Homework 7, Solutions:

1. The material gain of an optical amplifier has a parabolic shape with the FWHM bandwidth of $\Delta f = 30\text{nm}$. Neglect gain saturation effect for simplicity, please find the optical bandwidths when the peak optical gains of the amplifier are 10dB, 20dB and 30dB, respectively.

Solution:
For the peak optical gain of 10dB, 20dB and 30dB, $g_0L = \ln(G_0) = 2.3026, 4.6052$ and 6.9078, respectively.

$$B_0 = \Delta f \sqrt{\frac{\ln 2}{g_0L - \ln 2}} = 19.69\text{nm}, 12.63\text{nm} \text{ and } 10\text{nm}, \text{ respectively.}$$

In general, with the increase of the peak optical gain, the bandwidth will be reduced.

2. For an optical amplifier with the small-signal optical gain $G_0 = 30\text{dB}$ at the peak gain wavelength, and the saturation optical power is $P_{\text{sat}} = 10\text{mW}$. Find optical gain of this amplifier for the input signal optical power of -30dBm, -20dBm and 0dBm at the peak gain wavelength. (note: numerical method has to be used).

Solution:
Based on Equation 5.1.10,

$$\ln\left(\frac{G}{G_0}\right) = \frac{P(0)}{P_{\text{sat}}}$$

For the input power $P(0) = -30\text{dBm}$,

$$\ln\left(G \times 10^{-3}\right) = 10^{-4}$$

One needs to find at which value of $G$, $\ln\left(G \times 10^{-3}\right) - 10^{-4}(1 - G)$ is minimum, so that $G = 913$.

For $P(0) = -2\text{dBm}$ and 0dBm, the optical gains are: $G = 567$, and $G = 35$

3. An optical amplifier operates in the 1550nm wavelength window with a spontaneous emission factor of $n_{sp} = 2.5$ and an optical gain of 30dB. What is the optical noise power within 0.1nm optical bandwidth?

Solution:
0.1nm optical bandwidth in the 1550nm wavelength window is equivalent to 12.5GHz, and the photon energy is,

$$\frac{hc}{\lambda} = 6.626 \times 10^{-34} \times 3 \times 10^8 / 1550 \times 10^{-9} = 1.283 \times 10^{-19} \text{J}, \text{ and thus,}$$

$$P_{\text{ASE}} = 2n_{sp}hf\left[G(f) - 1\right]B_0 = 2 \times 2.5 \times 1.283 \times 10^{-19} \times 999 \times 12.5 \times 10^9 = 8\mu\text{W}$$

4. Consider an optical amplifier in the 1550nm wavelength window with 6dB noise figure and optical gain $G \gg 1$. The optical signal at the out of this amplifier is detected by a photodiode as shown in the following figure. If there is a bandpass optical filter (BPF)
with the bandwidth $B_o = 1\text{nm}$ before the photodiode, what is the signal optical power at which the signal-ASE beat noise is equal to the ASE-ASE beat noise? (assume that electric bandwidth is much less than the optical bandwidth)

Solution:

In the electric domain, signal-ASE noise and ASE-ASE beat noise PSD are,

$$S_{S-ASE} = 2R^2GP_in\rho_{ASE} \quad \text{and} \quad S_{ASE-ASE} = \frac{9R^2\rho_{ASE}^2}{2}(B_o - f_{ele}) \approx \frac{9R^2\rho_{ASE}^2}{2}B_o$$

$$n_{sp} = F/2 = 2$$

When they are equal, $$P_{in} = \frac{\rho_{ASE}B_o}{4G} = \frac{n_{sp}hf}{2} \left(\frac{G-1}{G}\right)B_o \approx n_{sp}hfB_o/2$$

$$P_{in} = 1.283 \times 10^{-19} \times 125 \times 10^9 = 1.6 \times 10^{-8} W \approx -48dBm$$

When the optical signal is higher than -48dBm, signal-ASE beat noise will be higher than the ASE-ASE beat noise.

5. There is an optical pre-amplifier in an optical receiver before the photodiode. The input optical power to the optical amplifier is $P_{in} = -20\text{dBm}$, and the optical amplifier has a 6dB noise figure. Other parameters are, operation temperature $T = 300k$, load resistance $R_L = 50\Omega$, photodiode responsivity $R = 0.9\text{A/W}$, and the operation wavelength $\lambda = 1550\text{nm}$.

(a) if the gain of the optical amplifier is $G = 30\text{dB}$, please find the noise power spectral densities of thermal noise, shot noise and signal-ASE beat noise

(b) What is the required optical gain of the amplifier so that signal-ASE beat noise is 10dB higher than the thermal noise after photo-detection?

Solution:

(a) thermal noise: $$\sigma_{th}^2 = \frac{4kT}{R_L} = \frac{4 \times 1.38 \times 10^{-23} \times 300}{50} = 3.312 \times 10^{-22} W / Hz$$

shot noise: $$\sigma_{th}^2 = 2qRGP_{in} = 2 \times 1.6 \times 10^{-19} \times 0.9 \times 10^{-3} \times 10^{-5} = 2.9 \times 10^{-21} W / Hz$$

Signal-ASE beat noise:

$$\sigma_{S-ASE}^2 = 4R^2P_{in}Ghf\rho_{sp}(G-1)$$

$$= 4 \times 0.81 \times 10^{-5} \times 10^3 \times 1.283 \times 10^{-19} \times 2 \times 999 = 8.3 \times 10^{-18} W / Hz$$
(b) When signal-ASE beat noise has to be 10 times higher than thermal noise,

\[49R^2 P_{in} h f n_{sp} G(G - 1) = 10 \times 4kT / R_L,\]

that is,

\[G \approx \sqrt{\frac{10kT}{R_L 9R^2 P_{in} h f n_{sp}}} = \sqrt{\frac{10 \times 1.38 \times 10^{-23} \times 300}{50 \times 0.81 \times 10^{-5} \times 1.283 \times 10^{-19} \times 2}} = 20 = 13dB\]

Therefore, if the optical amplifier gain is higher than 13dB, thermal noise contribution in the receiver can be negligible.