Chapter 3
Light Sources for Optical Communications

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In optical systems, signals are carried by photons and therefore an optical source is an essential part of every optical system. Although there are various types of optical sources, semiconductor-based light emitting diodes (LED) and laser diodes (LD) are most often used in fiber-optic systems because their miniature size and high power efficiency and superior reliability. Semiconductor LD and LED are based on forward-biased PN junctions, and the output optical powers are proportional to the injection electric currents. Thus optical power emitted from a LED or a LD can be rapidly modulated by the electrical injection current, which is commonly referred to as direct modulation. As semiconductor materials have energy bands instead of discrete energy levels as in the case of gases, transition between these energy bands allows photon emission across relatively wide wavelength window. Thus, semiconductor based laser sources are uniquely suited to support wavelength division multiplexing (WDM) in fiber optic systems to make efficient use of the wide bandwidth of the fiber. On the flip side, the broadband nature of semiconductor material also makes it difficult to achieve single frequency operation of a laser diode, and thus requiring special structural design such as DFB (distributed feedback), DBR (distributed Bragg reflector), or external cavity to ensure single longitudinal mode operation with narrow spectral linewidth. As the majority of optical communication systems use semiconductor lasers as the light sources, technological advances in semiconductor lasers have tremendous impacts in the architectural design and practical application of optical systems and networks. Historically, the evolution of communication wavelengths from 800nm to 1310nm and 1550nm were largely dictated by the availability of laser sources. Recent industrial development in coherent optical communication systems was also enabled by the availability of miniaturized narrow linewidth diode laser modules.
3.1. Properties of semiconductor materials for light sources

3.1.1. PN junction and energy diagram

In a doped semiconductor, the Fermi level $E_F$ depends on the doping density as,

$$E_F = E_{Fi} + kT \ln\left(\frac{n}{n_i}\right) \quad (3.1.1)$$

where, $k$ is the Boltzmann's constant, $T$ is the absolute temperature, $n$ is electron density, $E_{Fi}$ is the intrinsic Fermi level, and $n_i$ is the intrinsic electron density. Equation (3.1.1) can also be expressed as the function of the hole density $p$ and the intrinsic hole density $p_i$ as, $E_F = E_{Fi} - kT \ln\left(\frac{p}{p_i}\right)$. This is because under thermal equilibrium, the product of electron and hole densities is a constant, $np = n_i^2$, and the intrinsic electron and hole densities are always equal, $n_i = p_i$. As intrinsic electron (hole) density is usually low at room temperature, carrier density of a typical doped semiconductor is determined by the doping densities, such that $n \approx N_d$ for $n$-type doping and $p \approx N_a$ for $p$-type doping, where $N_d$ and $N_a$ are electron and hole doping densities, respectively. As the result, the Fermi level of a doped semiconductor is largely determined by doping condition, that is, $E_F = E_{Fi} + kT \ln\left(\frac{N_d}{n_i}\right)$, or $E_F = E_{Fi} - kT \ln\left(\frac{N_a}{p_i}\right)$. As illustrated in Figure 3.1.1(a), the Fermi level is closer to the conduction band in a $n$-type semiconductor because $N_d >> n_i$, and it is closer to the valence band in a $p$-type semiconductor because $N_a >> p_i$.

A pn junction is formed when there the n-type and the p-type semiconductors have intimate contact. Under thermal equilibrium, the Fermi level will be unified across the pn junction structure as shown in Figure 3.1.1(b). This happens because high-energy free electrons diffuse from n-side to p-side and low-energy holes diffuse in the opposite direction; as a result, the energy level of the p-type side is increased compared to that in the n-type side. Meanwhile, because free electrons migrate from n-type side to p-type-side, uncovered protons left over at the edge of the n-type semiconductor create a positively charged layer on the n-type side. Similarly, a negatively charged layer is created at the edge of the p-type semiconductor due to the loss of holes. Thus built-in electrical field and thus a potential barrier is created at the pn junction, which pulls the diffused free electronics back to the n-type side and holes back to the p-type-side, a process commonly referred to as carrier drift. Because of this built-in electrical field, neither free electrons nor holes exist at the junction.
region, and therefore this region is called the *depletion region* or *space charged region*. Without an external bias, there is no net carrier flow across the pn junction due to the exact balance between carrier diffusion and carrier drift. The built-in electrical potential barrier across the pn junction is related to the electron doping density $N_d$ on the n-type side and hole doping density $N_a$ on the p-type side, and can be expressed as,

$$\Delta V_{pm} = \frac{kT}{q} \left( \ln \left( \frac{N_d}{n_i} \right) + \ln \left( \frac{N_a}{p_i} \right) \right) = \frac{kT}{q} \ln \left( \frac{N_d N_a}{n_i^2} \right)$$

where, $q$ is the electron charge.

When the pn-junction is forward-biased as illustrated in Figure 3.1.1(c), excess electrons and holes are injected into the n-type and the p-type sections, respectively. This carrier injection reduces the built-in potential barrier so that the balance between the electron drift and diffusion broken. The external biasing source pushes excess electrons and holes to diffuse across the junction area. In this process, excess electrons and holes recombine inside the depletion region to generate photons. The recombination can be *non-radiative* which turns the energy difference between the participating electron and the hole into heat. It can also be *radiative*, in such a case a photon is created and the photon energy is equal to the energy of the bandgap, which will be discussed in the next section.
3.1.2. Direct and indirect semiconductors

One important rule of radiative recombination process is that both energy and momentum must be conserved. Depending on the shape of their band structure,
semiconductor materials can be generally classified as having direct bandgap or indirect bandgap, as illustrated in Figure 3.1.2, where $E$ is the energy and $k$ is the momentum.

Figure 3.1.2 Illustration of direct bandgap (a) and indirect bandgap (b) of semiconductor materials.

For direct semiconductors, holes at the top of the valence band have the same momentum as the electrons at the bottom of the conduction band. In this case, electrons directly recombine with the holes to emit photons, and the photon energy is equal to the bandgap,

$$hv = \frac{hc}{\lambda} = E_g \quad (3.1.2)$$

where, $h$ is the Planck's constant, $E_g$ is the bandgap of the semiconductor material, $\nu = \frac{c}{\lambda}$ is the optical frequency, $c$ is the speed of light, and $\lambda$ is the optical wavelength. Examples of direct bandgap semiconductor materials include GaAs, InAS, InP, AlGaAs, and InGaAsP. The desired bandgap can be obtained through compositional design of the material, as well as structural engineering in the nanometer scales such as quantum well and quantum dots.

For indirect semiconductors, on the other hand, holes at the top of the valence band and electrons at the bottom of the conduction band have different momentum. Any recombination between electrons in the conduction band and holes in the valence band would require significant momentum change. Although a photon can have considerable energy $hv$, its momentum $hv/c$ is much smaller, which cannot compensate for the momentum mismatch between the electrons and the holes.
Therefore radiative recombination is considered impossible in indirect semiconductor materials unless a third particle (for example, a phonon created by crystal lattice vibration) is involved and provides the required momentum. Silicon is a typical indirect bandgap semiconductor material, and therefore it cannot be directly used to make light emitting devices.

3.1.3 Spontaneous emission and stimulated emission

As discussed, radiative recombination between electrons and holes creates photons, but this is a random process. The energy is conserved in this process, which determines the frequency of the emitted photon as \( v = \Delta E / h \), where \( \Delta E \) is the energy gap between the conduction band electron and the valence band hole that participated in the process. \( h \) is Planck’s constant. However, the phase of the emitted lightwave is not predictable. Indeed, since semiconductors are solids, energy of carriers are not on discrete levels; instead they are continuously distributed within energy bands following the Fermi-Dirac distribution, as illustrated by Figure 3.1.3. Although the nominal value of \( \Delta E \) is in the vicinity of the material bandgap \( E_g \), its distribution depends on the width of the associated energy bands and the Fermi-Dirac distribution of carriers within these bands.

Figure 3.1.3 (a) shows that different electron-hole pairs may be separated by different energy gaps and \( \Delta E_i \) might not be equal to \( \Delta E_j \). Recombination of different electron-hole pairs will produce emission at different wavelengths. The spectral width of the emission is determined by the statistic energy distribution of the carriers, as illustrated by Figure 3.1.3(b).

![Figure 3.1.3](image-url)

Figure 3.1.3 Illustration of an energy band in semiconductors and the impact on the spectral width of radiative recombination. (a) Energy distributions of electrons and holes. (b) Probability distribution of the frequency of emitted photons.
Spontaneous emission is created by the spontaneous recombination of electron-hole pairs. The photon generated from each recombination event is independent, although statistically the emission frequency falls into the spectrum shown in Figure 3.1.3(b). The frequencies, the phases, and the direction of propagation of the emitted photons are not correlated. This is illustrated in Figure 3.1.4(a).

Stimulated emission, on the other hand, is created by stimulated recombination of electron-hole pairs. In this case the recombination is induced by an incoming photon, as shown in Figure 3.1.4(b). Both the frequency and the phase of the emitted photon are identical to those of the incoming photon. Therefore photons generated by the stimulated emission process are coherent, which results in narrow spectral linewidth.

![Figure 3.1.4 Illustration of spontaneous emission (a) and stimulated emission (b).](image)

3.1.4 Carrier confinement

In addition to the requirement of using direct bandgap semiconductor material, another important requirement for LEDs is the carrier confinement. In early LEDs, materials with the same bandgap were used at both sides of the pn junction, as shown in Figure 3.1.1. This is referred to as homojunction. In this case, carrier recombination happened over the entire depletion region with the width of 1~10 μm depending on the diffusion lengths of the electrons and the holes. This wide depletion region makes it difficult to achieve high spatial concentration of carriers. To overcome this problem, double heterojunction was introduced in which a thin layer of semiconductor material with a slightly narrower bandgap is sandwiched in the middle of the junction region between the p-type and the n-type sections. This concept is illustrated in Figure 3.1.5, where $E_g' < E_g$. 
In this structure, the thin layer with slightly smaller bandgap than other regions attracts the concentration of carriers when the junction is forward biased; therefore this layer is referred to as the *active region* of the device. The carrier confinement is a result of band gap discontinuity. The width, $W$, of the low bandgap layer can be precisely controlled in the layer deposition process which is typically on the order of ~0.1\( \mu \)m. This is several orders of magnitude thinner than the depletion region of a homojunction; therefore very high levels of carrier concentration can be realized at a certain injection current.

In addition to providing carrier confinement, another advantage of double heterostructure is the ability of providing efficient photon confinement which is essential for initiating stimulated emission. By using a material with slightly higher refractive index for the sandwich layer, a dielectric waveguide is formed. This dielectric optical waveguide provides a mechanism to maintain spatial confinement of photons within the active layer, and therefore very high photon density can be achieved.

### 3.2 Light-Emitting Diodes (LEDs)

Light emission in an LED is based on the spontaneous emission of forward-biased semiconductor pn junction. The basic structures are shown in Figure 3.2.1 for surface-emitting and edge-emitting LEDs.

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*Figure 3.1.5 Illustration of semiconductor double heterostructure.*
For surface-emitting diodes, light emits in the perpendicular direction of the active layer. The brightness of the active area is usually uniform and the emission angle is isotropic, which is commonly referred to as *Lambertian*. In such a case, optical power emission patterns can be described by \( P(\theta) = P_0 \cos \theta \), where \( \theta \) is the angle between the emitting direction and the surface normal of the emitter, and \( P_0 \) is the optical power viewed from the direction of surface normal. By properly design the shape of the bottom metal contact, the active emitting area can be made circular to maximize the coupling efficiency to an optical fiber.

For edge-emitting diodes, on the other hand, light emits in the same direction as the active layer. In this case, a waveguiding mechanism is usually required in the design, where the active layer has slightly higher refractive index than the surrounding layers, so that a higher power density in the active layer can be achieved. Compared to surface-emitting diodes, the emitting area of edge-emitting diodes is usually much
smaller and asymmetric, which is determined by the width and thickness of the active layer, or the cross section of the waveguide.

3.2.1 P~I curve

For an LED, the emitting optical power $P_{opt}$ is linearly proportional to the injected electrical current, as shown in Figure 3.2.2. This is commonly referred to as the $P$--$I$ curve. In the idea case, the recombination of each electron-hole generates a photon. If we define the power efficiency $dP_{opt}/dI$ as the ratio between the emitting optical power $P$ and the injected electrical current $I$, we have:

$$\frac{dP_{opt}}{dI} = \frac{hv}{q} = \frac{hc}{\lambda q}$$

(3.2.1)

where, $q$ is the electron change, $h$ is Plank’s constant, $c$ is the speed of light, $v$ is the optical frequency, and $\lambda$ is the wavelength. In a practical light emitting device, not all recombination events are radiative, and some of them do produce photons, and thus, an internal quantum efficiency can be defined as:

$$\eta_q = \frac{R_r}{R_r + R_{nr}}$$

(3.2.2)

where $R_r$ and $R_{nr}$ are the rates of radiative and nonradiative recombinations, respectively. A higher recombination rate is equivalent to a shorter carrier lifetime and thus the radiative and nonradiative carrier times are defined as $\tau_r = 1/R_r$ and $\tau_{nr} = 1/R_{nr}$, respectively, and the overall carrier lifetime is $\tau = (1/\tau_r + 1/\tau_{nr})^{-1}$.

Another factor that reduces the slope of the $P$--$I$ curve is that not all the photons generated through radiative recombination are able to exit the active layer. Various effects contribute to this efficiency reduction, such as internal material absorption, and total interface reflection at certain regions of emission angle. This is characterized as an external efficiency defined by:

$$\eta_{ext} = \frac{R_{emit}}{R_r}$$

(3.2.3)

where, $R_{emit}$ is the rate of the generated photons that actually exit the LED.
Figure 3.2.2 LED emitting power is linearly proportional to the injection current.

Considering both internal quantum efficiency and external efficiency, the slope of the P–I curve should be:

$$\frac{dP_{opt}}{dI} = \eta_q \eta_{ext} \frac{hc}{\lambda q}$$  \hspace{1cm} (3.2.4)

Because of the linear relationship between optical power and the injection current, the emitted optical power of an LED is:

$$P_{opt} = \eta_q \eta_{ext} \frac{hc}{\lambda q} I$$  \hspace{1cm} (3.2.5)

In general, the internal quantum efficiency of an LED can be on the order of 70 percent. However, since an LED is based on spontaneous emission and the photon emission is isotropic, its external efficiency is usually less than 5 percent. As a rule of thumb, for $\eta_q = 75\%$, $\eta_{ext} = 2\%$, and the wavelength of $\lambda = 1550\text{nm}$, the output optical power efficiency is approximately $12\mu\text{W/mA}$.

**Example 3.1,**

For a surface emitting LED shown in the following figure, what is the probability of a spontaneously generated photon inside the active layer to escape into the air above the LED? Assume the $n_1 = 3.5$, $n_2 = 3.45$, and $n_0 = 1$. 

![Diagram of LED and optical indices](image-url)
Figure 3.2.3, illustration of LED external efficiency reduction due to total internal reflection

Solution:

Spontaneous emission has random emission angle which fills $4\pi$. Total internal reflection between active layer and air is $\theta = \sin^{-1}(n_o / n_i) = 0.29\text{rad.} = 16.6^\circ$

The probability of escaping is $\eta = \frac{2\pi (1 - \cos \theta)}{4\pi} = \frac{0.0418}{2} = 0.029 = 2.9\%$

Note that, the critical angle on the $n_1/n_2$ interface is much larger than that between $n_1$ and $n_0$. Thus no reflection from $n_1/n_2$ interface can turn back to escape through $n_1/n_0$ interface.

This example indicates that the external efficiency of an LED is usually very small, mainly because of the random emission angle of spontaneous emission and a relatively narrow escape angle due to total internal reflection of the active layer.

3.2.2 Modulation dynamics

Within the active layer of an LED, the increase of carrier population is proportional to the rate of external carrier injection minus the rate of carrier recombination. Therefore, the rate equation of carrier population $N_i$ is:

$$\frac{dN_i(t)}{dt} = \frac{I(t)}{q} - \frac{N_i(t)}{\tau}$$

(3.2.6)

where, $\tau$ is referred to as carrier lifetime. In general, $\tau$ is a function of carrier density, and the rate equation does not have a closed-form solution. To simplify, if we assume $\tau$ is a constant, Equation 3.2.6 can be easily solved in the frequency domain as:

$$\tilde{N}_i(\omega) = \frac{\tilde{I}(\omega)\tau / q}{1 + j\omega\tau}$$

(3.2.7)

where $\tilde{N}_i(\omega)$ and $\tilde{I}(\omega)$ are the Fourier transforms of $N_i(t)$ and $I(t)$, respectively.

Equation 3.2.7 demonstrates that carrier population can be modulated through injection current modulation, and the 3-dB modulation bandwidth is $B_{3\text{dB}} = 1/\tau$.

Because the photons are created by radiative carrier recombination, and the optical
power is proportional to the carrier population, and thus the modulation bandwidth of the optical power also has the bandwidth of $1/\tau$.

$$P_{\text{opt}}(\omega) = \frac{P_{\text{opt}}(0)}{\sqrt{1 + (\omega \tau)^2}} \quad (3.2.8)$$

where $P_{\text{opt}}(0)$ is the optical power at DC. Typical carrier lifetime of an LED is in the order of nanoseconds and therefore the modulation bandwidth is in the $100\text{MHz} \sim 1\text{GHz}$ level, depending on the structure of the LED.

It is worth noting that since the optical power is proportional to the injection current, the modulation bandwidth can be defined as either electrical or optical. Here comes a practical question: To fully support an LED with an optical bandwidth of $B_{\text{opt}}$, what electrical bandwidth $B_{\text{ele}}$ is required for the driver circuit? Since optical power is proportional to the electrical current, $P_{\text{opt}}(\omega) \propto I(\omega)$, and the driver electrical power is proportional to the square of the injection current, $P_{\text{ele}}(\omega) \propto I^2(\omega)$, therefore driver electrical power is proportional to the square of the LED optical power $P_{\text{ele}}(\omega) \propto P_{\text{opt}}^2(\omega)$, that is:

$$\frac{P_{\text{ele}}(\omega)}{P_{\text{ele}}(0)} = \frac{P_{\text{opt}}^2(\omega)}{P_{\text{opt}}^2(0)} \quad (3.2.9)$$

If at a frequency the optical power is reduced by 3dB compared to its DC value, the driver electrical power is supposed to be reduced by only 1.5dB at that frequency. That is, 3dB electrical bandwidth is equivalent to 6dB optical bandwidth.

**Example 3.2**

Consider an LED emitting at $\lambda = 1550\text{nm}$ wavelength window. The internal quantum efficiency is 70%, the external efficiency is 2%, the carrier lifetime is $\tau = 20\text{ns}$, and the injection current is 20mA. Find:

(a) the output optical power of the LED, (b) the 3dB optical bandwidth, and (c) the required driver electrical bandwidth

Solution:

(a) Output optical power is:
(b) To find the 3dB optical bandwidth, we use:

\[ P_{\text{opt}}(\omega) = \frac{P_{\text{opt}}(0)}{\sqrt{1 + \omega^2 \tau^2}} \]

at \( \omega = \omega_{3\text{dB, opt}} \), \( \frac{1}{\sqrt{1 + \omega_{3\text{dB, opt}}^2 \tau^2}} = \frac{1}{2} \), that is \( \omega_{3\text{dB, opt}}^2 \tau^2 = 3 \)

Therefore, the angular frequency of optical bandwidth is

\( \omega_{3\text{dB, opt}} = \sqrt{3} / \tau = 86.6 \text{Mrad/s} \), which corresponds to a circular frequency

\( f_{3\text{dB, opt}} = \sqrt{3} / (2\pi\tau) \approx 13.8 \text{MHz} \).

(c) To find the 3dB electric bandwidth, we use:

\[ P_{\text{ele}}(\omega) = |P_{\text{ele}}(\omega)|^2 = \frac{P_{\text{ele}}^2(0)}{1 + \omega^2 \tau^2} \]

at \( \omega = \omega_{3\text{dB, ele}} \), \( \frac{1}{1 + \omega_{3\text{dB, ele}}^2 \tau^2} = \frac{1}{2} \),

Therefore \( \omega_{3\text{dB, ele}} = 1 / \tau = 50 \text{Mrad/s} \) and \( f_{3\text{dB, ele}} \approx 8 \text{MHz} \).

The relation between electrical and optical bandwidth is

\( \omega_{3\text{dB, opt}} / \omega_{3\text{dB, ele}} = \sqrt{3} \)

3.3 Laser Diodes (LDs)

Semiconductor laser diodes are based on the stimulated emission of forward-biased semiconductor pn junction. Compared to LEDs, LDs have higher spectral purity and higher external efficiency because of the spectral and spatial coherence of stimulated emission.

3.3.1 Amplitude and phase conditions for self-sustained oscillation

One of the basic requirements of laser diodes is optical feedback. Consider an optical cavity of length \( L \) as shown in Figure 3.3.1, where the semiconductor material in the cavity provides an optical gain \( g \) and an optical loss \( \alpha \) per unit length, the
The lightwave travels back and forth in the longitudinal ($\pm z$) direction in the cavity. The ratio of optical field before and after each roundtrip in the cavity is:

$$\frac{E_{t+1}}{E_t} = \sqrt{R_1} \sqrt{R_2} \exp(\Delta G + j\Delta \Phi)$$

where the total roundtrip phase shift of the optical field is:

$$\Delta \Phi = \frac{2\pi}{\lambda} 2nL$$

and the net roundtrip optical gain coefficient of the optical field is:

$$\Delta G = (\Gamma g - \alpha)2L$$

where $g$ is the optical gain coefficient in [$cm^{-1}$] and $\alpha$ is the material absorption coefficient, also in [$cm^{-1}$]. $0<\Gamma<1$ is a confinement factor. Since not all the optical field is confined within the active region of the waveguide, $\Gamma$ is defined as the ratio between the optical field in the active region and the total optical field.

To support a self-sustained oscillation, the optical field has to repeat itself after each roundtrip, and thus,

$$\sqrt{R_1} \sqrt{R_2} \exp(\Delta G + j\Delta \Phi) = 1$$

This is a necessary condition of oscillation, which is also commonly referred to as the \textit{threshold condition}. Equation 3.3.4 can be further decomposed into a phase condition and a threshold gain condition.
The phase condition is that after each roundtrip the optical phase change must be multiples of \(2\pi\), that is \(\Delta \Phi = 2m\pi\), where \(m\) is an integer. One important implication of this phase condition is that it can be satisfied by multiple wavelengths,

\[
\lambda_m = \frac{2nL}{m}
\]  
(3.3.5)

This explains the reason that a laser may emit at multiple wavelengths, which are generally referred to as *multiple longitudinal modes*.

**Example 3.3**

For an InGaAsP semiconductor laser operating in 1550nm wavelength window, if the effective refractive index of the waveguide is \(n \approx 3.5\) and the laser cavity length is \(L = 300\mu m\), find the wavelength spacing between adjacent longitudinal modes.

Solution:

Based on Equation 3.3.5, the wavelength spacing between the \(m\)-th mode and the \((m+1)\)-th modes can be found as \(\Delta \lambda \approx \lambda_m^2 / (2nL)\). Assume \(\lambda_m = 1550nm\), this mode spacing is \(\Delta \lambda = 1.144nm\), which corresponds to a frequency separation of approximately \(\Delta f = 143GHz\).

The threshold gain condition is that after each roundtrip the amplitude of optical field does not change—that is, \(\sqrt{R_1} \sqrt{R_2} \exp[(\Gamma g_{th} - \alpha)2L] = 1\), where \(g_{th}\) is the optical field gain at threshold. Therefore, in order to achieve the lasing threshold, the optical gain has to be high enough to compensate for both the material attenuation and the optical loss at the mirrors,

\[
\Gamma g_{th} = \alpha - \frac{1}{4L} \ln(R_1R_2)
\]  
(3.3.6)

Inside a semiconductor laser cavity, the optical field gain coefficient is a function of the carrier density, which depends on the rate of carrier injection,

\[
g(N) = a(N - N_o)
\]  
(3.3.7)
In this expression, $N$ is carrier density in $[cm^{-3}]$ and $N_0$ is the carrier density required to achieve material transparency. $a$ is the differential gain coefficient in $[cm^2]$; it indicates the gain per unit length along the laser cavity per unit carrier density.

In addition, due to the Fermi-Dirac distribution of carriers and the limited width of the energy bands, the differential gain is a function of the wavelength, which can be approximated as parabolic,

$$a(\lambda) = a_0 \left\{ 1 - \left( \frac{\lambda - \lambda_0}{\Delta \lambda_g} \right)^2 \right\}$$ (3.3.8)

for $|\lambda - \lambda_0| \ll \Delta \lambda_g$, where $a_0$ is the differential gain at the central wavelength $\lambda = \lambda_0$ and $\Delta \lambda_g$ is the spectral bandwidth of the material gain.

It must be noted that material gain coefficient $g$ is not equal to the actual gain of the optical field. When an optical field travels along an active waveguide, the field propagation is described as,

$$E(z,t) = E_0 e^{(iGz-\gamma t)} e^{j(\omega t-\beta z)}$$ (3.3.9)

where $E_0$ is the optical field at $z = 0$, $\omega$ is optical frequency, and $\beta = 2\pi / \lambda$ is the propagation constant. The envelop optical power along the waveguide is then

$$P(z) = |E(z)|^2 = P_0 e^{2(Gz-\gamma z)}$$ (3.3.10)

Combine Equations 3.3.7, 3.3.8, and 3.3.10, we have

$$P(z,\lambda) = |E(z)|^2 = P(z,\lambda_0) \exp \left\{ -2G_0 \left( \frac{\lambda - \lambda_0}{\Delta \lambda_g} \right)^2 z \right\}$$ (3.3.11)

where $P(z,\lambda_0) = P(0,\lambda_0)e^{2(Gz-\gamma z)}$ is the peak optical power at $z$. 

As shown in Figure 3.3.2, although there are a large number of potential longitudinal modes that all satisfy the phase condition, the threshold gain condition can be reached only by a limited number of modes near the center of the gain peak.

### 3.3.2 Rate equations

Rate equations describe the nature of interactions between photons and carriers in the active region of a semiconductor laser. Useful characteristics such as output optical power versus injection current, modulation response, and spontaneous emission noise can be found by solving the rate equations.

\[
\frac{dN(t)}{dt} = \frac{J}{qd} - \frac{N(t)}{\tau} - 2\Gamma_v a(N - N_0)P(t) \tag{3.3.12}
\]

\[
\frac{dP(t)}{dt} = 2\Gamma_v a(N - N_0)P(t) - \frac{P(t)}{\tau_{ph}} + R_{sp} \tag{3.3.13}
\]

where \(N(t)\) is the carrier density and \(P(t)\) is the photon density within the laser cavity, they have the same unit \([cm^{-3}]\). \(J\) is the injection current density in \([C/cm^2]\), \(d\) is the thickness of the active layer, \(v_g\) is the group velocity of the lightwave in \([cm/s]\), and \(\tau\) and \(\tau_{ph}\) are electron and photon lifetimes, respectively. \(R_{sp}\) is the rate of spontaneous emission; it represents the density of spontaneously generated photons per second that coupled into the lasing mode, so the unit of \(R_{sp}\) is \([cm^{-3}s^{-1}]\).

On the right-hand side of Equation 3.3.12, the first term is the number of electrons injected into each cubic meter within each second time window; the second term is the electron density reduction per second due to spontaneous recombination; the third
term represents electron density reduction rate due to stimulated recombination, which is proportional to both material gain and the photon density.

The same term $2 \Gamma v_g a(N - N_0)P(t)$ also appears in Equation 3.3.13 due to the fact that each stimulated recombination event will generate a photon, and therefore the first term on the right-hand side of Equation 3.3.13 is the rate of photon density increase due to stimulated emission. The second term on the right-hand side of Equation 3.3.13 is the photon density decay rate due to both material absorption and photon leakage from the two mirrors. The escape of photons through mirrors can be treated equivalently as a distributed loss in the cavity with the same unit, [\text{cm}^{-1}], as the material attenuation. The equivalent mirror loss coefficient is defined as

$$\alpha_m = \frac{-\ln(R_1 R_2)}{4L} \quad (3.3.14)$$

In this way, the photon lifetime can be expressed as

$$\tau_{\text{ph}} = \frac{1}{2 v_g (\alpha + \alpha_m)} \quad (3.3.15)$$

where $\alpha$ is the material attenuation coefficient. Using this photon lifetime expression, the photon density rate equation (3.3.13) can be simplified as

$$\frac{dP(t)}{dt} = 2 v_g \left[ \Gamma g - (\alpha + \alpha_m) \right] P(t) + \frac{P}{\tau_{\text{ph}}} \quad (3.3.16)$$

where $g$ is the material gain as defined in Equation 3.3.7.

Rate equations 3.3.12 and 3.3.13 are coupled differential equations, and generally they can be solved numerically to predict static as well as dynamic behaviors of a semiconductor laser.

### 3.3.3 Steady state solutions of rate equations

In the steady state, $d/dt = 0$, rate Equations 3.3.12 and 3.3.13 can be simplified as

$$\frac{J}{q d} - \frac{N}{\tau} - 2 \Gamma v_g a(N - N_0)P = 0 \quad (3.3.17)$$

$$2 \Gamma v_g a(N - N_0)P \frac{P}{\tau_{\text{ph}}} + R_{sp} = 0 \quad (3.3.18)$$
With this simplification, the equations can be solved analytically, which will help understand some basic characteristics of semiconductor lasers.

(A) **Threshold carrier density and current density**

Assume that $R_{sp}$, $\tau_{ph}$ and $\alpha$ are constants. Equation 3.3.18 can be expressed as

$$P = \frac{R_{sp}}{1/\tau_{ph} - 2\Gamma v_g a(N - N_0)}$$

Equation 3.3.19 indicates that when $2\Gamma v_g a(N - N_0)$ approaches the value of $1/\tau_{ph}$, the photon density would approach infinite, and this operation point is defined as the **threshold**. Therefore the threshold carrier density can be found as

$$N_{th} = N_0 + \frac{1}{2\Gamma v_g a \tau_{ph}}$$

In order to ensure a positive value of the photon density, $2\Gamma v_g a(N - N_0) < 1/\tau_{ph}$ is necessary, which requires $N < N_{th}$. Practically, carrier density $N$ can be increased to approach the threshold carrier density $N_{th}$ by increasing the injection current density. However, the level of threshold carrier density, $N_{th}$, can never be reached. With the increase of carrier density, photon density will be increased. Especially when the carrier density approaches the threshold level, photon density will be increased dramatically and the stimulated recombination becomes significant, which, in turn, reduces the carrier density. Figure 3.3.3 illustrates the relationships among carrier density $N$, photon density $P$, and the injection current density $J$. 

![Graph](image_url)
Figure 3.3.3 Photon density $P(J)$ and Carrier density $N(J)$ as functions of injection current density $J$. $J_{th}$ is the threshold current density and $N_{th}$ is the threshold carrier density.

As shown in Figure 3.3.3, for a semiconductor laser, carrier density linearly increases with the increase of injection current density to a certain level. After that level the carrier density increase is quickly saturated due to the significant contribution of stimulated recombination. The current density corresponding to that saturation point is called threshold current density, above which the laser output is dominated by stimulated emission. However, below that threshold point spontaneous recombination and emission is the dominant mechanism similar to that in an LED, and the output optical power is usually small because of the low external efficiency. For a laser diode operating below threshold, stimulated recombination is negligible, and Equation 3.3.17 can be simplified as $J / qd = N / \tau$. Assume this relation is still valid at the threshold point, $N = N_{th}$, the threshold current density is can be found, by considering the threshold carrier density definition 3.3.20, as,

$$J_{th} = \frac{qd}{\tau_e} = \frac{qd}{\tau_e} \left( N_0 + \frac{1}{2 \Gamma \nu_g a \tau_{ph}} \right)$$  (3.3.21)

(B) $P$–$J$ relationship about threshold

In general, the desired operation region of a laser diode is well above the threshold, where high-power coherent light is generated by stimulated emission. Combining equations 3.3.17 and 3.3.18, we have

$$\frac{J}{qd} = \frac{N}{\tau} + \frac{P}{\tau_{ph}} - R_{sp}$$  (3.3.22)

As shown in Figure 3.3.3, in the operation region well above threshold, carrier density is approximately equal to its threshold value ($N \approx N_{th}$). In addition, since $N_{th} / \tau = J_{th} / qd$, we have

$$\frac{J}{qd} = \frac{J_{th}}{qd} + \frac{P}{\tau_{ph}} - R_{sp}$$  (3.3.23)

Thus the relationship between photon density and current density above threshold is
Apart from a spontaneous emission contribution term $\tau_{ph} R_{sp}$, which is usually very small, the photon density is linearly proportional to the injection current density for $J > J_{th}$ and the slope is $dP/dJ = \tau_{ph}/qd$.

Then, a question is how to relate the photon density inside the laser cavity to the output optical power of the laser? Assume that the waveguide of the laser cavity has a length $l$, width $w$, and thickness $d$, as shown in Fig. 3.3.4.

![Illustration of the dimension of a laser cavity.](image)

The output optical power is the flow of photons through the facet, which can be expressed as

$$P_{opt} = P \cdot (lwd)hv2\alpha_{m}v_{g} \quad (3.3.25)$$

where $P \cdot (lwd)$ is the total photon number and $P \cdot (lwd)hv$ is the total photon energy within the cavity. $\alpha_{m}$ is the mirror loss in $[cm^{-1}]$, which is the percentage of photons that escape from each mirror, and $\alpha_{m}v_{g}$ represents the percentage of photon escape per second. The factor 2 indicates that photons travel in both directions along the cavity and escape through two end mirrors.

Neglecting the contribution of spontaneous emission in Equation 3.3.24, combining Equations 3.3.25 and 3.3.15, with Equation 3.3.24, and considering that the injection current is related to current density by $I = J \cdot w l$, we have

$$P_{opt} = \frac{(I - I_{th})hv}{q} \cdot \frac{\alpha_{m}}{\alpha_{m} + \alpha} \quad (3.3.26)$$
This is the total output optical power exit from both laser end facets. Since $\alpha$ is the rate of material absorption and $\alpha_m$ is the photon escape rate through facet mirrors, $\alpha_m/(a + a_m)$ represents the external efficiency.

### 3.3.4 Side mode suppression ratio (SMR)

As illustrated in Figure 3.3.2, the phase condition in a laser diode can be satisfied by multiple wavelengths, which are commonly referred to as multiple longitudinal modes. The gain profile has a parabolic shape with a maximum in the middle, and one of the longitudinal modes closest to the material gain peak usually has the highest power, which is the main mode. However, the power in the modes adjacent to the main mode may not be negligible for many applications that require single-mode operation. To take into account the multimodal effect in a laser diode, the photon density rate equation of the $m$th longitudinal mode can be written as

$$
\frac{dP_m(t)}{dt} = 2v_g \Gamma g_m(N)P_m(t) - \frac{P_m(t)}{\tau_{ph}} + R_{sp}
$$

(3.3.27)

where $g_m(N) = a(N - N_0)$ is the optical field gain for the $m$-th mode. Since all the longitudinal modes share the same pool of carrier density, the rate equation for the carrier density is,

$$
\frac{dN(t)}{dt} = J_{sd} - \frac{N(t)}{\tau} - \sum_k 2\Gamma_k v_g g_k(N)P_k(t)
$$

(3.3.28)

Using a parabolic approximation for the material gain,

$$
g(N, \lambda) = g_0(N)\left\{1 - \left[\frac{\lambda - \lambda_0}{\Delta\lambda_e}\right]^2\right\}
$$

and let $\lambda_m = \lambda_0 + m\Delta\lambda_e$, where $\Delta\lambda_e$ is the mode spacing as shown in Figure 3.3.2.

There should be approximately $2M + 1$ modes if there are $M$ modes on each side of the main mode ($-M < m < M$), where $M \approx \Delta\lambda_e / \Delta\lambda_e$.

Therefore, the optical field gain for the $m$-th mode can be expressed as a function of the mode index $m$ as,

$$
g_m(N) = g_0(N)\left\{1 - \left(\frac{m}{M}\right)^2\right\}
$$

(3.3.29)

The steady state solution of the photon density rate equation of the $m$-th mode is
\[ P_m = \frac{R_{sp}}{1/\tau_{ph} - 2\Gamma v_g g_m(N)} \]  
(3.3.30)

and the photon density of the main mode \((m = 0)\) is

\[ P_0 = \frac{R_{sp}}{1/\tau_{ph} - 2\Gamma v_g g_0(N)} \]  
(3.3.31)

The power ratio between the main mode and the \(m\)-th mode can then be found as,

\[ SMR = \frac{P_0}{P_M} = 1 + \frac{P_0}{R_{sp}} 2\Gamma v_g g_0(N)(m^2/M^2) \]  
(3.3.32)

Equation 3.3.32 indicates that the side mode suppression ratio is proportional to \(m^2\) because high index modes are far away from the main mode and the gain is much lower than the threshold gain. In addition, the side mode suppression ratio is proportional to the photon density of the main mode. The reason is that at a high photon density level, stimulated emission is predominantly higher than the spontaneous emission; thus side modes that benefited from spontaneous emission become weaker compared to the main mode.

### 3.3.5 Modulation response

Electro-optic modulation is important functionality in an optical transmitter, which translates electrical signals into optical domain. The optical power of a laser diode is a function of the injection current, and thus a convenient way to convert an electrical signal into an optical signal is through the direct modulation of the injection current. Both on-off modulation and linear modulation can be performed. The characteristic of laser diode under direct current modulation is a practical issue for applications in optical communication systems.

Figure 3.3.5 illustrates the operating principle of direct intensity modulation of a semiconductor laser. To ensure that the laser diode operates above threshold, a DC bias current \(I_B\) is usually required. A bias-Tee combines the electrical current signal with the DC bias current to modulate the laser diode. The modulation efficiency is then determined by the slope of the laser diode \(P-I\) curve. Obviously, if the \(P-I\) curve is ideally linear, the output optical power is linearly proportional to the modulating current by

\[ P_{opt}(t) \approx R_c (I_B - I_{th}) + R_c I(t) \]  
(3.3.33)
where $I_{th}$ is the threshold current of the laser and $R_c = \Delta P_{opt} / \Delta I$ is the slope of the laser diode $P-I$ curve. Here we have neglected the optical power level at threshold.

![Diagram](image-url)  

Figure 3.3.5 Direct intensity modulation of a laser diode. TIA: transimpedance amplifier.

Frequency response of direct modulation mainly depends on the carrier dynamics of the laser diode and $>20$ GHz modulation bandwidth has been demonstrated. However, further increasing the modulation bandwidth to 40 GHz appears to be quite challenging, mainly limited by the carrier lifetime as well as the parasitic effect of the electrode. Another well-known property of direct modulation in a semiconductor laser is the associated frequency modulation, commonly referred to as frequency chirp, as discussed later in this section.

(A) Turn-on delay

In directly modulated laser diodes, when the injection current is suddenly switched on from below to above the threshold, there is a time delay between the signal electrical pulse and the generated optical pulse. This is commonly referred to as turn-on delay, which is mainly caused by the slow response of the carrier density below threshold. It needs a certain period of time for the carrier density to build up and to reach the threshold level.

To analyze this process, we have to start from the rate equation at the low injection level $J_B$ below threshold, where photon density is very small and the stimulated recombination term is negligible in Equation 3.3.12:
\[
\frac{dN}{dt} = \frac{J(t)}{qd} - \frac{N(t)}{\tau}
\]  
(3.3.34)

Suppose \( J(t) \) switches on from \( J_B \) to \( J_2 \) at time \( t = 0 \). If \( J_B \) is below and \( J_2 \) is above the threshold current level, the carrier density is supposed to be switched from a level \( N_1 \) far below the threshold to the threshold level \( N_{th} \), as shown in Figure 3.3.6.

Equation 3.3.34 can be integrated to find the time required for the carrier density to increase from \( N_B \) to \( N_{th} \):

\[
t_d = \int_{N_B}^{N_{th}} \left[ \frac{J(t)}{qd} - \frac{N(t)}{\tau} \right]^{-1} dN = \tau \ln \left[ \frac{J - J_B}{J - J_{th}} \right] = \tau \ln \left[ \frac{I - I_B}{I - I_{th}} \right]
\]  
(3.3.35)

where \( J_B = \frac{qdN_B}{\tau} \) and \( J_{th} = \frac{qdN_{th}}{\tau} \). Because laser operates above threshold only for \( t \geq t_d \), the actual starting time of the laser output optical pulse is at \( t = t_d \). In practical applications, this time delay \( t_d \) may limit the speed of digital optical modulation in optical systems. Since \( t_d \) is proportional to \( \tau \), a laser diode with shorter spontaneous emission carrier lifetime may help reduce the turn-on delay. Another way to reduce turn-on delay is to bias the low-level injection current \( J_B \) very close to the threshold. However, this may result in poor extinction ratio of the output optical pulse.

Figure 3.3.6 Illustration of the laser turn-delay. Injection current is turned on at \( t = 0 \) from \( J_B \) to \( J_2 \), but both photon density and carrier density will require a certain time delay to build up toward their final values.

(B) Small-signal modulation response
In Section 3.2.2 we show that the modulation speed of an LED is inversely proportional to the carrier lifetime. For a laser diode operating above threshold, the modulation speed is expected to be much faster than that of an LED thanks to the contribution of stimulated recombination. In fact when a laser diode is modulated by a small current signal $\delta I(t)$ around a bias point $J_B$: $J = J_B + \delta I(t)$, the carrier density will be $N = N_B + \delta N(t)$, where $N_B$ and $\delta N(t)$ are the static and small signal response, respectively, of the carrier density. Rate Equation 3.3.12 can be linearized for the small-signal response as,

$$
\frac{d\delta N(t)}{dt} = \frac{\delta I(t)}{qd} - \frac{\delta N(t)}{\tau} - 2\Gamma_v a P \delta N(t)
$$

(3.3.36)

Here for simplicity we have assumed that the impact of photon density modulation is negligible. Equation 3.3.36 can be easily solved in frequency domain as

$$
\tilde{\delta N}(\omega) = \frac{1}{qd} \frac{\tilde{\delta I}(\omega)}{j\omega + 1/\tau + 2\Gamma_v a P}
$$

(3.3.37)

where $\tilde{\delta I}(\omega)$ and $\tilde{\delta N}(\omega)$ are Fourier transforms of $\delta I(t)$ and $\delta N(t)$, respectively. If we define an effective carrier lifetime as,

$$
\tau_{eff} = \left( \frac{1}{\tau} + 2\Gamma_v a P \right)^{-1}
$$

(3.3.38)

the 3-dB modulation bandwidth of the laser will be $B_{3dB} = 1/\tau_{eff}$. For a laser diode operating well above threshold, stimulated recombination is much stronger than spontaneous recombination, i.e., $2\Gamma_v a P >> 1/\tau$, and therefore, $\tau_{eff} << \tau$. This is the major reason that the modulation speed of a laser diode can be much faster than that of an LED.

In this simplified modulation response analysis, we have assumed that photon density is a constant, and thus there is no coupling between the carrier density rate equation and the photon density rate equation. A more precise analysis has to solve coupled rate equations 3.3.12 and 3.3.13. A direct consequence of coupling between the carrier density and the photon density is that for an increase of injection current, the carrier density will first increase, which will increase the photon density. But the photon density increase tends to reduce carrier density through stimulated
recombination. Therefore there could be an oscillation of both carrier density and photon density immediately after a change of the injection current. This is commonly referred to as relaxation oscillation. Detailed analysis of laser modulation can be found in [G. P. Agrawal, 2001].

(C) Modulation chirp

For semiconductor materials, the refractive index is usually a function of the carrier density, and thus the emission wavelength of a semiconductor laser often depends on the injection current. In addition, a direct current modulation on a laser diode may introduce both an intensity modulation and a phase modulation of the optical signal, where the phase modulation is originated from the carrier density dependent refractive index of the material within the laser cavity. The ratio between the phase modulation and the intensity modulation is referred to as the modulation chirp.

Carrier dependent refractive index in a semiconductor laser is the origin of both adiabatic chirp and transient chirp. The adiabatic chirp is caused by the change of phase condition of laser cavity caused by the refractive index change of semiconductor material inside the laser cavity. As refractive index is a parameter of the laser phase condition in Equation 3.3.5, any change of refractive index will change the resonance wavelength of the laser cavity. Based on Equation 3.3.5, the wavelength of the $m^{th}$ mode is $\lambda_m = 2nL/m$, and a small index change $n \Rightarrow n + \delta n$ will introduce a wavelength change $\lambda_m \Rightarrow \lambda_m + \delta \lambda$. The linearized relation between $\delta \lambda$ and $\delta n$ is $\delta \lambda / \lambda_m = \delta n / n$. In a practical laser diode based on InGaAs, a 1mA change in the injection current would introduce an optical frequency change on the order of 1GHz, or 8pm in 1550nm wavelength window. This effect is often utilized for laser frequency adjustment and stabilization through feedback control of the injection current.

Transient chirp, on the other hand, is a dynamic process of carrier-induced optical phase modulation. As the optical field is related to the photon density and the optical phase,

$$E(t) = \sqrt{P(t)}e^{j\varphi(t)} = \exp[j\varphi(t) + 0.5\ln P(t)] \quad (3.3.39)$$

the ratio between phase modulation and amplitude modulation around a static bias point $P_B$ can be defined as
\[
\alpha_{lw} = \frac{d\phi(t)/dt}{d[0.5\ln P(t)]/dt} = 2P_B \frac{d\phi(t)/dt}{dP(t)/dt}
\] (3.3.40)

\(\alpha_{lw}\) is the chirp parameter of the laser, which is identical to a well-known linewidth enhancement factor first introduced by Henry in 1982 [C. H. Henry, 1982], as the spectral linewidth of the laser diode is also related to \(\alpha_{lw}\) which will be discussed in the next section.

The derivative of optical phase modulation through transient chirping is equivalent to an optical frequency modulation. This optical frequency modulation is linearly proportional to the chirp parameter \(\alpha_{lw}\) as,

\[
\delta f = \frac{d\phi(t)}{dt} = \frac{\alpha_{lw}}{2P_B} \frac{dP(t)}{dt}
\] (3.3.41)

\(\alpha_{lw}\) is an important parameter of a laser diode, which is determined both by the semiconductor material and by the laser cavity structure. For intensity modulation-based optical systems, lasers with smaller chirp parameters are desired to minimize the spectral width of the modulated optical signal. On the other hand, for optical frequency modulation-based systems such as frequency-shift key (FSK), lasers with large chirp will be beneficial for an increased optical frequency modulation efficiency.

### 3.3.6 Laser noises

When biased by a constant current, an ideal laser diode would produce a constant optical power at a single optical frequency. However, a practical laser diode is not an ideal optical oscillator, which has both intensity noise and phase noise even driven by an ideally continuous wave (CW) injection current. Both intensity noise and phase noise may significantly affect the performance of an optical communication system which uses laser diode as the optical source.

(A) Relative intensity noise (RIN)

For a semiconductor laser operating above threshold, although stimulated emission dominates the emission process, there are still a small percentage of photons that are generated by spontaneous emission. The optical intensity noise in a laser diode is primarily caused by the incoherent nature of these spontaneous photons. As a result, even when a laser diode is biased by an ideally CW injection current, the effect of
spontaneous emission makes both carrier density and photon density fluctuate around their equilibrium values [Agrawal 1986].

Figure 3.3.7 Illustration of optical power emitted a CW operated laser diode without (a), and with relative intensity noise (b)

In general, the intensity noise in a laser diode caused by spontaneous emission is random in time as illustrated in Figure 3.3.7, and thus wideband in the frequency domain. Intensity noise is an important measure of a laser diode quality, which is often a limiting factor for its application in optical communications. Instead of the absolute value of the intensity noise, the ratio between the intensity noise power and the average optical power is a commonly used parameter for laser specification, known as the relative intensity noise (RIN), which is defined as

\[
RIN(\omega) = F\left\{\frac{\left\langle (P(t) - P_{ave})^2 \right\rangle}{P_{ave}^2}\right\}
\]

where, \(P_{ave}\), is the average optical power, \(\left\langle (P(t) - P_{ave})^2 \right\rangle\) is the mean square intensity fluctuation (variance), and \(F\) denotes Fourier transform. Note that this definition is based on electrical domain parameters measured by a photodiode in a setup shown in Figure 3.3.8. As will be discussed in the next chapter, the photocurrent of a photodiode is proportional to the input optical power, the electrical power produced by the photodiode should be proportional to the square of the optical power. The Fourier transform of \(\left\langle (P(t) - P_{ave})^2 \right\rangle\) is the electrical noise power spectral density in [W/Hz], and \(P_{ave}^2\) is proportional to the average electrical power in [W] created in the photodiode corresponding to the spectral density at DC. Thus, 20dB of RIN defined in Equation 3.3.42 is equivalent to 10dB of actual ratio in the optical domain. The square-law photodetection will be discussed in Chapter 4.
In a practical measurement system, optical loss between the laser and the photodiode, and the gain of the electrical amplifier after the photodiode may not be calibrated. But this does not affect the accuracy of RIN measurement as a ratio between the noise and the signal. This makes RIN a commonly accepted parameter for laser quality specification which is independent of the optical power that comes into the receiver. The unit of RIN is $[Hz^{-1}]$ or $[dB/Hz]$.

![System block diagram to measure laser RIN.](image)

For a Gaussian statics of the RIN, the standard deviation of the laser power fluctuation can be found by integrating the RIN spectrum over the electrical bandwidth. Assume a flat spectral density of $RIN = -120dB/Hz$ over the electrical bandwidth $B = 10GHz$, the variance of power fluctuation is $\sigma^2 = 10^{-2} P_{ave}^2$. In this case the optical power fluctuation (standard deviation) is roughly 10% of the average power.

In a practical diode laser, the intensity noise can be amplified by the resonance of the relaxation oscillation in the laser cavity, which modifies the noise spectral density. Figure 3.3.9 shows that each single event of spontaneous emission will increase the photon density in the laser cavity. Meanwhile this increased photon density will consume more carriers in the cavity and create a gain saturation in the medium. As a result, the photon density will tend to be decreased due to the reduced optical gain. This, in turn, will increase the carrier density due to the reduced saturation effect. This resonance process is strong near a specific frequency $\Omega_p$ which is determined by the optical gain $G$, the differential gain $dG/dN$, and the photon density $P$ in the laser cavity as

$$\Omega_p^2 = G \cdot P \cdot \frac{dG}{dN}$$  \hspace{1cm} (3.4.43)
Due to relaxation oscillation, the intensity noise of a semiconductor laser becomes frequency dependent, and the normalized intensity noise spectral density can be fit by,

\[
H(\Omega) \propto \frac{\Omega_R^2 + B\Omega^2}{|j\Omega(j\Omega + \gamma + \Omega_R^2)|^2} \tag{3.3.44}
\]

where \(B\) and \(\gamma\) are damping parameters depending on the specific laser structure and the bias condition. Figure 3.3.10 shows examples of the normalized intensity noise spectral density with three different damping parameters. The relaxation oscillation frequency used in this figure is \(\Omega_R = 2\pi \times 10 \text{GHz}\). At frequencies much higher than the relaxation oscillation frequency, the dynamic coupling between the photon density
and the carrier density is weak and therefore the intensity noise becomes less dependent on the frequency.

(B) Phase noise

Phase noise is a measure of random phase variation of an optical signal which determines the spectral purity of the laser beam. Phase noise is an important issue in diode lasers, especially when they are used in coherent optical communication systems where signal optical phase is utilized to carry information. Although stimulated emission dominates in a laser diode operation above threshold, a small amount of spontaneous emission still exists. While Photons of stimulated emission are mutually coherent, spontaneous emission is not coherent. Figure 3.3.11 shows that a spontaneous emission event not only generates intensity variation but also produces phase variation. To make things worse, the material refractive index inside a semiconductor laser cavity is a function of the photon density, so that the phase noise is further enhanced by the intensity noise, which makes the phase noise of a semiconductor laser much higher than other types of lasers [C. H. Henry, 1986].

Figure 3.3.11 Optical field vector diagram. Illustration of optical phase noise generated due to spontaneous emission events.

Assume that the optical field in a laser cavity is

\[ E(t) = \sqrt{P(t)} \exp \{ j[\omega_0 t + \phi(t)] \} \]  

(3.3.45)

where \( P(t) \) is the photon density inside the laser cavity, \( \omega_0 \) is the central optical frequency, and \( \phi(t) \) is the time-varying part of optical phase. A differential rate equation that describes the phase variation in the time domain is [Henry 1983]

\[ \frac{d\phi(t)}{dt} = F_\phi(t) - \frac{\alpha_{be}}{2P} F_\nu(t) \]  

(3.3.46)
where $\alpha_{lw}$ is the well-known linewidth enhancement factor of a semiconductor laser, which accounts for the coupling between intensity and phase variations. $F_\phi(t)$ and $F_P(t)$ are Langevin noise terms for phase and intensity. They are random and their statistic measures are

$$\langle F_\phi(t)^2 \rangle = \frac{R_{sp}}{2P}$$

and

$$\langle F_P(t)^2 \rangle = \frac{R_{sp}}{2P}$$

$R_{sp}$ is the spontaneous emission factor of the laser.

Although we directly use Equation 3.3.46 without derivation, the physical meanings of the two terms on the right-hand-side of the equation are very clear. The first term is created directly due to spontaneous emission contribution to the phase variation. Each spontaneous emission event randomly emits a photon which changes the optical phase as illustrated in Figure 3.3.11. The second term in Equation 3.3.46 shows that each spontaneous emission event randomly emits a photon that changes the carrier density, and this carrier density variation, in turn, changes the index of the material. Then this index change will alter the resonance condition of the laser cavity and thus will introduce a phase change of the emitting optical field.

Equation 3.3.46 can be solved by integration:

$$\phi(t) = \int_0^t \frac{d\phi(t)}{dt} dt = \int_0^t F_\phi(t) dt - \frac{\alpha_{lw}}{2P} \int_0^t F_P(t) dt$$

If we take an ensemble average, the power spectral density of phase noise can be expressed as

$$\langle \phi(t)^2 \rangle = \left( \int_0^t F_\phi(t) dt - \frac{\alpha_{lw}}{2P} \int_0^t F_P(t) dt \right)^2 = \int_0^t \langle F_\phi(t)^2 \rangle dt - \frac{\alpha_{lw}}{2P} \int_0^t \langle F_P(t)^2 \rangle dt =$$

$$= \left[ \frac{R_{sp}}{2P} + \left( \frac{\alpha_{lw}}{2P} \right)^2 2R_{sp} P \right] P = \frac{R_{sp}}{2P} \left[ 1 + \alpha_{lw}^2 \right] P$$

Since $\phi(t)$ is a random Gaussian process.
\( \langle e^{i\phi(t)} \rangle = e^{-\frac{1}{2}\langle \phi(t)^2 \rangle} \)  

(3.3.51)

Then the optical power spectral density is

\[
S_{\text{op}}(\omega) = \int_{-\infty}^{\infty} \langle E(t) E^*(0) \rangle e^{-j\omega t} dt = P \int_{-\infty}^{\infty} e^{-j(\omega - \omega_0) t} e^{-\frac{1}{2}\langle \phi(t)^2 \rangle} dt
\]

(3.3.52)

The normalized optical power spectral density has a Lorentzian shape which can be found as

\[
S_{\text{op}}(\omega) = \frac{\left[ \frac{R_{\text{sp}}}{4P} (1 + \alpha_{\text{lw}}^2) \right]^2}{\left[ \frac{R_{\text{sp}}}{4P} (1 + \alpha_{\text{lw}}^2) \right]^2 + (\omega - \omega_0)^2}
\]

(3.3.53)

The FWHM linewidth of this spectrum is

\[
\Delta v = \frac{\Delta \omega}{2\pi} = \frac{R_{\text{sp}}}{4\pi P} (1 + \alpha_{\text{lw}}^2)
\]

(3.3.54)

As the well-known Scholow-Towns formula which describes the laser spectral linewidth is \( \Delta v = R_{\text{sp}}/(4\pi P) \), Equation 3.3.34 is commonly referred to as the modified Scholow-Towns formula because of the introduction of linewidth enhancement factor \( \alpha_{\text{lw}} \).

Spectral linewidth of a non-modulated laser is determined by the phase noise, which is a measure of coherence of an optical beam. Other measures such as coherence time and coherence length can all be related to the spectral linewidth. Coherence time is usually defined as

\[
t_{\text{coh}} = \frac{1}{\pi \Delta v}
\]

(3.3.55)

It is the time over which a lightwave may still be considered coherent. Or, in other words, it is the time interval within which the phase of a lightwave is still predictable. Similarly, coherence length is defined by

\[
L_{\text{coh}} = t_{\text{coh}} v_g = \frac{v_g}{\pi \Delta v}
\]

(3.3.56)
It is the propagation distance over which a lightwave signal maintains its coherence where $v_g$ is the group velocity of the optical signal. As a simple example, for a lightwave signal with 1 MHz linewidth, the coherence time is approximately 320 ns and the coherence length is about 95 m in free space.

(C) Mode partition noise

As discussed in section 3.3.4, the phase condition in a laser cavity can be simultaneously satisfied by multiple wavelengths, and thus the output from a laser diode can have multiple longitudinal modes if the material gain profile is broad enough as illustrated in Figure 3.3.2. All these longitudinal modes compete for the carrier density from a common pool. Those modes near the peak of the material gain profile have competition advantages which consume most of the carrier density, while the power of other modes further away from the material gain peak will be mostly suppressed. In most Fabry-Perot (FP) type of diode lasers without extra mode selection mechanisms, the optical gain seen by different FP modes near the material gain peak is usually very small. Perturbations due to spontaneous emission noise, external reflection, or temperature change may introduce random switches of optical power between modes. This random mode hopping is usually associated with the total power fluctuation of the laser output, which is known as mode partition noise. This is why relative intensity noise in a laser diode with multiple longitudinal modes is much higher than that with only a single mode. In addition, if the external optical system has wavelength-dependent loss, this power hopping between modes with different wavelength will inevitably introduce additional intensity noise for the system.

3.4 Single-frequency semiconductor lasers

So far we have only considered the laser diode where the resonator consists of two parallel mirrors. This simple structure is called a Fabry-Perot resonator and the lasers made with this structure are usually called Fabry-Perot lasers, or simply FP lasers. An FP laser diode usually operates with multiple longitudinal modes because a phase condition can be met by a large number of wavelengths and the reflectivity of the mirrors is not wavelength selective. In addition to mode partition noise, multiple longitudinal modes occupy wide optical bandwidth, which results in poor bandwidth efficiency and low tolerance to chromatic dispersion of the optical system.
The definition of a single-frequency laser can be confusing. An absolute single-frequency laser does not exist because of phase noise and frequency noise. A single-frequency laser diode may simply be a laser diode with a single longitudinal mode. A more precise definition of single-frequency laser is a laser that not only has a single mode but that mode also has very narrow spectral linewidth. To achieve single-mode operation, the laser cavity has to have a special wavelength selection mechanism. One way to introduce wavelength selectivity is to add a grating along the active layer, which is called distributed feedback (DFB). The other way is to add an additional mirror outside the laser cavity, which is referred to as the external cavity.

### 3.4.1 DFB and DBR laser structures

DFB laser diodes are widely used as single-wavelength sources in optical communication systems and optical sensors. Figure 3.14 shows the structure of a DFB laser, where a corrugating grating is written just outside the active layer, providing a periodic refractive index perturbation [Kogelnik and Shank, 1972]. Similarly to what happens in an FP laser, the lightwave resonating within the cavity is composed of two counter-propagating waves, as shown in Figure 3.4.1(b). However, in the DFB structure, the index grating creates a mutual coupling between the two waves propagating in opposite directions, and therefore mirrors on the laser surface are no longer needed to provide the optical feedback.

![DFB laser structure](image)

Figure 3.4.1 (a) Structure of a DFB laser with a corrugating grating just outside the active layer, and (b) an illustration of two counter-propagated waves in the cavity.

The coupling efficiency between the two waves propagating in the opposite directions is frequency dependent, which provides a mechanism for frequency
selection of optical feedback in the laser cavity. This frequency-dependent coupling can be modeled by coupled-mode equations which will be discussed in more details in Chapter 6 where we discuss fiber Bragg grating filters. Briefly, because the refractive index along the grating is varying periodically, constructive interference between the two waves happens only at certain wavelengths. To satisfy the resonance condition the wavelength has to match the grating period, and the resonance wavelength of a DFB structure is thus,

\[ \lambda_g = \frac{2n\Lambda}{m} \]  

(3.4.1)

This is called the Bragg wavelength, where \( \Lambda \) is the grating pitch, \( n \) is the effective refractive index of the optical waveguide, and \( m \) is the grating order. For wavelengths away from the Bragg wavelength, the two counter-propagated waves do not reinforce each other along the way; therefore, self-oscillation cannot be sustained for these wavelengths. For example, consider a DFB laser operating at 1550nm wavelength, and the refractive index of InGaAsP semiconductor material is approximately \( n = 3.4 \). Operating as a 1st-order grating \((m = 1)\), the grating period is \( \Lambda = \frac{\lambda_g}{(2n)} = 228nm \).

With this grating period, the 2nd-order Bragg wavelength would be at around \( \lambda_g = n\Lambda = 775nm \), which would be far outside of the bandwidth of the material gain.

Another way to understand this distributed feedback is to treat the grating as an effective mirror. As shown in Figure 3.4.2, for a grating of length \( L \), the frequency dependent reflectivity is

\[ R(f) \propto \frac{\sin[2\pi(f - f_B)\tau]}{2\pi(f - f_B)\tau} \]  

(3.4.2)

where, \( \tau = 2L/v_g \) is the roundtrip time of the grating length \( L \), \( v_g \) is the group velocity, and \( f_B = c/\lambda_g \) is the Bragg frequency. This was simply obtained by a Fourier transform of the step-function reflectivity in the spatial domain. At the Bragg frequency \( f = f_B \), the power reflectivity is \( R(f) = 1 \) and the transmissivity is \( T(f) = 0 \). At frequencies \( f = f_B \pm 1/2\tau \), the field reflectivity drop to 0, and the separation between these two frequencies gives the full bandwidth of the Bragg reflectivity, which is \( \Delta f = 1/\tau \), or in wavelength, \( \Delta \lambda = \frac{\lambda_g^2}{(2nL)} \). This tells that sharper frequency selectivity can be provided by a grating with a longer length.
Figure 3.4.2 (a) A Bragg grating with length $L$, and (b) normalized reflectivity $R(f)$ and transmissivity $T(f)$ of the Bragg grating.

A DFB structure can be regarded as a cavity formed between two equivalent mirrors. As shown in Figure 3.4.3, from the reference point in the middle of the cavity looking left and right, the effect of the grating on each side can be treated as an equivalent mirror, similar to that in a FP laser cavity. The effective transmissivity of each mirror is

$$ T_{\text{eff}} \propto 1 - \left[ \sin x / x \right]^2 , $$

where $x = 2\pi L c (\lambda - \lambda_B)/(v_g \lambda_B^2)$, $v_g$ is the group velocity, $c$ is the speed of light, and $L$ is the cavity length. The output optical power spectrum is selected by the frequency-dependent transmissivity of these effective mirrors.

Figure 3.4.3 (a) A uniform DFB grating and (b) a DFB grating with a quarter-wave shift in the middle.

If the grating is uniform, this effective transmissivity has two major resonance peaks separated by a deep stop-band. As a result, a conventional DFB laser intrinsically has two degenerate longitudinal modes and the wavelength separation between these two modes is equal to the width of the stop band $\Delta \lambda = \lambda_B^2/(n L)$, as illustrated in Figure 3.4.4(a). Single mode operation can be obtained with an additional mechanism to break up the precise symmetry of the transfer function so
that the one of the two degenerate modes can be suppress. This extra differentiation mechanism is usually introduced by the residual reflectivities from cleaved laser facets, or by the non-uniformity of the grating structure and current density. Another technique is to etch the Bragg grating inside the active region of the waveguide in the laser cavity so that both the refractive index and the gain (or loss) of the material vary periodically along the waveguide. This complex grating structure yields an asymmetric transfer function, so that single mode operation can be obtained in a more controllable manor.

A most popular technique to create single-mode operation is to add a quarter-wave shift in the middle of the Bragg grating, as shown in Figure 3.4.3(b). This $\lambda/4$ phase shift introduces a phase discontinuity in the grating and results in a strong transmission peak at the middle of the stop-band, as shown in Figure 3.4.4(b). This ensures single longitudinal mode operation in the laser diode at the Bragg wavelength.

![Figure 3.4.4 Spectrum of a DFB laser with a pair of degenerate modes, and (b) spectrum of a $\lambda/4$-shifted DFB laser with a dominant single mode.](image)

Distributed feedback Reflector (DBR) is another structure of single wavelength laser sources commonly used in optical communication systems. As shown in Figure 3.4.5, a DBR laser usually has three sections; one of them is the active section which provides the optical gain, another one is a grating section witch provides a wavelength selective reflection, and there is a phase control section placed in the middle which is used to adjust the optical phase of the Bragg reflection from the grating section. Both the phase control section and the grating sections are usually passive because no optical gain is required for these sections, and thus they are usually biased by reverse voltages only to adjust the index of refractions through the electro-optic effect.
Figure 3.4.5 Structure of a DBR laser with an active section $A$, phase control section $P$ and Bragg reflector section $B$. The laser cavity is formed between the left end facet with reflectivity $R_1$ and the effective reflection from the Bragg section.

Assume the length of the active cavity is $L_C$, the FP mode spacing is

$$
\Delta \lambda_{FP} \approx \frac{\lambda_B^2}{2nL_C},
$$

while the wavelength-selective Bragg reflectivity has the spectral width

$$
\Delta \lambda_{Bragg} = \frac{\lambda_B^2}{2nL_B}.
$$

Thus, to avoid multi longitudinal mode operation $\Delta \lambda_{Bragg} < \Delta \lambda_{FP}$ is required. Suppose the same semiconductor material is used in all sections of the laser with the same index of refraction $n$, the Bragg grating section has to be longer than the active section. DBR lasers can be made wavelength tunable through the index tuning in the Bragg reflector section to control the Bragg wavelength. The tuning of phase control section is necessary to satisfy the phase condition of lasing across a continuous region of wavelength.

### 3.4.2 External cavity laser diodes

The operation of a semiconductor is sensitive to external feedback [Lang and Kobayashi, 1980]. Even a -40dB optical feedback is enough to bring a laser from single-frequency operation into chaos. Therefore an optical isolator has to be used at the output of a laser diode to prevent optical feedback from external optical interfaces. On the other hand, precisely controlled external optical feedback can be used to create wavelength-tunable lasers with very narrow spectral linewidth.

The configuration of a grating-based external cavity laser is shown in Figure 3.4.6, where laser facet reflectivities are $R_1$ and $R_2$ and the external grating has a wavelength-dependent reflectivity of $R_3(\omega)$. In this complex cavity configuration, the reflectivity $R_2$ of the facet facing the external cavity has to be replaced by an effective reflectivity $R_{eff}$ as shown in Figure 3.4.6, as
\[ R_{\text{eff}}(\omega) = \left\{ \sqrt{R_1} + (1 - R_1) \sqrt{R_2(\omega)} \sum_{m=1}^{\infty} \left( R_3 R_4(\omega) \right)^{m-1} e^{i\omega} \right\}^2 \]  

(3.4.3)

Figure 3.4.6 Configuration of an external cavity semiconductor laser, where the external feedback is provided by a reflective grating.

If external feedback is small enough \((R_3 \ll 1)\), only one roundtrip needs to be considered in the external cavity. Then Equation 3.4.3 can be simplified as

\[
R_{\text{eff}}(\omega) \approx R_2 \left\{ 1 + \frac{(1 - R_1) \sqrt{R_3(\omega)}}{\sqrt{R_2}} \right\}^2
\]

(3.4.4)

Then the mirror loss \(\alpha_m\) shown in Equation 3.3.14 can be modified by replacing \(R_2\) with \(R_{\text{eff}}\). Figure 3.4.7 illustrates the contributions of various loss terms: \(\alpha_1\) is the reflection loss of the grating, which is wavelength selective with only one reflection peak (corresponding to a valley in \(\alpha_1\)), and \(\alpha_2\) and \(\alpha_3\) are resonance losses between \(R_1\) and \(R_2\) and between \(R_2\) and \(R_3\), respectively. Combining these three contributions, the total wavelength-dependent loss \(\alpha_m\) has only one strong low loss wavelength, which determines the lasing wavelength. In practical external cavity laser applications, an antireflection coating is used on the laser facet facing the external cavity to reduce \(R_2\), and the wavelength dependency of both \(\alpha_1\) and \(\alpha_2\) can be made very small compared to that of the grating; therefore, a large wavelength tuning range can be achieved by rotating the angle of the grating while maintaining single longitudinal mode operation.
Figure 3.4.7 Illustration of resonance losses between $R_1$ and $R_2$ ($\alpha_2$) and between $R_2$ and $R_3$ ($\alpha_3$). $\alpha_1$ is the reflection loss of the grating and $\alpha_m$ is the combined mirror loss. Lasing threshold is reached only by one mode at $\lambda_\parallel$.

External optical feedback not only helps to obtain wavelength tuning, it also changes the spectral linewidth of the emission. The linewidth of an external cavity laser can be expressed as,

$$\Delta \nu = \frac{\Delta \nu_0}{1 + k \cos(\omega_0 \tau_x + \tan^{-1}(\alpha_{lw}))}$$  \hspace{1cm} (3.4.5)$$

where $\Delta \nu_0$ is the linewidth of the laser diode without external optical feedback, $\omega_0$ is the oscillation angular frequency, $\alpha_{lw}$ is the linewidth enhancement factor, and $k$ represents the strength of the optical feedback. When the feedback is not very strong, this feedback parameter can be expressed as

$$k = \frac{\tau_x (1 - R_2) \sqrt{R_3}}{\tau \sqrt{R_2} \sqrt{1 + \alpha_{lw}^2}}$$  \hspace{1cm} (3.4.6)$$

$\tau = 2nL/c$, $\tau_x = 2n_e L_e/c$ are roundtrip delays of the laser cavity and the external cavity, respectively, with $L$ and $L_e$ the lengths of the laser cavity and the external cavity. $n$ and $n_e$ are refractive indices of the two cavities.
Equation 3.4.5 shows that the linewidth of the external cavity laser depends on the phase of the external feedback. To obtain narrow linewidth, precise control of the external cavity length is critical; a mere $\lambda/2$ variation in the length of external cavity can change the impact of optical feedback from linewidth narrowing to linewidth enhancement. This is why an external cavity has to have a very stringent mechanical stability requirement.

An important observation from Equation 3.4.6 is that the maximum linewidth reduction is proportional to the ratio of the cavity length ratio $L_e/L$. This is because there is no optical propagation loss in the external cavity and the photon lifetime is increased by increasing the external cavity length. In addition, when photons travel in the external cavity, there is no power-dependent refractive index; this is the reason for including the factor $\sqrt{1 + \alpha_{lw}^2}$ in Equation 3.4.6.

In fact, if the antireflection coating is perfect such that $R_2 = 0$, this ideal external cavity laser can be defined as an extended-cavity laser because it becomes a two-section one, with one of the sections passive. With $R_2 = 0$, $\alpha_2$ and $\alpha_3$ in Figure 3.4.7 will be wavelength-independent and in this case, the laser operation will become very stable and the linewidth is no longer a function of the phase of the external optical feedback. The extended-cavity laser diode linewidth can simply be expressed as [Hui, 1989]

$$\Delta v = \frac{\Delta v_0}{1 + \frac{\tau_c}{\tau} \sqrt{1 + \alpha_{lw}^2}}$$  \hspace{1cm} (3.4.7)

Grating-based external cavity lasers are commercially available and they are able to provide >60nm continuous wavelength tuning range in a 1550nm wavelength window <100kHz spectral linewidth. Rapid technological advancement in micro-optics, precision mechanics and micro mechanical machining have enabled miniaturized and low-cost external cavity lasers with sub-100kHz linewidth and high reliability. This in turn revived the interest in the research and development of coherent optical communication systems which utilize optical phase as an information carrier. More details of coherent communication will be discussed in Chapter 9.

In practical optical systems using semiconductor lasers, external optical back reflections often exist from fiber connectors and interfaces of optical components. The
strengths and phases of these unwanted optical feedbacks vary randomly, affected by the temperature and mechanical stress of the optical fiber. Thus, laser linewidth can be made very unstable, especially when the optical feedback is originated from a distance far away from the laser diode. This can be understood through Equation (3.4.6): a long external cavity length results in a longer delay $\tau_e$ and a larger feedback parameter $k$. Sufficiently high optical feedback level can result in the coherence collapse [Schunk, 1988] of the laser emission and also significantly enhanced intensity noise. In order to avoid this performance degradation, a single-frequency laser diode used in high speed optical communication system usually has to have an optical isolator inserted between the laser diode and the output optical fiber pigtail, which allows the optical signal to travel only in one direction (see Figure 3.5.4).

3.5 Laser diode biasing and packaging

From a practical application point of view, as the optical power of a diode laser is linearly proportional to the injection current when biased above the threshold, the driving electronic circuit has to be a current source instead of a voltage source. The equivalent electrical circuit of a diode laser is an ideal diode with a series resistance, as shown in Figure 3.5.1(a).

\[ I_D = I_s \exp\left[\frac{qV_D}{nk_BT}\right] \]  

Figure 3.5.1 (a) Equivalent electrical circuit of a laser diode, and (b) voltage versus current relation of a diode laser with 0.1Ω series resistance.

It is well-known that the relation between the current, $I_D$, and the voltage, $V_D$, of a diode is,
where \( I_s \) is the saturation current, \( q \) is the electron charge, \( T \) is the absolute temperature, \( k_B \) is the Boltzmann's constant, and \( n \) is an ideality factor of the diode. When a series resistance \( R_s \) is introduced, the external current \( (I_D) \) voltage \( (V_B) \) relation is then,

\[
I_D = I_s \exp \left( q \frac{V_B - R_s I_D}{n k_B T} \right)
\]

(3.5.2)

The series resistance \( R_s \) is primarily caused by the Ohmic contact of the electrode, which is usually less than \( 1 \Omega \). For a practical laser diode used for telecommunication transmitter, the voltage \( V_B \) is on the order of \( 1.5V \) at the operation point with a reasonable output optical power. For example, assume a saturation current \( I_s = 10^{-13} A \), an ideality factor \( n = 2 \), and a series resistance \( R_s = 0.1 \Omega \), current-voltage relation is shown in Figure 3.5.1 (b). An injection current increase from 0.1A to 1A corresponds to a very small voltage increase of less than 0.2V, which is also sensitive to the operation temperature of the laser diode. As the optical power of a laser diode is proportional to the injection current, the electrical driver needs to be a current source instead of a voltage source.

Figure 3.5.2 (a) Illustration of diode laser P-I curve at different junction temperatures, and (b) Threshold current as a function of junction temperature

Threshold current and optical power efficiency are two key parameters of a diode laser, both of them are sensitive to the junction temperature \( T \). The threshold current \( I_{th} \) increases with the junction temperature exponentially as \( I_{th} = A \exp(T / T_0) \), where \( A \) is a proportionality factor, and \( T_0 \) is the characteristic temperature ranging from \( 40K \) to \( 120K \) depending on the material and laser structure. Figure 3.5.2 (a) illustrates the laser \( P-I \) curves at different junction temperatures, and Figure 3.5.2 (b) shows the
threshold current as the function of the temperature, assuming $A = 0.1\text{mA}$, and $T_0 = 60K$ were assumed.

Figure 3.5.2 (a) shows the picture of a laser diode (LD) chip on mounted on a heat sink. As the size of the electrode on top of the LD chip is very small and fragile, it is usually connected to a staging metal pad through very thin gold wires, and the wire for external connection to the driver electrical circuit is soldered to this metal pad. The heat sink can also be mounted on a larger heat sink which helps dissipate the heat generated by the diode laser chip when driven by a large injection current. Figure 3.5.2(b) shows the picture of a sealed cylindrical package for a diode laser. A collimating lens can be added in front of the laser diode so that the laser beam emitted from the exit window is collimated.

Figure 3.5.3 (a) Picture of diode laser on heat sink and (b) picture of cylindrical packaged diode laser

For lasers used for high speed optical communication systems, the requirement of temperature stabilization is more stringent than for other applications. The reason is that the junction temperature of a laser diode not only affects the threshold current and power efficiency, but also significantly affects the emission wavelength through the temperature sensitivity of the refractive index. As a rule of thumb, for an InGaAsP based DFB laser operating in 1550nm wavelength window, each °C change of junction temperature may introduce as much as 0.1nm change of the emission wavelength. Thus, a temperature stability of better than 0.1°C is usually required for lasers diodes used for wavelength division multiplexed (WDM) optical systems, in which each channel has its assigned narrow wavelength window.

Figure 3.5.4 (a) shows the configuration of a fiber pigtailed laser diode typically used in fiber-optic communication systems. In this configuration, the laser chip is mounted
on a heat sink, and the emission is collimated and passes through an optical isolator to avoid the impact of external optical reflection on the laser diode performance. After the isolator, the collimated optical beam is refocused and coupled into an optical fiber through another lens. A photodiode mounted on the left side of the laser chip is used to monitor the optical power emitted from the back facet of the laser diode, which is in fact linearly proportional to the power emitted from the right side laser facet.

Figure 3.5.4 (a) Configuration of a diode laser package used for fiber-optic communication systems, and (b) Pin assignment of a standard 14-pin butterfly package of a laser diode.

The temperature of the heat sink is monitored by a thermistor burred inside the heat sink, and the reading from the thermistor is used to control the thermal electrical cooler (TEC), known as a Peltier, through a electronic feedback circuit. All the elements are hermetically sealed inside a metal case to ensure the mechanical and optoelectronic stability, and reliability for commercial applications. The pin assignment of the standard 14-pin butterfly package is shown in Figure 3.5.4(b). Although the fabrication of diode laser chips can be cost-effective and each processed 6-inch semiconductor wafer can be cleaved into a large number of laser diodes, packaging appears much more challenging for cost reduction. The packaging including fiber coupling, optical isolation, temperature control and power monitoring usually constitutes more than 80% of the total cost of a laser diode used in the transmitter of a fiber-optic communication system.