |A_k|^2 \) and \( p_j = |A_j|^2 \) are optical powers of the pump and the probe, respectively. Due to
Chromatic dispersion, the pump and the probe waves generally travel at different speeds, and this difference must be taken into account in the calculation of XPM because it introduces the walk-off between the two waves. Here we use $v_j$ and $v_k$ to represent the group velocities of these two channels.

On the right-hand-side of Equation 8.5.36, the first term represents the effect of attenuation, the second term is the linear propagation group delay, the third term accounts for chromatic dispersion, the fourth term is responsible for SPM, and the fifth term is the XPM on the probe signal $j$ induced by the pump signal $k$. The strength of XPM is proportional to the optical power of the pump and the fiber nonlinear coefficient. To simplify our analysis and focus the investigation on the effect of XPM-induced interchannel crosstalk, the interaction between SPM and XPM has to be neglected, assuming that these two act independently. We also assume that the probe is operated in CW, whereas the pump is modulated with a sinusoid at a frequency $\Omega$. Although the effects of SPM for both the probe and the pump channels are neglected in the XPM calculation, a complete system performance evaluation can take into account the effect of SPM and other nonlinear effects separately. This approximation is valid as long as the pump signal waveform is not appreciably changed by the SPM-induced distortion within the nonlinear length of the fiber. Under this approximation the fourth term on the right side of Equation 8.5.36 can thus be neglected. Using variable substitutions $T = t - z / v_j$ and $A_j(t,z) = E_j(T,z)\exp(-\alpha z / 2)$, Equation 8.5.36 becomes,

$$\frac{\partial E_j(T,z)}{\partial z} = -i\beta_2 \frac{\partial^2 E_j(T,z)}{\partial T^2} + i\gamma_j 2p_k(T - d_{jk} z,0)\exp(-\alpha z)E_j(T,z) \quad (8.5.37)$$

where $d_{jk} \equiv (1/v_j) - (1/v_k)$ is the relative pump/probe walk-off, which can be linearized as $d_{jk} = D\Delta \lambda_{jk}$ if the channel separation $\Delta \lambda_{jk}$ is not too wide. $D = -2\pi\beta_2 / \lambda^2$ is the fiber dispersion coefficient, $\lambda$ is the average wavelength, and $c$ is the light velocity. This linear approximation of $d_{jk}$ neglected higher-order dispersion effects.
In general, dispersion and nonlinearity act together along the fiber. However, as illustrated by Figure 8.5.13, in an infinitesimal fiber section $dz$, we can assume that the dispersive and the nonlinear effects act independently, the same idea as used in the split-step Fourier method discussed in the last section. Let $E_j(T,z) = |E_j| \exp[i \phi_j(T,z)]$, where $|E_j|$ and $\phi_j$ are the amplitude and the phase of the probe channel optical field. Taking into account the effect of XPM alone, at $z = z'$, the nonlinear phase modulation in the probe signal induced by the pump power in the small fiber section $dz$ can be obtained as

$$d\phi_j(T,z') = \gamma_j 2p_k(T-d_{jk}z',0) \exp(-\alpha z') dz$$

The Fourier transformation of this phase variation gives

$$d\tilde{\phi}_j(\Omega,z') = 2\gamma_j p_k(\Omega,0) e^{(-\alpha+i\Omega d_{jk})z'} dz$$ \hspace{1cm} (8.5.38)

Neglecting the intensity fluctuation of the probe channel, this phase change corresponds to a change in the electrical field, $\overline{E}_j \exp[i d\phi_j(T,z')] \approx \overline{E}_j[1 + i d\phi_j(T,z')]$, or, in the Fourier domain, $\overline{E}_j[1 + i d\tilde{\phi}_j(\Omega,z')]$, where $d\tilde{\phi}_j(\Omega,z')$ is the Fourier transform of $d\phi_j(T,z')$, and $\overline{E}_j$ represents the average field amplitude.

Due to chromatic dispersion of the fiber, the phase variation generated at location $z = z'$ is converted into an amplitude variation at the end of the fiber $z = L$. Taking into account a source term of nonlinearity-induced phase perturbation at $z = z'$ and the effect of chromatic dispersion, the Fourier transform of Equation 8.5.37 becomes
\[
\frac{\partial \tilde{E}_j(\Omega, z)}{\partial z} = \frac{i\beta_2 \Omega^2}{2} \cdot \tilde{E}_j(\Omega, z) + \tilde{E}_j[1 + id\phi(\Omega, z')] \delta(z - z')
\]

where the Kronecker delta \( \delta(z-z') \) is introduced to take into account the fact that the source term exists only in an infinitesimal fiber section at \( z = z' \). Therefore, at the fiber output \( z = L \), the probe field is

\[
\tilde{E}_j(\Omega, L) = \tilde{E}_j + id\phi(\Omega, z') \tilde{E}_j \exp[i\beta_2 \Omega^2 (L - z') / 2]
\]

Figure 8.5.13 Illustration of elementary contribution of XPM from a short fiber section.

The optical power variation caused by the nonlinear phase modulation created in the short section \( dz \) at \( z = z' \) is thus

\[
\Delta \tilde{\alpha}_{jk}(\Omega, z', L) = \left| \tilde{E}_j(\Omega, L) \right|^2 - \tilde{E}_j^2 = -2 \tilde{E}_j^2 d\phi(\Omega, z') \sin[\beta_2 \Omega^2 (L - z') / 2]
\]

where a linearization has been made considering that \( d\phi \) is infinitesimal. Using \( E_j(T, z) = A_j(T + z / v_j, z) \exp(\alpha z / 2) \) and Equation 8.5.38, integrating all nonlinear phase contributions along the fiber, the accumulated intensity fluctuation at the end of the fiber can be obtained as

\[
\Delta \tilde{\alpha}_{jk}(\Omega, L) = -4\gamma_j p_j(0)e^{-(\alpha - \kappa v_j)L} \int_0^L p_k(\Omega, 0) \sin[\beta_2 \Omega^2 (L - z') / 2]e^{-(\alpha - \Omega \Delta d_{jk})z'} dz' \tag{8.5.39}
\]

where \( \Delta \tilde{\alpha}_{jk}(\Omega, L) = \Delta \tilde{\alpha}_{jk}(\Omega, L)e^{-\alpha L} \) represents the fluctuation of \( A_j \) at frequency \( \Omega \). After integration, we have

\[
\Delta \tilde{\alpha}_{jk}(\Omega, L) = 2p_j(L)\gamma_j e^{\alpha v_j L} \left\{ \frac{p_k(\Omega, 0)\left[ \exp(i\beta_2 \Omega^2 L / 2) - \exp(-\alpha + i\Omega d_{jk})L \right]}{i(\alpha - i\Omega d_{jk} + i\beta_2 \Omega^2 / 2)} - \frac{\exp(-i\beta_2 \Omega^2 L / 2) - \exp(-\alpha + i\Omega d_{jk})L}{i(\alpha - i\Omega d_{jk} - i\beta_2 \Omega^2 / 2)} \right\} \tag{8.5.40}
\]
where \( p_j(0) \) and \( p_j(L) \) are the probe optical powers at the input and the output of the fiber, respectively. If the fiber length is much longer than the nonlinear length, \( \exp(-\alpha L) \ll 1 \), and the modulation bandwidth is much smaller than the channel spacing, i.e., \( d_{jk} \gg \beta_2 \Omega / 2 \). A much simpler frequency domain description of the XPM-induced intensity fluctuation can be derived for the probe channel:

\[
\Delta \tilde{S}_{jk}(\Omega, L) = 4\gamma_j p_j(L)p_k(\Omega,0) \frac{\sin(\beta_2 \Omega^2 L / 2)}{\alpha - i\Omega d_{jk}} e^{i\Omega v_j L} \quad (8.5.41)
\]

Equation 8.5.41 can be further generalized to analyze multispan optically amplified systems, where the total intensity fluctuation at the receiver is the accumulation of XPM contributions created by each fiber span, as illustrated in Figure 8.5.14. For a system with \( N \) amplified fiber spans, the nonlinear phase modulation created in the \( m \)-th span produces an intensity modulation \( \Delta \tilde{S}^{(m)}_{jk}(\Omega, L_N) \) at the end of the system. Even though the phase modulation creation depends only on the pump power and the pump/probe walk-off within the \( m \)-th span, the phase-to-intensity conversion depends on the accumulated dispersion of the fibers from the \( m \)-th to the \( N \)-th fiber spans, and therefore

\[
\Delta \tilde{S}^{(m)}_{jk}(\Omega, L_N) = 4\gamma_j p_j(L_N)p_k^{(m)}(\Omega,0) \exp[i\Omega \sum_{n=1}^{m-1} d_{jk}^{(m)} L^{(n)} / 2] \frac{\sin[\Omega^2 \sum_{n=m}^{N} \beta_2^{(n)} L^{(n)} / 2]}{\alpha - i\Omega d_{jk}^{(m)}} \exp(i\Omega L_N / v_j)
\]

(8.5.42)

where \( L_N = \sum_{n=1}^{N} L^{(n)} \) is the total fiber length in the system, \( L^{(m)} \) and \( \beta_2^{(m)} \) are fiber length and dispersion of the \( m \)-th span (where \( L^{(0)} = 0 \)), \( p_k^{(m)}(\Omega,0) \) is the pump signal input power spectrum in the \( m \)-th span, and \( d_{jk}^{(m)} \) is the relative walk-off between two channels in the \( m \)-th span (where \( d_{jk}^{(0)} = 0 \)). To generalize the single-span XPM expression Equations 8.5.41 to 8.5.42, which represents a multispan system, the term \( \sin(\beta_2 \Omega^2 L / 2) \) in Equation 8.5.41 has to be replaced by \( \sin[\Omega^2 \sum_{n=m}^{N} \beta_2^{(n)} L^{(n)} / 2] \) in Equation 8.5.42 to take into account the linear accumulation of dispersion. Another important effect that has to be taken into account is the different propagation speeds between the pump and the probe.
wavelengths. The phase difference between the pump and the probe at the input of the \( m \)-th span is different from that at the input of the first span. The walk-off-dependent term 
\[ \exp[\Omega \sum_{n=1}^{m} d_{jk}^{(n)} L^{(n)}] \] in Equation 8.5.42 takes into account the walk-off between the probe and the pump channels before they both enter into the \( m \)-th fiber span.

Finally, contributions from all fiber spans add up, as illustrated in Figure 8.5.14, and therefore the intensity fluctuation induced by the XPM of the whole system can be expressed as

\[
\Delta \overline{S}_{jk}(\Omega, L_N) = \sum_{m=1}^{N} \Delta \overline{S}_{jk}^{(m)}(\Omega, L_N) \tag{8.5.43}
\]

In the time domain, the output probe optical power with XPM-induced intensity crosstalk is

\[
p_{jk}(t, L_N) = p_j(L_N) + \Delta S_{jk}(t, L_N) \tag{8.5.44}
\]

where \( \Delta S_{jk}(t, L_N) \) is the inverse Fourier transform of \( \Delta \overline{S}_{jk}(\Omega, L_N) \) and \( p_j(L_N) \) is the probe output without XPM. \( \Delta S_{jk}(t, L_N) \) has a zero mean.

![Multispan amplified optical system](image)

Figure 8.5.14 Linear superposition of XPM contributions from each amplified span.

When the probe signal reaches an optical receiver, the electrical power spectral density at the output of the photodiode is the Fourier transform of the autocorrelation of the time domain optical intensity waveform. Therefore we have

\[
\rho_j(\Omega, L_N) = \eta^2 \left\{ p_j^2(L_N) \delta(\Omega) + \left| \Delta \overline{S}_{jk}(\Omega, L_N) \right|^2 \right\} \tag{8.5.45}
\]
where $\delta$ is the Kronecker delta and $\eta$ is the photodiode responsivity. For $\Omega > 0$, the XPM-induced electrical domain power spectral density in the probe channel, normalized to its power level without an XPM effect, can be expressed as

$$
\Delta p_{ji}(\Omega, L_N) = \frac{\eta^2}{\eta^2 p_j^2(L_N)} \left[ \sum_{i=1}^{N'} 4\gamma_j p_j^{(i)}(\Omega,0) \exp[i\Omega \sum_{n=1}^{i-1} d_{jk}^{(n)} L^{(n)}] \sin[\Omega^2 \sum_{n=1}^{i-1} \beta_n^{(n)} L^{(n)} / 2] \right] \left[ \alpha - i\Omega \epsilon_j^{(i)} \right]
$$

(8.5.46)

$\Delta p_{ji}(\Omega, L_N)$ can be defined as a normalized XPM power transfer function, which can be directly measured by a microwave network analyzer. It is worth noting that in the derivation of Equation 8.5.46, the waveform distortion of the pump signal has been neglected. This is indeed a small signal approximation, which is valid when the XPM-induced crosstalk is only a weak perturbation to the probe signal [Shtaif 1997]. In fact, if this crosstalk level is less than, for example, 20 percent of the signal, the second-order effect caused by the small intensity fluctuation through SPM in the pump is considered negligible.

To characterize the XPM-induced interchannel crosstalk and its impact on optical system performance, it is relatively easy to perform a frequency-domain transfer function measurement. A block diagram of the experimental setup is shown in Figure 8.5.15. Two external-cavity tunable semiconductor lasers (ECL) emitting at $\lambda_j$ and $\lambda_k$, respectively, are used as sources for the probe and the pump. The probe signal is CW and the pump signal is externally modulated by a sinusoid signal from a microwave network analyzer. The two optical signals are combined by a 3 dB coupler and then sent to an EDFA to boost the optical power. A tunable optical filter is used before the receiver to select the probe signal and suppress the pump signal. After passing through an optical preamplifier, the probe signal is detected by a wideband photodiode, amplified by a microwave amplifier, and then sent to the receiver port of the network analyzer. The transfer function measured in this experiment is the relationship between the frequency-swept input pump and its crosstalk into the output probe.
As an example, Figure 8.5.16 shows the normalized XPM frequency response measured at the output of a fiber link consisting of a single 114 km span of nonzero dispersion-shifted fiber (NZDSF). The channel spacings used to obtain this figure were 0.8 nm ($\lambda_j=1559$ nm, $\lambda_k=1559.8$ nm) and 1.6 nm ($\lambda_j=1559$ nm, $\lambda_k=1560.6$ nm). Corresponding theoretical results obtained from Equation 8.5.46 are also plotted in the same figure. To have the best fit to the measured results, parameters used in the calculation were chosen to be $\lambda_0 = 1520.2$ nm, $S_0 = 0.075$ ps/km/nm$^2$, $n_2 = 2.35 \cdot 10^{-20}$ m$^2$/W, $A_{eff} = 5.5 \cdot 10^{-11}$ m$^2$, and $\alpha = 0.25$ dB/km. These values agree with nominal parameter values of the NZDSF used in the experiment. Both the probe and the pump signal input optical powers were 11.5 dBm, and the pump channel modulation frequency was swept from 50 MHz to 10 GHz. To avoid significant higher-order harmonics generated from the LiNbO$_3$ Mach-zehnder intensity modulator, the modulation index is chosen to be approximately 50 percent. High-pass characteristics are clearly demonstrated in both curves in Figure 8.5.16. This is qualitatively different from the frequency dependence of phase-modulation described in the next section, where the conversion from phase modulation to intensity modulation through fiber dispersion does not have to be accounted for and the phase variation caused by the XPM process has a lowpass characteristic [Chiang 1996]. In an ideal IMDD system, the phase modulation of the probe signal by itself does not affect the system performance. However, when a non-ideal optical filter is involved, it may convert the phase noise to intensity noise. This is significant in the low frequency part where XPM-induced probe phase modulation is
The discrepancy between theoretical and experimental results in the low-frequency part of Figure 8.5.16 is most likely caused by electrostriction nonlinearity and the transverse acoustic wave resonating between the center of the fiber core and the circumference of the fiber cladding which causes resonance in the frequency regime lower than 1GHz [Hui, electrostriction, JLT 2015].

Figure 8.5.16 XPM frequency response in the system with single span (114 km) nonzero dispersion shifted fiber. Stars: 0.8 nm channel spacing ($\lambda_{\text{probe}}=1559$ nm and $\lambda_{\text{pump}}=1559.8$ nm), open circles: 1.6 nm channel spacing ($\lambda_{\text{probe}}=1559$ nm and $\lambda_{\text{pump}}=1560.6$ nm). Continuous lines are corresponding theoretical results.

To demonstrate the effect of XPM in multispan systems, Figure 8.5.17 shows the XPM frequency response of a two-span NZDSF system (114 km for the first span and 116 km for the second span), where, again, two channel spacings of 0.8 nm and 1.6 nm were used and the optical power launched into each fiber span was 11.5 dBm. In this multispan case, the detailed shape of the XPM frequency response is strongly dependent on the channel spacing, and the ripples in the XPM spectral shown in Figure 8.5.17 are due to interference between XPM-induced crosstalk created in different fiber spans. For this two-span system, the notch frequencies in the spectrum can be found from Equation 8.5.46 as approximately

$$1 + e^{i\alpha d_{\mu} L_1} = 0 \quad (8.5.47)$$
The frequency difference between adjacent notches in the spectrum is thus, 
\[ \Delta f = \frac{1}{(d_{jk} L_1)} \], where \( L_1 \) is the fiber length of the first span.

Figure 8.5.17 XPM frequency response in the system with two spans (114 km and 116 km) of NZDSF. Stars: 0.8 nm channel spacing (\( \lambda_{\text{probe}} = 1559 \) nm and \( \lambda_{\text{pump}} = 1559.8 \) nm), open circles: 1.6 nm channel spacing (\( \lambda_{\text{probe}} = 1559 \) nm and \( \lambda_{\text{pump}} = 1560.6 \) nm). Continuous lines are corresponding theoretical results.

Because the resonance structure of the XPM transfer function is caused by interactions between XPM created in various fiber spans, different arrangements of fiber system dispersion maps may cause dramatic changes in the overall XPM transfer function. As another example, the XPM frequency response measured in a three-span system is shown in Figure 8.5.18, where the first two spans are 114 km and 116 km of NZDSF and the third span is 75 km of standard SMF. In this experiment, the EDFAs are adjusted such that the optical power launched into the first two spans of NZDSF is 11.5 dBm and the power launched into the third span is 5 dBm. Taking into account the larger spot size, \( A_{\text{eff}} = 80 \mu m^2 \), of the standard SMF (about 55 \( \mu m^2 \) for NZDSF) and the lower pump power in the third span, the nonlinear phase modulation generated in the third span is significantly smaller than that generated in the previous two spans.
Comparing Figure 8.5.18 with Figure 8.5.17, we see that the level increase in the crosstalk power transfer function in Figure 8.5.18 is mainly due to the high dispersion in the last standard SMF span. This high dispersion results in a high efficiency of converting the phase modulation, created in the previous two NZDSF spans, into intensity modulation. As a reasonable speculation, if the standard SMF were placed at the first span near the transmitter, the XPM crosstalk level would be much lower.

![Power transfer function (dB)](image)

Figure 8.5.18 XPM frequency response in a system with two spans (114 km and 116 km) of NZDSF and one span (75km) of normal SMF. Stars: 0.8 nm channel spacing ($\lambda_{\text{probe}}=1559$ nm and $\lambda_{\text{pump}}=1559.8$ nm), open circles: 1.6 nm channel spacing ($\lambda_{\text{probe}}=1559$ nm and $\lambda_{\text{pump}}=1560.6$ nm).

So far we have discussed the normalized frequency response of XPM-induced intensity crosstalk and the measurement technique. It would also be useful to find its impact on the performance of optical transmission systems. Even though the CW waveform of the probe simulates only the continuous "1"s in an NRZ bit pattern, the results may be generalized to pseudo-random signal waveforms. It is evident in Equation 8.5.41 that the actual optical power fluctuation of the probe output caused by XPM is directly proportional to the unperturbed optical signal of the probe channel. Taking into account the actual waveforms of both the pump and the probe, XPM-induced crosstalk from the pump to the probe can be obtained as
where $m_j(t)$ is the normalized probe waveform at the receiver and $m_k(t)$ is the normalized pump waveform at the transmitter. For pseudo-random bit patterns, $m_{j,k}(t) = u_{j,k}(t)/2P_{j,k}^{av}$ with $u_{j,k}$ the real waveforms, and $P_{j,k}^{av}$ the average optical powers $F$ and $F^{-1}$ indicate Fourier and inverse Fourier transformations. $H_j(\Omega)$ is the receiver electrical power transfer function for the probe channel.

It is important to mention here that the expression of $\Delta p_{j,k}(\Omega, L)$ in Equation 8.5.46 was derived for a CW probe, so that Equation 8.5.48 is not accurate during probe signal transitions between 0s and 1s. In fact, XPM during probe signal transitions may introduce an additional time jitter, which is neglected in this analysis. It has been verified experimentally that XPM-induced time jitter due to a probe pattern effect was negligible compared to the XPM-induced eye closure at signal 1 in a system with NRZ coding; therefore the CW probe method might still be an effective approach [Eiselt 1999]. Another approximation in this analysis is the omission of pump waveform distortion during transmission. This may affect the details of the XPM crosstalk waveforms calculated by Equation 8.5.48. However, the maximum amplitude of $C_{j,k}(t)$, which indicates the worst-case system penalty, will not be affected as long as there is no significant change in the pump signal optical bandwidth during transmission. In general, the impact of XPM crosstalk on system performance depends on the bit rate of the pump channel, XPM power transfer function of the system, and the baseband filter transfer function of the receiver for the probe channel.

To understand the impact of XPM on the system performance, it is helpful to look at the time-domain waveforms involved in the XPM process. As an example, trace (a) in Figure 8.5.19 shows the normalized waveform (optical power) of the pump channel, which is a 10 Gb/s ($2^7$-1) pseudo-random bit pattern, band-limited by a 7.5 GHz raised-cosine filter. Suppose that the probe is launched into the same fiber as a CW wave and its amplitude is normalized to 1. Due to XPM, the probe channel is intensity modulated by the pump, and the waveforms created by the XPM process for two different system configurations are shown by traces (b) and (c) in Figure 8.5.19. Trace (b) was obtained in
a single-span system with 130 km NZDSF, whereas trace (c) shows the XPM crosstalk waveform calculated for a three-span system with 130 km NZDSF + 115 km NZDSF + 75 km standard SMF. Looking at these time-domain traces carefully, we can see that trace (b) clearly exhibits a simple highpass characteristic that agrees with the similar waveform measured and reported in [Rapp 1997] in a single-span fiber system. However, in multispan systems, XPM transfer functions are more complicated. Trace (c) in Figure 8.5.19 shows that the amplitude of the crosstalk associated with periodic 0 1 0 1 patterns in the pump waveform has been significantly suppressed.

Figure 8.5.19 Time-domain waveforms. Trace a: input pump signal (10 Gb/s (2^7-1) pseudo-random bit pattern). Trace b: XPM crosstalk of the probe channel in a single-span 130km NZDSF system. Trace c: XPM crosstalk of the probe channel in a three-span system with 130 km NZDSF + 115 km NZDSF + 75 km normal SMF.

To help you better understand the features in the time-domain waveforms obtained with various system configurations, Figure 8.5.20 shows the XPM power transfer functions in the frequency-domain corresponding to trace (b) and trace (c) in Figure 8.5.19. In the single-span case, the crosstalk indeed has a simple highpass characteristic. For the three-span system, the XPM power transfer function has a notch at the frequency close to the half bit rate, which suppresses the crosstalk of 0 1 0 1 bit patterns in the time domain.
Figure 8.5.20 XPM power transfer functions: (b) corresponds to trace (b) in Figure 8.5.19 and (c) corresponds to trace (c) in Figure 8.5.19.

It is worth mentioning that the crosstalk waveforms shown in Figure 8.5.19 were calculated before an optical receiver. In practice, the transfer function and the frequency bandwidth of the receiver will reshape the crosstalk waveform and may have a strong impact on system performance. After introducing a receiver transfer function, XPM-induced eye closure \( ECL \) in the receiver of a system can be evaluated from the amplitude in the crosstalk waveform for the probe channel. The worst-case eye closure happens with \( m_j(t) = 1 \), where \( ECL_{(m_j=1)} = \frac{\max[C_{jk}(t)] - \min[C_{jk}(t)]}{2} \). It is convenient to define this eye closure as a *normalized XPM crosstalk*. In a complete system performance evaluation, this normalized XPM crosstalk penalty should be added on top of other penalties, such as those caused by dispersion and SPM. Considering the waveform distortion due to transmission impairments, the received probe waveform typically has \( m_j(t) \leq 1 \), especially for isolated 1s. Therefore normalized XPM crosstalk gives a conservative measure of system performance.

In WDM optical networks, different WDM channels may have different data rates. The impact of probe channel bit rate on its sensitivity to XPM-induced crosstalk is affected by the receiver bandwidth. Figure 8.5.21(a) shows the normalized power crosstalk levels versus the probe channel receiver electrical bandwidth for 2.5 Gb/s, 10 Gb/s, and 40 Gb/s bit rates in the pump channel. This figure was obtained for a single-span system of 100 km with a dispersion of 2.9 ps/nm/km, launched optical power of 11.5 dBm, and a
channel spacing of 0.8 nm. In this particular system, we see that for a bit rate of higher than 10 Gb/s, the XPM-induced crosstalk is less sensitive to the increase in the pump bit rate. This is because the normalized XPM power transfer function peaks at approximately 15 GHz for this system. When the pump spectrum is wider than 15 GHz, the XPM crosstalk efficiency is greatly reduced. This is the reason that the difference in the XPM-induced crosstalk between 40 Gb/s and 10 Gb/s systems is much smaller than that between 10 Gb/s and 2.5 Gb/s systems.

Typical receiver bandwidths for 2.5 Gb/s, 10 Gb/s, and 40 Gb/s systems are 1.75 GHz, 7.5 GHz, and 30 GHz, respectively. Figure 8.5.21(a) indicates that when the receiver bandwidth exceeds the bandwidth of the pump channel, there is little increase in the XPM-induced crosstalk level with further increasing of the receiver bandwidth. In principle, the crosstalk between high bit rate and low bit rate channels is comparable to the crosstalk between two low bit rate channels. An important implication of this idea is in hybrid WDM systems with different bit rate interleaving; for example, channels 1, 3, and 5 have high bit rates and channels 2, 4, and 6 have low bit rates. The XPM-induced crosstalk levels in both high and low bit rate channels are very similar and are not higher than the crosstalk level in the system with the low bit rate. However, when the channel spacing is too low, XPM crosstalk from channel 3 to channel 1 can be bigger than that

![Figure 8.5.21 Normalized power crosstalk levels versus the receiver bandwidth for 2.5 Gb/s, 10 Gb/s, and 40 Gb/s bit rates in the pump channel. (a) The system has a 130 km single fiber span with fiber dispersion of 2.9 ps/nm/km and optical channel spacing of 0.8 nm. Launched pump optical power at each span is 11.5 dBm. (b) The system has five fiber spans (100 km/span) with fiber dispersion of 2.9 ps/nm/km and optical channel spacing of 0.8 nm. Launched pump optical power at each span is 8.5 dBm.](image-url)
from channel 2 with a low bit rate. Figure 8.5.21(b) shows the normalized crosstalk levels versus receiver electrical bandwidth in a five-span NZDSF system with a 100 km/span. The fiber dispersion is 2.9 ps/nm/km and the launched optical power at each span is 8.5 dBm. There is little difference in the crosstalk levels for the 10 Gb/s system and the 40 Gb/s system. This is because in systems with higher accumulated dispersion, the XPM power transfer function peaks at a lower frequency and the high-frequency components are strongly attenuated.

Figure 8.5.22 shows the normalized crosstalk versus fiber dispersion for the same system used to obtain Figure 8.5.21(b). The fixed receiver bandwidths used for 40 Gb/s, 10 Gb/s, and 2.5 Gb/s systems are 30 GHz, 7.5 GHz, and 1.75 GHz, respectively. The worst-case XPM crosstalk happens at lower dispersion levels with higher signal bit rates. It is worth noting that for the 10 Gb/s system, the worst-case XPM crosstalk happens when the fiber dispersion parameter is 2.5 ps/nm/km, and therefore the total accumulated dispersion of the system is 1250 ps/nm, which is about the same as the dispersion limit for an uncompensated 10 Gb/s system.

Figure 8.5.22 Normalized power crosstalk levels versus the fiber dispersion for 2.5 Gb/s, 10 Gb/s, and 40 Gb/s bit rates. Five cascaded fiber spans (100 km/span). Optical channel spacing 0.8 nm; 8.5 dBm launched pump optical power at each span.

It needs to be pointed out that, for simplicity, in both Figures 8.5.21(a) and (b), the signal optical powers were chosen to be the same for systems with different bit rates. However, in practice, a higher power level is normally required for a system with a
higher bit rate. A generalization of these results to the case with different signal power levels can be made using the simple linear dependence of XPM crosstalk on the launched power level, as shown in Equation 8.5.41.

Although most people would think that XPM crosstalk was significant only in low dispersion fibers, Figure 8.5.22 clearly indicates that for uncompensated systems, before the system dispersion limit, higher dispersion generally produces more XPM crosstalk. On the other hand, in dispersion compensated optical systems, high local dispersion helps reduce the XPM-induced phase modulation and low accumulated system dispersion will reduce the phase noise to intensity noise conversion.

One important way to reduce the impact of XPM-induced crosstalk in a fiber system is to use dispersion compensation [Saunders 1997]. The position of dispersion compensator in the system is also important. The least amount of dispersion compensation is required if the compensator is placed in front of the receiver. In this position, the dispersion compensator compensates XPM crosstalk created in all fiber spans in the system. The optimum amount of dispersion compensation for the purpose of XPM crosstalk reduction is about 50 percent of the accumulated dispersion in the system [Saunders 1997]. Although this lumped compensation scheme requires the minimum amount of dispersion compensation, it does not give the best overall system performance.

Figure 8.5.23 shows the normalized power crosstalk levels versus the percentage of dispersion compensation in a 10 Gb/s system with six amplified NZDSF fiber spans of 100 km/span. The dispersion of transmission fiber is 2.9 ps/nm/km and the launched optical power into each fiber span is 8.5 dBm. Nonlinear effects in the dispersion-compensating fibers are neglected for simplicity. Various dispersion compensation schemes are compared in this figure. Trace (1) is obtained with compensation in each span. In this scheme XPM-induced crosstalk created from each span can be precisely compensated, so at 100 percent of compensation the XPM crosstalk is effectively eliminated. Trace (2) was obtained with the dispersion compensator placed after every other span. In this case, the value of dispersion compensation can only be optimized for either the first span or the second span but not for both of them. The residual XPM crosstalk level is higher in this case than that with compensation in each span. Similarly,
trace (3) in Figure 8.5.23 was obtained with a dispersion compensator placed after every three spans, and trace (4) is with only one lumped compensator placed in front of the receiver. Obviously, when the number of dispersion compensators is reduced, the level of residual XPM crosstalk is higher and the optimum value of dispersion compensation is closer to 50 percent of the total system dispersion. Therefore, in systems where XPM-induced crosstalk is a significant impairment, per-span dispersion compensation is recommended. However, this will increase the number of dispersion compensators and thus increase the cost.

Figure 8.5.23 Normalized power crosstalk levels versus the percentage of dispersion compensation in a 10 Gb/s, six-span system (100 km/span) with fiber dispersion of 2.9 ps/nm/km. An 8.5 dBm launched pump optical power at each fiber span. (1) Dispersion compensation after each span, (2) dispersion compensation after every two spans, (3) dispersion compensation after every three spans, and (4) one lumped dispersion compensation in front of the receiver.

8.5.3.2 XPM-induced phase modulation

In the last section, we discussed the intensity modulation introduced by XPM in which the phase modulation is converted into intensity modulation through chromatic dispersion. This XPM-induced intensity crosstalk is a source of performance degradation in IMDD systems and introduces eye closure penalty. On the other hand, if the system is phase modulated, most likely the most relevant impact of XPM is the nonlinear phase modulation itself. In fact, from Equation 8.5.38, if we directly integrate the elementary contribution of nonlinear phase created over the entire fiber, the overall contribution is
\[ \tilde{\phi}_j(\Omega) = 2\gamma_j p_k(\Omega,0) \int_0^L e^{(-\alpha + i\Omega d_{jk})z}dz = 2\gamma_j p_k(\Omega,0)\sqrt{\eta_{XPM}} L_{eff} e^{i\theta} \quad (8.5.49) \]

where \( L_{eff} = (1 - e^{-\alpha d_{jk}})/\alpha \) is the effective nonlinear length, and

\[ \eta_{XPM} = \frac{\alpha^2}{\alpha^2 + \Omega^2 d_{jk}^2} \left[ 1 + \frac{4\sin^2(\Omega d_{jk} L / 2)e^{-\alpha d_{jk} L}}{(1 - e^{-\alpha d_{jk} L})^2} \right] \quad (8.5.50) \]

is the XPM-induced phase modulation efficiency, which is obviously a function of the modulation frequency \( \Omega \). The phase term in Equation 8.5.49 is,

\[ \theta = -\tan^{-1}\left( \frac{\Omega d_{jk}}{\alpha} \right) - \tan^{-1}\left[ \frac{e^{-\alpha d_{jk} L} \sin(\Omega d_{jk} L)}{1 - e^{-\alpha d_{jk} L} \cos(\Omega d_{jk} L)} \right] \quad (8.5.51) \]

In time domain, the XPM-induced phase variation of the probe signal can be expressed as [Chiang 1996]

\[ \phi_{XPM}(L,t) = \gamma_j \left[ \tilde{P}_j(0) + 2\tilde{P}_j(0)L_{eff} + |\tilde{\phi}_j(\Omega)| \cos \left[ \Omega \left( t - \frac{L}{v_j} \right) + \theta \right] \right] \quad (8.5.52) \]

where the first term on the right side is a constant nonlinear phase shift with \( \tilde{P}_j(0) \) and \( \tilde{P}_j(0) \) the average input powers of the probe and the pump, respectively. In this case the conversion from phase modulation to intensity modulation through chromatic dispersion has been neglected.

If the system has more than one amplified fiber span, the overall effect of cross-phase modulation is the superposition of contributions from all fiber spans,

\[ \phi_{XPM} \left( \sum_{l=1}^N L^{(l)} , t \right) = \sum_{l=1}^N |\tilde{\phi}^{(l)}_j(\Omega)| \cos \left[ \Omega \left( t - \sum_{n=1}^N L^{(n)}_{v_j} + \sum_{n=1}^{l-1} L^{(n)} d^{(n)}_{jk} + \theta^{(l)} \right) \right] \quad (8.5.53) \]

where \( N \) is the total number of amplified fiber spans, and each term of summation on the right hand side of Equation 8.5.53 represents the XPM-induced phase shift created in the corresponding fiber span. The additional phase term \( \Omega \sum_{n=1}^{l-1} L^{(n)} d^{(n)}_{jk} \) represents the effect of pump-probe phase walk-off before the \( l \)-th span.
XPM-induced phase modulation (PM) can be measured with phase sensitive techniques such as coherent detection which will be discussed in details in the next chapter. Figure 8.5.24 shows examples of the measured XPM-induced phase modulation indices in two-span and three-span amplified fiber-optic systems [Chang 1996]. Similar to that shown in Figure 8.5.15, in this measurement two tunable lasers are used as the probe and the pump at the wavelengths of $\lambda_j$ and $\lambda_k$, respectively. The pump laser is intensity modulated by a sinusoid wave at frequency $\Omega$ through an external modulator. The probe and the pump are combined and sent to an optical system with multiple amplified fiber spans. At the output of the optical system, a narrowband optical filter is used to select the probe wave at $\lambda_j$ while rejecting the pump. Instead of measuring the intensity modulation created by the XPM process, this measurement measures the optical phase modulation created on the probe through XPM using a phase sensitive detection technique. As the average pump power level $P_k(0)$ at the input of each amplified fiber span was equal, the XPM index was defined as a normalized phase modulation efficiency $\phi_{\text{XPM}} / P_k(0)$. The wavelength spacing between the pump and the probe in this measurement was 3.7 nm.

![Figure 8.5.24 XPM index for (a) a two-span system and (b) a three-span system [Chiang 1996].](image)

Figure 8.5.24(a) was obtained in a system with two fiber spans, each having 25 km standard SMF. For Figure 8.5.24(b) there were three fiber spans in the system, again with 25 km standard SMF in each span. In both cases, optical power at the input of each span was set to 7 dBm. If there is no optical amplifier between fiber spans, the XPM index
versus modulation frequency decreases monotonically showing a lowpass characteristic. This is shown in Figure 5.6.15 as the dotted lines that were calculated with a single 50km (a) and 75 km (b) fiber span. When an optical amplifier was added at the beginning of each 25 km fiber span, the XPM indices varied significantly over the modulation frequency. Clearly, this is due to coherent interferences between XPM-induced phase modulations produced in different fiber spans. Compared to the XPM-induced intensity modulation, XPM-induced phase modulation tends to diminish at high modulation frequency, especially when the chromatic dispersion of the fiber is high. From this point of view, XPM-induced intensity modulation discussed in the last section is probably the most damaging effect on high-speed optical systems.

It is important to point out that XPM is the crosstalk originated from the intensity modulation of the pump, which results in the intensity and phase modulations of the probe. In optical systems based on phase modulation on all wavelength channels, XPM will not exist, in principle, because the pump has a constant optical power. However, the phase-coded optical signals carried by the pump wave can be converted into intensity modulation through fiber chromatic dispersion. Then this intensity modulation will be able to produce an XPM effect in the probe channels, thus causing system performance degradation [Ho 2003].

8.5.3.3 FWM-induced crosstalk in IMDD optical systems

Four-wave mixing (FWM) is a parametric process that results in the generation of signals at new optical frequencies:

\[ f_{jkl} = f_j + f_k - f_i \]  \hspace{1cm} (8.5.54)

where \( f_j, f_k \) and \( f_i \) are the optical frequencies of the contributing signals. There will be system performance degradation if the newly generated FWM frequency component overlaps with a signal channel in a WDM system, and appreciable FWM power is delivered into the receiver. The penalty will be greatest if the frequency difference between the FWM product and the signal, \( f_{jkl} - f_i \), lies within the receiver bandwidth. Unfortunately, for signals on the regular ITU frequency grid in a WDM system, this overlapping is quite probable.
Over an optical cable span in which the chromatic dispersion is constant, there is a
closed form solution for the FWM product-power-to-signal-power ratio:

\[
x_s = \frac{P_{jkl}(L_s)}{P_l(L_s)} = \eta L_{\text{eff}}^2 \chi^2 \gamma^2 P_j(0)P_k(0)
\]  
(8.5.55)

where \( L_{\text{eff}} = (1 - \exp(-\alpha L_s))/\alpha \) is the nonlinear length of the fiber span, \( L_s \) is the span
length, \( P_j(0), P_k(0) \) and \( P_l(0) \) are contributing signal optical powers at the fiber input,
\( \chi = 1, 2 \) for nondegenerate and degenerate FWM, respectively, and the efficiency is

\[
\eta = \rho \frac{\alpha^2}{\Delta k_{jkl}^2 + \alpha^2} \left[ 1 + \frac{4e^{-\alpha L_s} \sin(\Delta k_{jkl} L_s / 2)}{(1 - e^{-\alpha L_s})^2} \right]
\]  
(8.5.56)

where the detune is

\[
\Delta k_{jkl} = \frac{2 \pi c}{f_m^2} D(\lambda_m) \left[ (f_j - f_m)^2 - (f_i - f_m)^2 \right]
\]  
(8.5.57)

0 < \( \rho < 1 \) is the polarization mismatch factor, and \( \lambda_m \) is the central wavelength,
corresponding to the frequency of \( f_m = \frac{f_j + f_k}{2} = \frac{f_i + f_{jkl}}{2} \).

In practice, in a multispan WDM system, the dispersion varies not only from span to
span but also between fiber cabling segments, typically a few kilometers in length, within
each span. The contributions from each segment can be calculated using the analytic
solution described earlier, with the additional requirement that the relative phases of the
signals and FWM product must be taken into account in combining each contribution
[Inoue 1995]. The overall FWM contribution is a superposition of all FWM contributions
throughout the system:

\[
a_F = \sum_{\text{spans}} \sum_{\text{segments}} \exp(i \Delta \phi_{jkl}) \frac{P_{jkl}(z)}{P_i(z)}
\]  
(8.5.58)

where \( \Delta \phi_{jkl} \) is the relative phase of FWM generated at each fiber section. The magnitude
of the FWM-to-signal-ratio is quite sensitive, not only to the chromatic dispersions of the
cable segments but also to their distribution, the segment lengths, and the exact optical
frequencies of the contributing signals. Because of the random nature of the relative
phase in each fiber section, statistic analysis may be necessary for system performance evaluation.

In the simplest case when the system has only two wavelength channels, degenerate FWM exists and the new frequency components created on both sides of each original optical signal frequency. However, for a WDM system with multiple equally spaced wavelength channels, there will be a large number of FWM components and almost all of them overlap with the original optical signals to create inter-channel crosstalls. For example, in a WDM system with four equally spaced wavelength channels, there are 10 FWM components which overlap with the original signal channels, as illustrated in Figure 8.5.25(a), where $f_1, f_2, f_3,$ and $f_4$ are the frequencies of the signal optical channels and $f_{ijkl}$ ($j, k, l = 1, 2, 3, 4$) are the FWM components created by the interactions among signals at $f_j, f_k,$ and $f_l$. In system performance evaluation, the power ratio between the original signal channel and the FWM crosstalk created at the same frequency is an important measure of the crosstalk. The spectral overlap between them makes the crosstalk evaluation in the spectral domain difficult. One way to overcome this overlapping problem is to deliberately remove one of the signal channels and observe the power of the FWM components generated at that wavelength, as illustrated in Figure 8.5.25(b). This obviously underestimates the FWM crosstalk because the FWM contribution that would involve that empty channel is not considered. For example, in the four-channel case, if we remove the signal channel at $f_3$, FWM components at frequency $f_3$ would only be $f_{223}$ and $f_{142}$, whereas $f_{243}$ would not exist. But for a WDM system with a large number of signal channels, removing one of the channels does not introduce significant difference in terms of FWM.

\[\text{FWM} \quad \text{WDM channels}\]

\[\text{f}\]

(a)

(b)
Figure 8.5.25 (a) Illustration of FWM components in a four-channel system and (b) evaluating FWM crosstalk in a WDM system with one empty channel slot.

Figure 8.5.26 shows an example of the measured FWM-to-carrier power ratio defined by Equation 8.5.55. The system consists of five amplified fiber spans with 80 km fiber in each span, and the per-channel optical power level at the beginning of each fiber span ranges from 6 dBm to 8 dBm in a random manner. The fiber used in the system has zero-dispersion wavelength at $\lambda_0 = 1564$ nm, whereas the average signal wavelength is approximately $\lambda = 1558$ nm. In this system, three channels were used with frequencies at $f_1$, $f_2$, and $f_3$, respectively. The horizontal axis in Figure 8.5.26 is the frequency separation between $f_1$ and $f_3$, and the frequency of the FWM component $f_{132}$ is 15 GHz away from $f_2$, that is, $f_2 = (f_1 + f_3)/2 - 7.5$ GHz. Because there is no frequency overlap between $f_{132}$ and $f_2$, the FWM-to-carrier power ratio $P_{132}/P_2$ can be measured. Figure 8.5.26 clearly demonstrates that FWM efficiency is very sensitive to the frequency detune, and there can be more than 10 dB efficiency variation, with the frequency change only on the order of 5 GHz. This is mainly due to the rapid change in the phase match conditions as well as the interference between FWM components created at different locations in the system.
The instantaneous four-wave mixing efficiency is also sensitive to the relative polarization states of the contributing signals. In this system, the fiber DGD averaged over the wavelength range 1550–1565 nm is \( \tau = 0.056 \text{ps/}\sqrt{\text{km}} \). From this DGD value, we can estimate the rate at which signals change their relative polarizations (at the system input they are polarization-aligned). For signal launch states that excite both principle states of polarization (PSP) of the fiber, the projection of the output SOP rotates around the PSP vector at a rate of \( d\phi/d\omega = \tau \), where \( \tau \) is the instantaneous DGD and \( \omega \) is the (radian) optical frequency. In crude terms, we expect the signals to remain substantially aligned within the nonlinear interaction length of around 20 km. There may be a significant loss of SOP alignment between adjacent spans.

In practical long-distance optical transmission systems, frequency dithering on the laser source is often used to reduce the effect of stimulated Brillouin scattering (SBS). The SBS dither effectively increases the transmission efficiency of the fiber by decreasing the power loss due to SBS while simultaneously smoothing out the phase match peaks. However, Figure 8.5.26 indicates that SBS dithering only moderately reduces the level of FWM.

![Figure 8.5.27](image_url)

Figure 8.5.27 (a) and (b) WDM optical spectrum with eight equally spaced channels and the measured eye diagram; (c) and (d) WDM optical spectrum with eight unequally spaced channels and the corresponding eye diagram [Forghieri 1995].
An effective way to reduce the FWM crosstalk is to use unequal channel spacing in a WDM system. Figure 8.5.27 shows a comparison between WDM systems with equal and unequal channel spacing through the measurement of both the optical spectra and the corresponding eye diagrams [Forghieri 1995]. This figure was obtained in a system with a single-span 137 km dispersion shifted fiber that carries eight optical channels at 10 Gb/s data rate per channel. In an optical system with equally spaced WDM channels, as shown in Figure 8.5.27 (a) and (b), severe degradation on the eye diagram can be observed at the per-channel signal optical power of 3dBm at the fiber input. This eye closure penalty can be significantly reduced if the wavelengths of the WDM channels are unequally spaced as shown in Figure 8.5.27(c) and (d), although a higher per-channel signal optical power of 5 dBm was used. This is because the FWM components do not overlap with the signal channels so that they can be spectrally filtered out at the receiver. However, non-equal channel spacing is rarely used in commercial optical transmission systems. Instead, standardized ITU wavelength grid is commonly adopted for interoperability of product from different manufacturers and system operators.

Since FWM crosstalk is generated from nonlinear mixing between optical signals, it behaves more like a coherent crosstalk than a noise. To explain the coherent crosstalk due to FWM, we can consider that a single FWM product is created at the same wavelength as the signal channel $f_i$. Assuming that the power of the FWM component is $p_{f_{jkl}}$, which coherently interferes with an amplitude-modulated optical signal whose instantaneous power is $p_s$, the worst-case eye closure occurs when all the contributing signals are at the high level (digital 1). Due to the mixing between the optical signal and the FWM component, the photocurrent at the receiver is proportional to

$$I' \propto \sqrt{p_s + p_{f_{jkl}}} \cos(\Delta \phi + 2\pi(f_i - f_{jkl}))$$

As a result, the normalized signal 1 level becomes

$$A = \frac{p_s + p_{f_{jkl}} \pm 2 \sqrt{p_s p_{f_{jkl}}}}{p_s} \approx 1 \pm 2 \sqrt{\frac{p_{f_{jkl}}}{p_s}}$$

(8.5.60)
This is a “bounded” crosstalk because the worst-case closure to the normalized eye diagram is $2\sqrt{p_{\text{fwm}}/p_s}$.

In a high-density multichannel WDM system there will be more than one FWM product that overlaps with each signal channel. In general, these FWM products will interfere with the signal at independent beating frequencies and phases. The absolute worst-case eye closure can be found by superimposing the absolute value of each contributing FWM product. However, if the number of FWM products is too high, the chance of reaching the absolute worst case is very low. The overall crosstalk then approaches Gaussian statistics as the number of contributors increases [Eiselt 1999].

8.5.3.4 Modulation instability and its impact in WDM optical systems

In addition to nonlinear crosstalk between signal channels such as XPM and FWM discussed above, crosstalk may also happen between optical signal and the broadband ASE noise in the system to cause performance degradations. This can be seen as FWM between the optical signal and the broadband noise, commonly referred to as parametric gain, or modulation instability (MI) [Yu 1995]. The mechanism for system performance degradation caused by MI depends on system type. In optical phase coded transmission systems, degradation is mainly caused by the broadening of the optical spectrum [Mecozzi 1994]. On the other hand, for an intensity-modulated system with direct detection (IMDD), the performance degradation can be introduced by phase noise to intensity noise conversion through chromatic dispersion in the optical fiber. The increased relative intensity noise (RIN) within the receiver baseband is responsible for this degradation.

Since MI is caused by the Kerr effect nonlinearity, its analysis can be based on the nonlinear Schrödinger equation, which was given in Equation 8.5.23 and we repeat here for your convenience,

$$\frac{\partial A(t,z)}{\partial z} + i\beta_z \frac{\partial^2 A(t,z)}{\partial t^2} + \frac{\alpha}{2} A(t,z) - i\gamma |A(t,z)|^2 A(t,z) = 0$$

(8.5.61)

where $A(z,t)$ is the electrical field, $\gamma = \omega_0 n_2 / c A_{\text{eff}}$ is the nonlinear coefficient of the fiber, $\omega_0$ is the angular frequency, $n_2$ is the nonlinear refractive index of the fiber, $c$ is the speed
of light, $A_{\text{eff}}$ is the effective fiber core area, $\beta_2$ is the fiber dispersion parameter, and $\alpha$ is the fiber attenuation. High-order dispersions were ignored here. The $z$-dependent steady-state solution of Equation 8.5.61 is:

$$A_0(z) = \sqrt{P_0} \exp\left(i \gamma |A_0(z)|^2 z \right) \exp\left(-\frac{\alpha}{2} z \right) \tag{8.5.62}$$

Since the signal optical power can vary significantly along the fiber in practical optical systems because of the attenuation, a simple mean-field approximation over the transmission fiber is usually not accurate enough. To obtain a semi-analytical solution, the fiber can be divided into short sections, as illustrated in Figure 8.5.28, and a mean-field approximation can be applied within each section. For example, in the $j$-th section with length $\Delta z_j$, Equation 8.5.61 becomes:

$$\frac{\partial A_j(z,t)}{\partial z} = -\frac{i}{2} \beta_2 \frac{\partial^2 A_j(z,t)}{\partial t^2} + i \gamma_j |A_j(z,t)|^2 A_j(z,t) \tag{8.5.63}$$

where

$$\gamma_j = \frac{1 - \exp(-\alpha \Delta z_j)}{\alpha \Delta z_j} \gamma \tag{8.5.64}$$

Figure 8.5.28 Illustration of dividing fiber into short sections for transfer matrix analysis.

With the assumption that noise power at fiber input is much weaker than pump power, the solution of Equation 8.5.63 can be written as

$$A_j(z,t) = [A_{0j} + \tilde{a}_j(z,t)] \exp\left(i \gamma |A_{0j}|^2 z \right) \tag{8.5.65}$$

where $A_{0j}$ is the steady-state solution of Equation 8.5.63, $\tilde{a}_j(z,t)$ is a small perturbation and $\tilde{a}_j(z,t) \ll A_{0j}$ is assumed. With linear approximation of the noise term, the nonlinear Schrodinger Equation 8.5.63 becomes

82
\[
\frac{\partial \tilde{a}_j(z,t)}{\partial z} = -i \frac{\beta_z}{2} \frac{\partial^2 \tilde{a}_j(z,t)}{\partial t^2} + i \gamma_j \left[ A_{0j} \tilde{a}_j(z,t) + A_{0j}^* \tilde{a}_j^*(z,t) \right] \quad (8.5.66)
\]

where the symbol \(*\) denotes complex conjugate. We need to emphasize that the linearization used to obtain Equation 8.5.66 is valid only when the perturbation is small enough such that the effect of pump depletion can be neglected.

By converting Equation 8.5.66 into frequency domain through Fourier transform [Marcus 1994], the following two equations can be obtained for the real and the imaginary parts of \( \tilde{a}_j(z,t) \), respectively:

\[
\frac{\partial a_j(\omega, z)}{\partial z} = \frac{i}{2} \omega^2 \beta_z a_j(\omega, z) + i \gamma_j \left[ A_{0j} \tilde{a}_j(\omega, z) + A_{0j}^* a_j^*(-\omega, z) \right] \quad (8.5.67)
\]

\[
\frac{\partial a_j^*(-\omega, z)}{\partial z} = -\frac{i}{2} \omega^2 \beta_z a_j^*(-\omega, z) - i \gamma_j \left[ A_{0j} \tilde{a}_j^*(-\omega, z) + A_{0j}^* a_j(\omega, z) \right] \quad (8.5.68)
\]

The formal solution of linear differential Equations 8.5.67 and 8.5.68 can be expressed in a matrix format:

\[
\begin{bmatrix}
  a_{j+1}(\omega, z_{j+1}) \\
  a_{j+1}^*(-\omega, z_{j+1})
\end{bmatrix}
= \begin{bmatrix}
  M_{11}^{(j)} & M_{12}^{(j)} \\
  M_{21}^{(j)} & M_{22}^{(j)}
\end{bmatrix}
\begin{bmatrix}
  a_{j+1}(\omega, z_j) \\
  a_{j+1}^*(-\omega, z_j)
\end{bmatrix}
\]

When we further take into account the linear attenuation of the signal in each section, a factor \( \exp(-\alpha \Delta z_j / 2) \) has to be added so that

\[
\begin{bmatrix}
  a_{j+1}(\omega, z_{j+1}) \\
  a_{j+1}^*(-\omega, z_{j+1})
\end{bmatrix}
= \begin{bmatrix}
  M_{11}^{(j)} & M_{12}^{(j)} \\
  M_{21}^{(j)} & M_{22}^{(j)}
\end{bmatrix}
\begin{bmatrix}
  a_{j+1}(\omega, z_j) \\
  a_{j+1}^*(-\omega, z_j)
\end{bmatrix}
\exp\left(-\frac{\alpha}{2} \Delta z_j \right) \quad (8.5.69)
\]

where \( \Delta z_j = z_{j+1} - z_j \) and

\[
M_{11}^{(j)} = \frac{e^{ik_j z_j} - r_{bj} r_{bj} e^{-ik_j z_j}}{1 - r_{bj} r_{bj}} \quad (8.5.70)
\]

\[
M_{12}^{(j)} = r_{bj} \left( e^{-ik_j z_j} - e^{ik_j z_j} \right) \frac{1}{1 - r_{bj} r_{bj}} \quad (8.5.71)
\]
\[
M^{(j)}_{21} = \frac{r_f (e^{i k z_j} - e^{-i k z_j})}{1 - r_f r_b}
\]  
(8.5.72)

\[
M^{(j)}_{22} = \frac{e^{-i k z_j} - r_f r_b e^{i k z_j}}{1 - r_f r_b}
\]  
(8.5.73)

\[
r_f = \frac{k_j - \beta \omega^2 - \gamma_j |A_{0j}|^2}{\gamma_j A_{0j}^2} = \frac{-\gamma_j A_{0j}^2}{k_j + \beta \omega^2 + \gamma_j |A_{0j}|^2}
\]  
(8.5.74)

\[
r_b = \frac{k_j - \beta \omega^2 - \gamma_j |A_{0j}|^2}{\gamma_j A_{0j}^2} = \frac{-\gamma_j A_{0j}^2}{k_j + \beta \omega^2 + \gamma_j |A_{0j}|^2}
\]  
(8.5.75)

Equations 8.5.74 and 8.5.75 have two eigenmodes whose propagation constants are equal in magnitude and opposite in sign, and they are given by

\[
k_j = \pm \sqrt{ \left( \frac{\beta_2 \omega^2 + \gamma_j |A_{0j}|^2}{2} \right)^2 - \left( \gamma_j |A_{0j}|^2 \right)^2}
\]  
(8.5.76)

The parameters \(r_f\) and \(r_b\) can be regarded as effective reflectivities for the two eigenmodes; therefore the sign of \(k\) should be chosen such that \(|r_f| \leq 1|\) and \(|r_b| \leq 1|\).

The evolution of the noise along the fiber can then be calculated simply by matrix multiplication:

\[
\begin{bmatrix}
  a(\omega, L) \\
  a^*(-\omega, L)
\end{bmatrix} =
\begin{bmatrix}
  B_{11} & B_{12} \\
  B_{21} & B_{22}
\end{bmatrix}
\begin{bmatrix}
  a(\omega, 0) \\
  a^*(-\omega, 0)
\end{bmatrix}
\exp\left( -\frac{\alpha L}{2} \right)
\]  
(8.5.77)

with

\[
\begin{bmatrix}
  B_{11} & B_{12} \\
  B_{21} & B_{22}
\end{bmatrix} = \prod_{j=1}^N
\begin{bmatrix}
  M^{(j)}_{11} & M^{(j)}_{12} \\
  M^{(j)}_{21} & M^{(j)}_{22}
\end{bmatrix}
\]  
(8.5.78)

where \(L\) is the fiber length and \(N\) is the total number of sections.

Let us first look at the power spectral density of the optical field, according to the Wiener-Khintchine theorem it is proportional to the square of the modulus of the Fourier
transformation of the complex field amplitude. If the optical field is sampled over a time interval $T$, this optical power spectral density is

$$S_i(\omega, z) = \left[ \frac{1}{T} \int_{-T/2}^{T/2} A(z, t) \exp(i\omega t) dt \right]^2 \text{ as } T \to \infty$$

Separating the optical field into CW and stochastic components, we have,

$$S_i(\omega, z) = \left\langle |a(\omega, L)|^2 \right\rangle + |A_0(L)|^2 \delta(\omega)$$

where $\langle \rangle$ denotes ensemble average, and normalization by the sample interval. $\delta(\omega)$ is the Kronecker delta function. The noise term has zero mean, so $\langle a(\omega, z) \rangle = 0$ and the cross terms vanish. It is convenient to remove the CW contribution from the power spectrum, so we define

$$S(\omega, L) = S_i(\omega, L) - |A_0(L)|^2 \delta(\omega) = \left\langle |a(\omega, L)|^2 \right\rangle$$

(8.5.79)

Using Equation 8.5.77 we can find

$$S(\omega, L) = \left\langle |a(\omega, L)|^2 \right\rangle \exp(-\alpha L)$$

(8.5.80)

Because $a(\omega, 0)$ is a random process, amplitudes at distinct frequencies are uncorrelated, so $\langle a(\omega, 0) a^*(\omega, 0) \rangle = 0$ and

$$S(\omega, L) = |B_{11}|^2 S(\omega, 0) + |B_{12}|^2 S(-\omega, 0) \exp(-\alpha L)$$

(8.5.81)

where $S(\omega, 0) = \langle a(\omega, 0) a^*(\omega, 0) \rangle$ is the power spectrum of $\tilde{a}(t, 0)$, which is the input noise. To simplify the analysis, we assume that the input noise spectrum is symmetric around the carrier (e.g., white noise): $S(\omega, 0) = S(-\omega, 0)$. Equation 8.5.81 becomes

$$S(\omega, L) = |B_{11}|^2 + |B_{12}|^2 S(\omega, 0) e^{-\alpha L}$$

(8.5.82)

A linear system can be treated as a special case with nonlinear coefficient $\gamma = 0$. In this case, $k = \beta_2 \sigma^2 / 2$, $r_j = r_b = 0$, $|B_{11}| = 1$, $B_{12} = 0$, and

$$S_i(\omega, L) = S(\omega, 0) e^{-\alpha L}$$

(8.5.83)
Using $S_2$ as a normalization factor so that the normalized optical gain or optical noise amplification in the nonlinear system is

$$S_{OG}(\omega, L) = \frac{S(\omega, L)}{S_2(\omega, L)} = |B_{11}|^2 + |B_{12}|^2$$  \hspace{1cm} (8.5.84)

Figure 8.5.29 shows the normalized optical spectra versus fiber length in a single-span system using dispersion shifted fibers (DSF) with anomalous (a) and normal (b) dispersions. Fiber parameters used to obtain Figure 8.5.28 are: loss coefficient $\alpha = 0.22$ dB/km, input signal optical power $P_{in} = 13$ dBm, nonlinear coefficient $\gamma = 2.07 \text{ W}^{-1}\text{km}^{-1}$ and fiber dispersion $D = 2 \text{ ps/nm/km}$ for Figure 8.5.28(a), and $D = -2 \text{ ps/nm/km}$ for Figure 8.5.28(b) with $D$ defined as $D = 2\pi c \beta_2/\lambda^2$. In both cases of Figure 8.5.28(a) and (b), optical noise is amplified around the carrier. The difference is that in an anomalous dispersion regime, optical spectrums have two peaks at each side of the carrier, whereas in the normal dispersion regime, spectra are single peaked. The amplification of optical spectra near the carrier can be explained as the spectrum broadening of the carrier caused by the nonlinear phase modulation between the signal and the broadband ASE. Figure 8.5.28(c) and (d) show nonlinear amplifications of ASE in a 100 km fiber with positive (c) and negative (d) chromatic dispersions. Input power level used is $P_{in} = 15$ dBm. Three different dispersion values are used in each figure, which are solid line: $D = \pm 1 \text{ ps/nm/km}$, dashed line: $D = \pm 0.5 \text{ ps/nm/km}$, and dashed-dotted line: $D = \pm 0.05 \text{ ps/nm/km}$. 
Figure 8.5.29 (a) and (b) Nonlinear amplification of ASE along the longitudinal direction of a single-span fiber. Input optical power $P_{in}=13\text{dBm}$, fiber nonlinear coefficient $\gamma = 2.07 \text{ W}^{-1}\text{km}^{-1}$, and fiber loss $\alpha = 0.22 \text{ dB/km}$. (c) and (d) Nonlinear amplification of ASE in a 100 km fiber for positive (c) and negative (d) dispersions. Input power $P_{in} = 15\text{dBm}$. Solid line: $D = \pm 1 \text{ ps/nm/km}$, dashed line: $D = \pm 0.5 \text{ ps/nm/km}$, and dash-dotted line: $D = \pm 0.05 \text{ ps/nm/km}$.

In the case of coherent optical transmission, the entire optical spectrum is moved to the intermediate frequency after beating with the local oscillator. The frequency components beyond the range of the baseband filter will then be removed, which may cause receiver power reduction. Therefore the broadening of the signal optical spectrum is the major source of degradation in coherent optical transmission systems. For IMDD optical systems, on the other hand, the photodiode detects the total optical power without wavelength discrimination, and the relative intensity noise (RIN) of the optical signal is the major source of degradation related to MI.

Let us then look at the electric noise power spectral density after a direct-detection optical receiver. After the square-law detection of a photodiode, the photo current can be expressed as
\[ I(t) = \eta |A_0 + \tilde{\alpha}(t, L)|^2 = \eta \left[ |A_0|^2 + A_0^* \tilde{\alpha}^*(t, L) + A_0^* \tilde{\alpha}(t, L) \right] \quad (8.5.85) \]

where \( \eta \) is the photodetector responsivity. For simplicity, second and higher orders of small terms have been omitted in the derivation of Equation 8.5.85.

In an IMDD system, the receiver performance is sensitive only to the amplitude noise of the photocurrent, which can be obtained from Equation 8.5.85 as

\[ \delta I(t) = I(t) - I_0 = \eta [A_0^* \tilde{\alpha}^*(t, L) + A_0^* \tilde{\alpha}(t, L)] \quad (8.5.86) \]

where \( I_0 = \eta |A_0|^2 \) is the photocurrent generated by the CW optical signal.

The power spectrum of the noise photocurrent is the Fourier transformation of the autocorrelation of the time-domain noise amplitude:

\[ \rho_n(\omega) = \eta^2 \left( |A_0^* B_{11} + A_0 B_{12}|^2 S(\omega,0) + |A_0^* B_{12} + A_0 B_{22}|^2 S(-\omega,0) \right) e^{-\alpha L} \quad (8.5.87) \]

where \( S(\omega,0) \) is the power spectral density of \( \tilde{\alpha}(t,0) \).

Under the same approximation as we used in the optical spectrum calculation, the input optical noise spectrum is assumed to be symmetric around zero frequency: \( S(\omega,0) = S(-\omega,0) \), Equation 8.5.87 becomes

\[ \rho_n(\omega) = \eta^2 \left( |B_{11}|^2 + |B_{12}|^2 + |B_{22}|^2 \right) A_0^2(L)^2 S(\omega,0) e^{-\alpha L} \quad (8.5.88) \]

Using Equations 8.5.70–8.5.76, it is easy to prove that \( |B_{11} + B_{21}|^2 = |B_{12} + B_{22}|^2 \), and therefore Equation 8.5.88 can be written as

\[ \rho_n(\omega) = 2 P_{in} \eta^2 |B_{11} + B_{21}| S(\omega,0) e^{-2\alpha L} \quad (8.5.89) \]

where \( P_{in} \) is the input signal power such that \( |A_0(L)|^2 = P_{in} \exp(-\alpha L) \).

Again, a linear system can be treated as a special case of a nonlinear system with the nonlinear coefficient \( \gamma = 0 \). In this case, \( k = \beta_2 \sigma_\delta^2/2 \), \( r_f = r_b = 0 \), \( |B_{11}| = 1 \), and \( B_{21} = 0 \). The electrical noise power spectral density in the linear system is then

\[ \rho_n(\omega) = 2 P_{in} \eta^2 S(\omega,0) e^{-2\alpha L} \quad (8.5.90) \]
Using $\rho_0(\omega)$ as the normalization factor, the normalized power spectral density of the receiver electrical noise, or the normalized RIN, caused by fiber Kerr effect is therefore

$$R(\omega) = \left| B_{11} + B_{21} \right|^2$$

(8.5.91)

Comparing Equation 8.5.91 to Equation 8.5.84, it is interesting to note that these two spectra are fundamentally different: The relative phase difference between $B_{11}$ and $B_{21}$ has no impact on the amplified optical noise spectrum of Equation 8.5.84, but this phase difference is important in the electric domain RIN spectrum as given in Equation 8.5.91.

Figure 8.5.30 shows the normalized RIN spectra versus fiber length in a single-span system with anomalous dispersion (a) and normal dispersion (b). Fiber parameters used in Figure 8.5.30 are the same as those used for Figure 8.5.29. In the anomalous fiber dispersion regime [Figure 8.5.30(a)], two main side peaks of noise grow along $z$, the peak frequencies become closer to the carrier and the widths become narrower in the process of propagating along the fiber. On the other hand, if the fiber dispersion is in the normal regime, as shown in Figure 8.5.30(b), noise power density becomes smaller than in the linear case in the vicinity of the carrier frequency. This implies a noise squeezing [Hui 1996] and a possible system performance improvement. In either regime, the system performance is sensitive to the baseband electrical filter bandwidth. For the purpose of clearer display, Figure 8.5.30(c) and (d) show nonlinear amplifications of RIN over 100 km fiber with positive (c) and negative (d) chromatic dispersions. The input power level is $P_{in} = 15$ dBm. Three different dispersion values are used in both Figure 8.5.30(c) and (d), they are solid line: $D = \pm 1$ ps/nm/km, dashed line: $D = \pm 0.5$ ps/nm/km and dashed-dotted line: $D = \pm 0.05$ ps/nm/km.
Figure 8.5.30 (a) and (b) Nonlinear amplification of RIN along the longitudinal direction of a single-span fiber. Input optical signal power $P_{in} = 13$ dBm, fiber nonlinear coefficient $\gamma = 2.07 \text{ W}^{-1}\text{km}^{-1}$ and fiber loss $\alpha = 0.22 \text{ dB/km}$. (c) and (d) Nonlinear amplification of RIN of 100 km fiber for positive (c) and negative (d) dispersions. Input power $P_{in} = 15$ dBm. Solid line: $D = \pm 1 \text{ ps/nm/km}$, dashed line: $D = \pm 0.5 \text{ ps/nm/km}$, and dashed-dotted line: $D = \pm 0.05 \text{ ps/nm/km}$.

It is worthwhile to notice the importance of taking into account fiber loss in the calculation. Without considering fiber loss, the calculated RIN spectra would be qualitatively different. For example, in the case of normal fiber dispersion, the normalized RIN spectra were always less than 0 dB in the mean-field approximation [Marcus 1994], which is in fact not accurate, as can be seen in Figure 8.5.30(b) and (d).

After discussing the nonlinearly amplified optical noise spectrum due to MI, and the impact in the electric domain RIN spectrum after a direct-detection receiver, one wonders if dispersion compensation (DC) would affect the impact of MI. It is well known that DC is an important way to reduce eye closure penalty due to chromatic dispersion in fiber systems. It can also reduce XPM-induced nonlinear crosstalk in IMDD system, as
discussed in the earlier. The impact of DC on the effect of MI in optical systems is also an important consideration in optical system design and performance evaluation. Neglecting the nonlinear effect of the DC module, its transfer function can be represented by a conventional Jones Matrix:

$$\begin{bmatrix} \exp[i\Phi(\omega)] & 0 \\ 0 & \exp[-i\Phi(\omega)] \end{bmatrix}$$  \quad (8.5.92)

If the DC is made by a piece of optical fiber, the phase term is related to the chromatic dispersion, \(\Phi(\omega) = \beta_2 \omega^2 z / 2\).

The effect of DC on the optical system can be evaluated by simply multiplying the Jones matrix of Equation 8.5.92 to the MI transfer matrix of Equation 8.5.77. The RIN spectra at the IMDD optical receiver are sensitive not only to the value of DC but to the position at which the DC module (DCM) is placed in the system. Let’s take two examples to explain the reason. First, if the DCM is positioned after the nonlinear transmission fiber (at the receiver side), the combined transfer function becomes:

$$\begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} \exp[i\Phi(\omega)] & 0 \\ 0 & \exp[-i\Phi(\omega)] \end{bmatrix} \begin{bmatrix} B_{11}' & B_{12}' \\ B_{21}' & B_{22}' \end{bmatrix} = \begin{bmatrix} B_{11}' \exp(i\Phi) & B_{12}' \exp(i\Phi) \\ B_{21}' \exp(-i\Phi) & B_{22}' \exp(-i\Phi) \end{bmatrix}$$

The normalized RIN spectrum in Equation 8.5.91 is then:

$$R(\omega) = \left| B_{11}' \exp(i\Phi) + B_{21}' \exp(-i\Phi) \right|^2$$

where \(B_{ij}' (i = 1, 2, j = 1, 2)\) are the transfer function elements of the nonlinear transmission fiber only. Obviously, the normalized RIN spectrum is sensitive to the amount of DC represented by the phase shift \(\Phi\). Figure 5.7.4 shows an example of normalized RIN spectra without dispersion compensation (solid line), with 50 percent of compensation (dashed-dotted line), and with 100 percent compensation (dashed line). It is interesting to note that 100 percent dispersion compensation does not necessarily bring the RIN spectrum to the linear case.

On the other hand, if the DCM is placed before the nonlinear transmission fiber (at the transmitter side), the total transfer matrix is
The normalized RIN spectrum in Equation 8.5.91 then becomes

\[
R(\omega) = |B'_{11} \exp(i\Phi) + B'_{21} \exp(i\Phi)|^2 = |B'_{11} + B'_{21}|^2
\]

In this case, it is apparent that the phase term \(\Phi\) introduced by dispersion compensation does not bring any difference into the normalized RIN spectrum.

Figure 8.5.31 Normalized RIN spectra for 0 percent (solid line), 50 percent (dash-dotted line), and 100 percent (dashed line) dispersion compensations. Fiber length \(L = 100\) km, fiber nonlinear coefficient \(\gamma = 2.07\) \(\text{W}^{-1}\text{km}^{-1}\), fiber loss \(\alpha = 0.22\) dB/km, \(P_{\text{in}} = 15\) dBm, and \(D = 1\) ps/nm/km [Hui 1997].

Another important observation is that although the RIN spectrum in the electrical domain after photodetection can be affected by the DC, the optical spectrum is not affected by the use of DC, regardless of the position of the DC module (DCM). The reason for this can be found in Equation 8.5.84, where the normalized optical spectrum is related to the absolute values of \(B_{11}\) and \(B_{12}\). It does not matter where the DCM is placed in the system; DC has no effect on the normalized optical spectrum.

So far, we have discussed the effect of MI in a single-span fiber optic system, and it is straightforward to extend the analysis to multi-span fiber systems with in-line optical amplifiers. In an optical fiber system of \(N\) spans with \(N\) EDFAs (one post-amplifier and \(N-1\) line amplifiers), the fiber loss per span is compensated by the optical gain of the
EDFA. Suppose that all EDFAs have the same noise figure; the ASE noise power
espectral density generated by the \( i \)-th EDFA is

\[
S_i = h\nu \left( F_{G_i} - 1 \right) \quad (8.5.93)
\]

where the ASE noise optical spectrum is supposed to be white within the receiver optical
bandwidth. After transmission through fibers, amplified by EDFAs and detected by the
photodiode, the power spectrum of the detected RIN can be obtained by the
multiplication of the transfer function of each span of optical fiber, supposing that ASE
noise generated by different EDFAs are uncorrelated:

\[
\rho(\omega) = 2h\nu P_m \sum_{m=1}^{N} \left( F_{G_{N-m+1}} - 1 \right) \left| B_{11}^{(N-m+1)} + B_{21}^{(N-m+1)} \right|^2 \quad (8.5.94)
\]

where \( B_{ij}^{(k)} (i, j, = 1, 2) \) are matrix elements defined as

\[
B_{ij}^{(k)} = \prod_{m=1}^{k} \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}^{m} \quad (8.5.95)
\]

and the matrix

\[
\begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}
\]

represents the IM transfer function of the \( m \)-th fiber span.

Setting \( \gamma = 0 \) to obtain the normalization factor, the normalized RIN spectrum is then

\[
R(\omega) = \frac{\sum_{m=1}^{N} \left( F_{G_{N-m+1}} - 1 \right) \left| B_{11}^{(N-m+1)} + B_{21}^{(N-m+1)} \right|^2}{\sum_{m=1}^{N} \left( F_{G_{N-m+1}} - 1 \right)} \quad (8.5.96)
\]

Assuming Gaussian statistics, the change of the standard deviation of the noise caused by
fiber MI can be expressed in a simple way:

\[
\delta \sigma = \frac{\sigma^2}{\sigma_0^2} = \int_{-\infty}^{\infty} R(\omega) |f(\omega)|^2 d\omega \quad (8.5.97)
\]
where $\sigma$ and $\sigma_0$ are noise standard deviations in the nonlinear and linear cases, respectively, and $f(\omega)$ is the receiver baseband filter transfer function.

Extending from the $Q$-definition in Equation 8.1.13, in a direct-detection optical receiver with the effect of MI taken into account, the quality factor $Q$ can be expressed as

$$Q = \frac{\Re(P_1 - P_0)}{\sqrt{\sigma_{\text{sh}}^2 + \sigma_{\text{th}}^2 + \sigma_{\text{sp-sp}}^2 + \sigma_{s-sp}^2 \delta\sigma_1 + \sqrt{\sigma_{\text{sh}0}^2 + \sigma_{\text{th}0}^2 + \sigma_{\text{sp-sp}0}^2 + \sigma_{s-sp}^2 \delta\sigma_0}}}$$

(8.5.98)

where $P$ is the signal level, $\sigma_{\text{sh}}$, $\sigma_{\text{th}}$, $\sigma_{\text{sp-sp}}$, and $\sigma_{s-sp}$ are, respectively, the standard deviations of shot noise, thermal noise, ASE-ASE beat noise, and signal-ASE beat noise in the absence of MI. Subscripts 1 and 0 indicate the symbols at signal logical 1 and logical 0, respectively. In practice, MI introduces a signal-dependent noise that affects more on signal during logical 1 than that during logical 0. The change in the RIN spectrum causes system performance degradation primarily due to the increase of signal-ASE beat noise through the ratio $\delta\sigma_1$. However, considering that if the signal extinction ratio is not infinite, signal power at logical 0 may also introduce MI through $\delta\sigma_0$, although this effect should be very relatively small. Meanwhile, optical spectrum change due to MI may also introduce $Q$ degradation through ASE-ASE beat noise, but it is expected to be a second-order small effect in an IMDD receiver.

Equation 8.5.98 indicates that, in general, system performance degradation due to MI depends on the proportionality of signal-ASE beat noise to other noises. Multispan optical amplified fiber systems, where signal-ASE beat noise predominates, are more sensitive to MI in comparison to unamplified optical systems. For signal-ASE beat noise limited optical receiver, to the first-order approximation, the system $Q$ degradation caused by MI can be expressed in a very simple form:

$$10\log(\delta Q) - 10\log(Q_0) = -510\log(\delta\sigma)$$

(8.5.99)

where $Q_0$ is the receiver $Q$-value in the linear system without considering the impact of MI, and $\delta\sigma$ is the change of noise standard deviations due to MI.

As the major impact of MI in an IMDD optical system is the increase of the RIN at the receiver due to parametric amplification of ASE noise, it is straightforward to measure
MI and its impact in optical transmission systems in frequency domain. To provide experimental evidence and verify the theoretical analysis of MI discussed above, Figure 8.5.32 shows the detailed experimental setup for the MI measurement in a dispersion-compensated multi-span fiber system with optical amplifiers.

Figure 8.5.32 Experimental setup. $L_1$: 84.6 km (1307 ps/nm), $L_2$: 84.6 km (1291 ps/nm), $L_3$: 90.4 km (1367 ps/nm). Optical powers at the output of EDFA5, EDFA6, and EDFA7 are less than 0 dBm. DCM: dispersion compensation module.

To make sure that the measured receiver RIN is dominated by signal-ASE beat noise, one can increase the broadband ASE noise from EDFAs intentionally. The role of the first optical amplifier in the link was to inject optical noise. It had a noise figure of 7.5 dB and the input optical power of –32 dBm. The narrowband optical filter that followed kept the total ASE power less than that of the signal so as to decrease the ASE-ASE beat noise. The signal power was typically 0.5 dB less than the total power emerging from the EDFA, and this was taken into account when setting the output power of the line amplifiers. The first EDFA was the dominant source of ASE arriving at the PIN detector. The three line amplifiers had output power adjustable by computer control. The fiber spans all had loss coefficient measured by OTDR of 0.2 dB/km. The output power of the next two EDFAs was set below 0 dBm to avoid nonlinear effects in the DCM. The narrowband filter after EDFA suppressed ASE in the 1560 nm region, which would have led to excessive ASE-ASE beat noise. The last optical amplifier was controlled to an output power of 4 dBm, just below the overload level of the PIN detector.

First, a calibration run was made, with an attenuator in place of the system. This measurement was subtracted from all subsequent traces so as to take out the effect of the frequency response of the detection system. The calibration trace was >20 dB higher than the noise floor for the whole 0–18 GHz band.
The RIN spectra at the output of the three-span link are shown in Figure 8.5.33 with line amplifier output power (signal power) controlled at 8, 10, 12, and 14 dBm, for inverted triangles, triangles, squares, and circles, respectively. In Figure 8.5.33(a), open points represent the measured spectra with the system configuration described in Figure 8.5.32 except no dispersion compensation was used. Continuous lines in the same figures were calculated using Equation 8.5.91. Similarly, Figure 8.5.33(b) shows the measured and calculated RIN spectra with the dispersion compensation of –4070 ps/nm at the receiver side, as shown in Figure 8.5.32. To obtain the theoretical results shown in Figure 8.5.33, the fiber nonlinear coefficient used in the calculation was \( \gamma = 1.19 \text{ W}^{-1}\text{km}^{-1} \), and other fiber parameters such as length and dispersion were chosen according to the values of standard single-mode fiber used in the experiment, as shown in the caption of Figure 8.5.32. Very good agreement between measured and calculated results in the practical power range assures the validity of the two major approximations used in the transfer matrix formulation, namely, the linear approximation to the noise term and the insignificance of pump depletion.

![Normalized RIN (5dB/div.) vs Frequency (GHz)](image)

Figure 8.5.33 Measured (open points) and calculated (solid lines) RIN spectra in the three-span standard SMF as described in Figure 5.7.6. The optical power at the output of EDFA2, EDFA3, and EDFA4 is 8 dBm (triangles-down), 10 dBm (triangles-up), 12 dBm (squares), and 14 dBm (circles). Curves are shifted for 10 dB between one and another for better display. (a) without dispersion compensation and (b) with –4070 ps/nm dispersion compensation [Hui 1997].

Although the RIN spectra are independent of the signal data rate, the variance of the noise depends on the bandwidth of the baseband filter as explained in Equation 8.5.97.
Figure 8.5.34 shows the effect of dispersion compensation on the ratio of noise standard deviation between nonlinear and linear cases for the three-span fiber system described in Figure 8.5.32. The optical power was 12 dBm at the input of each fiber span and raised-cosine filters were used with 8 GHz bandwidth. Both theoretical and experimental results demonstrate that $\delta \sigma$ approaches its minimum when the DC is approximately 70 percent of the total system dispersion. Generally, the optimum level of dispersion compensation depends on the number of spans, electrical filter bandwidth, optical power levels, and the dispersion in each fiber span.

![Dispersion compensation (ps/nm) vs. $\delta \sigma$](image)

Figure 8.5.34 Comparison of $\delta \sigma$ between calculation (solid line) and measurement (diamonds) for the three-span system described in Figure 5.7.6 with optical power $P_{in} = 12$ dBm [Hui 1997].

As far as MI is concerned, in the anomalous fiber dispersion regime, system performance always becomes worse with increasing signal power. On the other hand, in the normal dispersion regime, system sensitivity may be improved by the nonlinear process. This is an interesting phenomenon that was explained as a noise squeezing [Hui, 1996]. It can be readily understood from Figure 8.5.33(b), where if the signal optical power is higher than 12 dBm, the noise level is reduced at the low-frequency region. If the receiver bandwidth is less than 7 GHz, the total noise level will be lower compared to the linear system. However, for systems with higher bit rate, sensitivity degradations may also be possible in the normal dispersion regime with high input signal powers. This
degradation is caused by the increased noise level at higher frequencies, which happen to be within the receiver baseband.

To conclude this section, MI is in a unique category of fiber nonlinearity. In contrast to nonlinear crosstalk due to XPM and FWM, MI is a single-channel process. The high power of the optical signal amplifies the ASE noise through the parametric gain process. The increased RIN due to MI is originated from the nonlinear interaction between the signal and the broadband ASE noise. This effect is also different from SPM because the consequence of MI is the amplification of intensity noise rather than the deterministic waveform distortion.

Conclusion:

Optical system design and characterization requires extensive knowledge in both system and component levels. In this chapter we have concentrated in the discussion of basic performance evaluation criteria of binary intensity modulated optical communication systems with direct detection. High order modulation and phase modulated optical systems with coherent detection will be introduced later in Chapters 9 and 10. Bit error rate (BER) is an ultimate measure of the signal quality at the receiver, which is often represented by a Q-value. Sources of BER degradation in an optical system can be categorized by waveform distortion and noise. Waveform distortion is usually deterministic which can be caused by chromatic distortion, linear and nonlinear crosstalk between channels, whereas noise is random which is caused by accumulated amplified spontaneous emission (ASE) along the system when optical amplifiers as well as noises created by the photodetector such as thermal noise and shot noise.

In a traditional optical system without inline optical amplifiers, receiver thermal noise and shot noise are major limits to the transmission performance, and receiver sensitivity, defined as the minimum signal optical power required to achieve a certain BER, is often used to specify the system. For optical systems employing inline optical amplifiers, on the other hand, signal optical power reaching the photodiode can be easily amplified to a high level. However BER in the optically amplified system is mainly determined by the signal-ASE beat noise in the receiver, and thus required optical signal-to-noise ratio (R-OSNR) is a more relevant measure of the system performance.
With the knowledge of the noise statistics, the major impact of noise on BER degradation can often be evaluated analytically. However, waveform distortion depends on specific system configuration as well as waveforms and optical power levels of modulated optical signals. Especially in wavelength division multiplexed (WDM) optical systems, linear and nonlinear crosstalk can become major limiting factors in the transmission performance. Numerical simulators solving nonlinear Schrödinger equations based on split-step Fourier method are powerful tools for predicting optical system performance, which can be used to guide system design and performance evaluation. Analytical and semi-analytical methods are also indispensable for the understanding of physical origins of eye closure penalties.

The impact of cross-phase modulation (XPM), four-wave mixing (FWM) and modulation instability (MI) are specifically presented at the end of this chapter as examples of nonlinear crosstalks. Semi-analytic methods have been used for the analysis, emphasizing the importance of small-signal perturbation approximation and linearization. Good understanding of various mechanisms introducing system performance degradation helps system design and performance optimization through optimizing dispersion maps, channel spacing, as well as choosing the optimum power levels for optical signals.