Erbium-doped Fiber Amplifier (EDFA)

- Erbium-doped fiber amplifier (EDFA) uses optical pumping (in contrast to SOA which uses electric pumping through current injection)
- Short wavelength pump transfers its energy to the longer wavelength signal along the erbium-doped fiber (EDF), so that the signal can be amplified
- Erbium ions doped in the EDF provide mechanism enabling this energy transfer
Energy diagrams of erbium ions

3-level energy diagram

- **Excited state**: τ = 1μs
- **Meta-stable**: τ = 10ms
- **Ground state**: 980nm
- **Pump**: 980nm
- **Radiative**: 1480nm
- **Non-radiative**: 1580nm
- **Emission**: 1550nm

Simplified 2-level energy diagram

- **Ground state**: 1460nm
- **Pump**: 1480nm
- **Emission**: 1550nm

Ideally, if one pump photon at wavelength \( \lambda_p \) generates one signal photon at wavelength \( \lambda_s \) through carrier generation and recombination, energy conversion efficiency is:

\[
\eta_{\text{max}} = \frac{hf_s}{hf_p} = \frac{hc}{\lambda_s} = \frac{\lambda_p}{\lambda_s}
\]

This efficiency cannot exceed unity (\( \lambda_p < \lambda_s \) is required)

- Example: for \( \lambda_p = 980\text{nm} \), \( \lambda_s = 1550\text{nm} \), \( \eta_{\text{max}} = 63\% \)
- for \( \lambda_p = 1480\text{nm} \), \( \lambda_s = 1550\text{nm} \), \( \eta_{\text{max}} = 95\% \)
Absorption cross-section

*Absorption efficiency:* probably that a carrier in the ground state is pumped up to the metastable state per second:

\[ W_p = \sigma_a \frac{P_p}{hf_p A} \quad \text{Unit} \ [s^{-1}] \]

- \( f_p \): pump optical frequency,
- \( A \): effective fiber core cross section \([cm^2]\)
- \( P_p \): pump power \([W] = [J/s]\)

*Absorption rate:*

\[ W_p N_1 = \sigma_a \frac{P_p N_1}{hf_p A} \quad \text{Unit} \ [cm^{-3}s^{-1}] \]

- \( P_p / (hf_p) \): number of pump photons flowing per second \([s^{-1}] \) (flow rate)
- \( P_p / (hf_p A) \): density of pump photon flow rate \([cm^2s^{-1}] \)
- \( N_1 \): ground state carrier density \([cm^{-3}] \)

\( \sigma_a \): *Absorption cross-section*, is a coefficient indicating how efficiently the fiber can absorb pump photons: (it is a property of the EDF, independent of the operation condition)

\[ \sigma_a = W_p \left( \frac{P_p}{hf_p A} \right) \quad \text{Unit:} \ [s^{-1}]/[cm^2s^{-1}] = [cm^2] \]

The name of “absorption cross-section” came from its unit of \([cm^2]\). But its physical meaning is an *efficiency* instead of geometric cross-section.

Actual absorption per unit length along the EDF is proportional to the absorption cross-section and ground state carrier density

\[ \Gamma(f_p) \sigma_a N_1 \quad \text{Unit:} \ [cm^{-1}] \]

- \( \Gamma \): confinement factor \([\text{unitless}] \)

Only a certain area in the fiber core is erbium-doped

R. Hui
**Emission cross-section**

**Emission efficiency**: probably that a carrier in the metastable state drops to the ground state per second to generate emission:

\[
W_s = \sigma_e \frac{P_s}{h\nu_s A}
\]

Unit [s^{-1}]

- \(f_s\): signal optical frequency,
- \(A\): effective fiber core cross section [cm^2]
- \(P_s\): signal power [W] = [J/s]

**Emission rate**:

\[
W_sN_2 = \sigma_e \frac{P_s N_2}{h\nu_s A}
\]

Unit [cm^{-3} s^{-1}]

- \(P_s/(h\nu_s)\): number of signal photons flowing per second [s^{-1}] (flow rate)
- \(P_s/(h\nu_s A)\): density of signal photon flow rate [cm^{-2} s^{-1}]
- \(N_2\): metastable state carrier density [cm^{-3}]

**\(\sigma_e\)**: **Emission cross-section**, is a coefficient indicating how efficiently a metastable state carrier can generate emission: (it is a property of the EDF, independent of the operation condition)

\[
\sigma_e = \frac{W_s}{\left(\frac{P_s}{h\nu_s A}\right)}
\]

Unit: [s^{-1}]/[cm^{-2} s^{-1}] = [cm^2]

The name of “emission cross-section” came from its unit of [cm^2]. But its physical meaning is an efficiency instead of geometric cross-section

Actual optical gain per unit length along the EDF is proportional to the emission cross-section and metastable state carrier density

\[
\Gamma(f_s)\sigma_e N_2
\]

Unit: [cm^{-1}]

- \(\Gamma\): confinement factor [unitless]
- Only a certain area in the fiber core is erbium-doped
**Emission and absorption cross-sections**

- Both absorption cross-section and emission cross-section are wavelength dependent: $\sigma_a(\lambda)$ and $\sigma_e(\lambda)$
- The EDF does not know which is “signal” which is “pump”, it only knows their wavelengths
- A channel at a wavelength where $\sigma_a(\lambda) > \sigma_e(\lambda)$ will see absorption ($N_1 \downarrow$ and $N_2 \uparrow$,
- A channel at a wavelength where $\sigma_e(\lambda) > \sigma_a(\lambda)$ will see gain ($N_2 \downarrow$ and $N_1 \uparrow$,

At an optical frequency $f_s$, the signal experiences both absorption and gain, and the net emission rate is:

$$R_e = W_s(\lambda_s)N_2 - W_p(\lambda_s)N_1 = \frac{\Gamma(f_s)\sigma_e(f_s)P_s}{h\nu f_s A}N_2 - \frac{\Gamma(f_s)\sigma_a(f_s)P_s}{h\nu f_s A}N_1$$

Depending on whether $\sigma_e(f_s) > \sigma_a(f_s)$ or $\sigma_a(f_s) > \sigma_e(f_s)$, the net emission can either be positive or negative

\[ R. Hui \]
Rate equations

Net optical gain (or loss) along the EDF in $[cm^{-1}]$: 
\[ g_n(\lambda, z) = \Gamma(\lambda)\left[\sigma_e(\lambda)N_2(z) - \sigma_a(\lambda)N_1(z)\right] \]

Note: \(\sigma_e(\lambda)\) and \(\sigma_a(\lambda)\) are only wavelength-dependent; \(N_1(z)\) and \(N_2(z)\) are only \(z\)-dependent.

Carrier conservation: 
\[ N_1(z) + N_2(z) = N_T \]

\(N_T\) is the total erbium doping density in EDF (independent of \(z\)).

EDFA optical gain: 
\[ G(\lambda) = e^{\int_0^L g_n(\lambda, z)dz} \]

Since \(\Gamma(\lambda)\), \(N_T\), \(\sigma_e(\lambda)\) and \(\sigma_a(\lambda)\) are known EDF parameters, the only variable is \(N_2(z)\).

Then, the question is how to find \(N_2(z)\)?

For two \(\lambda\)-channels \(P_s\) at \(\lambda_s\) and \(P_p\) at \(\lambda_p\) the upper level carrier density rate equation is,
\[ \frac{dN_2}{dt} = \frac{\Gamma(\lambda_p)\sigma_a(\lambda_p)P_p}{h\nu_p A} N_1 - \frac{\Gamma(\lambda_s)\sigma_e(\lambda_s)P_s}{h\nu_s A} \left( N_2 - \frac{\sigma_a(\lambda_s)}{\sigma_e(\lambda_s)} N_1 \right) - \frac{N_2}{\tau} \]

Pump \(P_p\) absorption \(\to N_2\uparrow\)
(here we assume \(\sigma_e(\lambda_p) = 0\))

Signal \(P_s\) induces stimulated recombination \(\to N_2\downarrow\).

2\textsuperscript{nd} term represents \(N_2\uparrow\) through \(P_s\) absorption as \(\sigma_a(\lambda_s) \neq 0\)

Spontaneous recombination

\[ R. Hui \]
Steady-state solution of rate equation

For steady state \( d/dt = 0 \)

\[
\frac{\Gamma(\lambda_p)\sigma_a(\lambda_p)P_p}{h_f_p A} N_1 - \frac{\Gamma(\lambda_s)\sigma_e(\lambda_s)P_s}{h_f_s A} \left( \frac{N_2 - \sigma_a(\lambda_s)}{\sigma_e(\lambda_s)} N_1 \right) - \frac{N_2}{\tau} = 0
\]

Solution:

\[
N_2 = \frac{\Gamma(\lambda_p)\sigma_a(\lambda_p)f_s P_p + \Gamma(\lambda_s)\sigma_a(\lambda_s)f_p P_s}{\Gamma(\lambda_p)\sigma_a(\lambda_p)f_s P_p + \Gamma(\lambda_s)\sigma_a(\lambda_s)f_p P_s + \frac{h_f_s f_p A}{\tau}} N_T
\]

These equations are coupled through \( N_2 \)

\[
\frac{dP_s(\lambda_s)}{dz} = g_s(z, \lambda_s)P_s(z)
\]

\[
\frac{dP_p(\lambda_p)}{dz} = g_p(z, \lambda_p)P_p(z)
\]

One has gain and the other has loss. \( g_s(z, \lambda_s) \) can be expressed as \( \alpha(z, \lambda) \)

These 4 coupled equations have to be solved together numerically to find \( N_2 \)

This can be extended to more than two wavelength channels, each one has its own propagation equation, and they jointly impact carrier density rate equation

\[
-\sum_{i=1}^{M} \frac{\Gamma(\lambda_i)\sigma_e(\lambda_i)P_i(\lambda_i)}{h_f_i A} \left( \frac{N_2 - \sigma_a(\lambda_i)}{\sigma_e(\lambda_i)} N_1 \right) - \frac{N_2}{\tau} = 0
\]

\[
\frac{dP_i(\lambda_i)}{dz} = g_n(z, \lambda_i)P_i(z)
\]

\( M \) wavelength channels \( P_i(\lambda_i) \), \( i = 1, 2, \ldots M \)

\[
g_n(\lambda, z) = \Gamma(\lambda)\sigma_e(\lambda)N_2(z) - \sigma_a(\lambda)N_T - N_2(z)
\]

R. Hui
Express cross-section in [dB/m]

Optical gain coefficient along the EDF in \([m^{-1}]\):
\[
g_n(\lambda, z) = \Gamma(\lambda)\left[\sigma_e(\lambda)N_2(z) - \sigma_a(\lambda)[N_T - N_2(z)]\right]
\]

Optical gain of EDFA per unit length \(L = 1m\)
\[
G(\lambda) = e^{\int_0^L g_n(\lambda, z)\,dz}
\]

If \(N_2 = N_T\) (total inversion):
\[
g(\lambda) = \Gamma(\lambda)\sigma_e(\lambda)N_T
\]
\[
\sigma_e(\lambda)_{dB} = 10 \times \log_{10}[e^{\Gamma(\lambda)\sigma_e(\lambda)}]
\]
Is the emission cross-section expressed in [dB/m]

If \(N_1 = N_T\) (no inversion):
Material gain
\[
g(\lambda) = -\Gamma(\lambda)\sigma_a(\lambda)N_T
\]
Material loss
\[
\alpha(\lambda) = \Gamma(\lambda)\sigma_a(\lambda)N_T
\]
\[
\sigma_a(\lambda)_{dB} = 10 \times \log_{10}[e^{\Gamma(\lambda)\sigma_a(\lambda)}]
\]
Is the absorption cross-section expressed in [dB/m]
Typical EDF specification

<table>
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<tr>
<th>Item #</th>
<th>ER30-4/125</th>
<th>ER110-4/125</th>
<th>ER16-8/125</th>
<th>ER80-8/125</th>
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<tbody>
<tr>
<td>Peak Core Absorption @ 1530 nm</td>
<td>30 ± 3 dB/m</td>
<td>110 ± 10 dB/m</td>
<td>16 ± 3 dB/m</td>
<td>8 ± 8 dB/m</td>
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<td>MFD</td>
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<td>6.5 ± 0.5 μm</td>
<td>9.5 ± 0.8 μm</td>
<td>9.5 ± 0.8 μm</td>
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<td>0.2</td>
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<td>Cut-Off Wavelength</td>
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<td>890 ± 90 nm</td>
<td>1100 - 1400 nm</td>
<td>1250 ± 150 nm</td>
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<tr>
<td>Cladding Diameter</td>
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<td>125 ± 2 μm</td>
<td>125 ± 2 μm</td>
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<td>Cladding Geometry</td>
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<tr>
<td>Coating (Second Cladding) Diameter</td>
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<td>245 ± 15 μm</td>
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<td>&gt;100 kpsi</td>
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<td>Cladding Index</td>
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</tbody>
</table>

R. Hui
Numerical simulation: taking into account saturation caused by broadband ASE noise

Carrier density equation:
\[
\frac{N_2}{\tau} + \int_{\lambda} \left[ \frac{P_{s}^f(\lambda, z)}{\text{Ahc} / \lambda} + \frac{P_{sp}^f(\lambda, z)}{\text{Ahc} / \lambda} + \frac{P_{sp}^b(\lambda, z)}{\text{Ahc} / \lambda} \right] g(\lambda, z) d\lambda = 0
\]

Power propagation equations:
\[
\begin{aligned}
\frac{dP_s^f(\lambda, z)}{dz} &= P_s^f(\lambda, z)g(\lambda, z) \quad \text{← M equations for the forward signal including pump} \\
\frac{dP_{ASE}^f(\lambda, z)}{dz} &= P_{ASE}^f(\lambda, z)g(\lambda, z) + 2n_{sp}(z)g(\lambda, z)hc / \lambda \quad \text{← forward propagating noise} \\
\frac{dP_{ASE}^b(\lambda, z)}{dz} &= -P_{ASE}^b(\lambda, z)g(\lambda, z) - 2n_{sp}(z)g(\lambda, z)hc / \lambda \quad \text{← backward propagating noise}
\end{aligned}
\]

Noise amplification Noise generation

Together with
\[
g(\lambda, z) = g_n(\lambda, z) = \Gamma(\lambda) \left\{ [\sigma_c(\lambda) + \sigma_a(\lambda)]N_2(z) - \sigma_a(\lambda)N_T \right\} \quad \text{← material gain coefficient}
\]
\[
n_{sp}(z) = \frac{N_2(z)}{N_2(z) - N_1(z)} = \frac{N_2(z)}{2N_2(z) - N_T} \quad \text{← Noise coefficient}
\]
Example

100mW forward pump, and 6 signal channels each with -20dBm power at the EDFA and 4nm channel spacing. The length of the EDF is $L = 25m$. 

(a) Power (dBm) vs. $z$ (m) showing pump, signal, forward ASE, and backward ASE. 

(b) Spectrum (dBm/nm) vs. Wavelength (µm) showing multiple signal channels.

(c) Backward ASE (dBm/nm) vs. Wavelength (µm) showing two peaks.

(d) Carrier population vs. $z$ (m) showing $N_2/N_T$ and $N_1/N_T$.
EDFA gain dynamics:

- As the carrier lifetime in an EDFA is $\tau \approx 10\text{ms}$ (versus ns in SOA), dynamic response of gain saturation in EDFA is considered very slow.
- EDFA gain variation due to saturation is not fast enough to follow the waveform of optical signal as long as the modulation speed is higher than MHz ($\mu$s time scale).
- This slow response of gain saturation avoids inter-channel crosstalk in multi-channel WDM system.
- EDFAs are most often used as in-line optical amplifiers in commercial WDM optical systems.
- In a high speed system, EDFA gain saturation only depends on the average signal optical power, and EDFA gain does not vary with time.