1. Consider a plane wave with a relative phase difference $\delta = \pi/3$ between the $E_x$ and $E_y$ components. If the magnitude of the $x$ component is twice as large as the $y$ component ($E_{0x} = 2 E_{0y}$), and assume $E_{0y} = 1$, please find the orientation angle $\phi$ of this polarization ellipse, and find the lengths of the long axis and the short axis of the ellipse.

Solution:

Based on Equation (2.2.5)

$$\phi = \frac{1}{2} \tan^{-1} \left( \frac{2 E_{0x} E_{0y} \cos \delta}{E_{0x}^2 - E_{0y}^2} \right) = \frac{1}{2} \tan^{-1} \left( \frac{2 \times 2 \times 0.5}{4 - 1} \right) = \frac{1}{2} \tan^{-1} \left( \frac{2}{3} \right) = 0.294 \text{ rad}. $$

Since $E_x = E_{0x} \cos \phi = 2 \cos(0.294) = 1.9142$

$E_y = E_{0y} \cos(\phi + \delta) = \cos(0.294 + \pi/3) = 0.2276$

The half length of the long axis is

$$\sqrt{E_{0x}^2 \cos^2 \phi + E_{0y}^2 \cos^2(\phi + \delta)} = \sqrt{1.9142^2 + 0.2276^2} = 1.7821$$

The half length of the short axis is

$$\sqrt{E_{0x}^2 \cos^2(\phi + \pi/2) + E_{0y}^2 \cos^2(\phi + \pi/2 + \delta)}$$

$$= \sqrt{4 \cos(0.294 + \pi/2)^2 + \cos(0.294 + 5\pi/6)^2} = 1.1332$$

Therefore the full lengths of the long and the short axes are, 3.564 and 2.2664, respectively.

2. Consider the air/water interface shown in the following figure.

(a) A collimated light beam launches from the air to the water at an angle $\theta = \pi/6$ with respect to the surface normal as shown in Figure E-2.1(a), where $h_a = 1.5$ m and $h_b = 1$ m. The refractive index of air is $n_0 = 1$, and assume the refractive index of water is $n_1 = 1.3$.

Please find the distance $x_1 = ?$

In order to eliminate reflection from the water surface, what should be the angle $\theta$, and in which direction the light should be polarized?

(b) If the light is launched from the bottom of the water tank as shown in E-2.1(b), and the incidence angle is still $\theta = \pi/6$, please find the distance $x_2 = ?$.

What is the angle $\theta$ to achieve total reflection on the water/air surface?
Solution:

(a) \( x_0 = h_a \tan \theta = 1.5 \tan(\pi / 6) = 0.866m \)

Diffraction angle \( \theta_2 = \sin^{-1}\left(\frac{n_0}{n_1} \sin \theta\right) = \sin^{-1}\left(\frac{1}{1.3} \sin\left(\frac{\theta}{6}\right)\right) = 0.395rad = 22.6^\circ \)

\( x_1 = x_0 + h_0 \tan \theta = 0.866 + 1 \times \tan(0.395) = 1.28m \)

When the incidence angle \( \theta \) is equal to the Brewster angle and the optical field polarization is parallel to the incidence plane, reflection from water surface can be eliminated. The Brewster angle is,

\( \theta = \theta_B = \tan^{-1}(n_2 / n_1) = \tan^{-1}(1.3) = 0.915rad = 52.4^\circ \)

(b) \( x_0 = h_b \tan \theta = 1 \times \tan(\pi / 6) = 0.5774m \)

Diffraction angle \( \theta_0 = \sin^{-1}\left(\frac{n_1}{n_0} \sin \theta\right) = \sin^{-1}\left(\frac{1.3}{1} \sin\left(\frac{\theta}{6}\right)\right) = 0.708rad = 40.5^\circ \)

\( x_2 = x_0 + h_a \tan \theta = 0.5774 + 1.5 \times \tan(0.708) = 1.86m \)

Total reflection happens when the incidence angle \( \theta \) is equal to the critical angle,

\( \theta = \sin^{-1}(n_0 / n_1) = \sin^{-1}(1/1.3) = 0.878rad = 50.28^\circ \)
3. A light beam is launched to a thin semiconductor film with a thickness of \(d=2 \, \mu\text{m}\). The refractive index of air is \(n_0 = 1\), and assume the semiconductor film has no loss and its refractive index is \(n_1 = 3.5\). The wavelength of the light is \(\lambda = 633\text{nm}\) which is parallel polarized on the plane of the paper, and the incidence angle is \(\theta = 45^\circ\).

Please find the relative amplitude \(|E_1|/|E_2|\) and the phase difference \([\text{Angle}(E_1) - \text{Angle}(E_2)]\) of the two reflected beams at the reference plane.

**Solution:**

For the parallel polarization, according equation 2.5, the reflectivity is

\[
\rho_p = \frac{-n_1^2 \cos \theta_1 + n_0 \sqrt{(n_1^2 - n_0^2 \sin^2 \theta_1)}}{n_1^2 \cos \theta_1 + n_0 \sqrt{(n_1^2 - n_0^2 \sin^2 \theta_1)}} = -0.4329, \text{ where } \theta_1 = \pi/4.
\]

which means that the phase shift is \(\pi\).

The field that penetrates into the film is \(\sqrt{1 - 0.4329^2}\)

For the 2nd beam, the beam angle into the glass is \(\theta_2 = \sin^{-1}\left(\frac{n_0}{n_1} \sin \theta_1\right) = 0.2034\text{rad}\)

on the bottom glass/air interface, the field reflection can be found from

\[
\rho_p = \frac{-n_0^2 \cos \theta_2 + n_1 \sqrt{(n_0^2 - n_1^2 \sin^2 \theta_2)}}{n_0^2 \cos \theta_2 + n_1 \sqrt{(n_0^2 - n_1^2 \sin^2 \theta_2)}} = 0.4329
\]

in which the phase shift is zero and the amplitude reflectivity is 0.4329

\[
E_2 = \sqrt{1 - 0.4329^2} \times 0.4329 \times \sqrt{1 - 0.4329^2} = 0.4329 \times (1 - 0.4329^2)
\]

No phase shift at surfaces, and thus,
$|E_1| = \frac{0.4329}{\sqrt{1 - 0.4329^2}} = \frac{1}{\sqrt{1 - 0.4329^2}} = 1.2306$

$|E_2| = \frac{0.4329}{\sqrt{1 - 0.4329^2}} = \frac{0.4329}{\sqrt{1 - 0.4329^2}}$

The phase difference is, $\Delta \Phi = -\pi - \frac{2\pi}{\lambda} \frac{2d}{\cos \theta_2} = -\pi \left( 1 + \frac{4d}{\lambda \cos \theta_2} \right) = -14\pi$

the 2nd term is roundtrip propagation phase delay $\frac{2\pi}{\lambda} \frac{2d}{\cos \theta_2}$ in the film

4. A light beam with $\lambda = 0.63 \, \mu m$ travels inside a slab optical waveguide by bouncing back and forth between the two interfaces. The refractive index of the waveguide is $n_1 = 1.5$, and the thickness is $d = 3 \, \mu m$. Assume the lightwave is vertically polarized ($E$ field goes into the paper) with respect to the incidence plane. For the incidence angle $\theta = 60^\circ$, what is the optical phase shift after each roundtrip? What is the evanescent field penetration depth near the glass/air interface?

Solution:
Based on Equation 2.1.8, the optical phase shift of each reflection is

$$\Delta \Phi_\perp = -2 \tan^{-1} \left( \frac{n_0 \sqrt{n_2^2 / n_0^2 \sin^2 \theta - 1}}{n_1 \cos \theta} \right) = -1.671 \, \text{rad.}$$

and the phase delay due to roundtrip propagation is,

$$\Delta \Phi_\parallel = \frac{2\pi}{\lambda \cos \theta} 2d = \frac{4d}{\lambda \cos \theta} = 116.34 \, \text{rad.}$$

Total phase shift is then, $\Delta \Phi = \Delta \Phi_\perp + \Delta \Phi_\parallel = 116.34 - 1.671 \times 2 = 113 \, \text{rad.} \approx 36\pi$

The penetration depth can be found as,

$$z_e = \frac{1}{\alpha} = \frac{\lambda}{2\pi \sqrt{n_1^2 \sin^2 \theta_1 - n_2^2}} = \frac{0.63}{5.21} = 0.121 \, \mu m$$