Homework 1,

1. Consider the air/water interface shown in the following figure.
   (a) A collimated light beam launches from the air to the water at an angle $\theta = \pi / 6$ with respect to the surface normal as shown in Figure E-2.1(a), where $h_a = 1.5$ m and $h_b = 1$ m. The refractive index of air is $n_0 = 1$, and assume the refractive index of water is $n_1 = 1.3$.

   Please find the distance $x_1 = ?$

   In order to eliminate reflection from the water surface, what should be the angle $\theta$, and in which direction the light should be polarized?

   (b) If the light is launched from the bottom of the water tank as shown in E-2.1(b), and the incidence angle is still $\theta = \pi / 6$, please find the distance $x_2 = ?$.

   What is the angle $\theta$ to achieve total reflection on the water/air surface?

Solution:

(a) $x_0 = h_a \tan \theta = 1.5 \tan(\pi / 6) = 0.866m$

   Diffraction angle $\theta_2 = \sin^{-1}\left(\frac{n_0 \sin \theta}{n_1}\right) = \sin^{-1}\left(\frac{1}{1.3} \sin\left(\frac{\pi}{6}\right)\right) = 0.395 rad = 22.6^\circ$

   $x_1 = x_0 + h_b \tan \theta_2 = 0.866 + 1 \times \tan(0.395) = 1.28m$
When the incidence angle $\theta$ is equal to the Brewster angle and the optical field polarization is parallel to the incidence plane, reflection from water surface can eliminated. The Brewster angle is,

$$\theta = \theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right) = \tan^{-1}(1.3) = 0.915\text{rad} = 52.4^\circ$$

(b) $x_0 = h_x \tan \theta = 1 \times \tan(\pi/6) = 0.5774m$

Diffraction angle $\theta_d = \sin^{-1}\left(\frac{n_1}{n_0} \sin \theta\right) = \sin^{-1}\left(\frac{1.3}{1} \sin\left(\frac{\theta}{6}\right)\right) = 0.708\text{rad} = 40.5^\circ$

$x_2 = x_0 + h_x \tan \theta_d = 0.5774 + 1.5 \times \tan(0.708) = 1.86m$

Total reflection happens when the incidence angle $\theta$ is equal to the critical angle,

$$\theta = \sin^{-1}\left(\frac{n_0}{n_1}\right) = \sin^{-1}(1/1.3) = 0.878\text{rad} = 50.28^\circ$$

2. A light beam is launched to a thin semiconductor film with a thickness of $d=2\ \mu m$. The refractive index of air is $n_0 = 1$, and assume the semiconductor film has no loss and its refractive index is $n_1 = 3.5$. The wavelength of the light is $\lambda = 633\text{nm}$ which is parallel polarized on the plane of the paper, and the incidence angle is $\theta = 45^\circ$.

Please find the relative amplitude $|E_1|/|E_2|$ and the phase difference $[\text{Angle}(E_1) - \text{Angle}(E_2)]$ of the two reflected beams at the reference plane.

Solution:

For the parallel polarization, according equation 2.5, the reflectivity is

$$\rho_\parallel = \frac{-n_1^2 \cos \theta_1 + n_0 \sqrt{(n_1^2 - n_0^2 \sin^2 \theta_1)}}{n_1^2 \cos \theta_1 + n_0 \sqrt{(n_1^2 - n_0^2 \sin^2 \theta_1)}} = -0.4329$$

which means that the phase shift is $\pi$.

For the 2nd beam, the beam angle into the glass is $\theta_2 = \sin^{-1}\left(\frac{n_0}{n_1} \sin \theta\right) = 0.2034\text{rad}$

and the reflectivity on the bottom glass/air interface can be found from

$$\rho_\parallel = \frac{-n_0^2 \cos \theta_2 + n_1 \sqrt{(n_0^2 - n_1^2 \sin^2 \theta_2)}}{n_0^2 \cos \theta_2 + n_1 \sqrt{(n_0^2 - n_1^2 \sin^2 \theta_2)}} = 0.4329$$

The phase shift is zero.
The amplitude ratio is thus,
\[
\left| \frac{E_1}{E_2} \right| = \frac{0.4329}{\sqrt{1 - 0.4329^2 \times 0.4329 \times \sqrt{1 - 0.4329^2}}} = \frac{1}{1 - 0.4329^2} = 1.2306
\]

The phase difference is, \( \Delta \Phi = -\pi \frac{2\pi}{\lambda} \frac{2d}{\cos \theta_2} = -\pi \left( 1 + \frac{4d}{\lambda \cos \theta_2} \right) = -14\pi \)

3. A light beam with \( \lambda = 0.63 \, \mu m \) travels inside a slab optical waveguide by bouncing back and forth between the two interfaces. The refractive index of the waveguide is \( n_1 = 1.5 \), and the thickness is \( d = 3 \, \mu m \). Assume the lightwave is vertically polarized (\( E \) field goes into the paper) with respect to the incidence plane. For the incidence angle \( \theta = 60^\circ \), what is the optical phase shift after each roundtrip? What is the evanescent field penetration depth near the glass/air interface?

Solution:
Based on Equation 2.1.8, the optical phase shift of each reflection is
\[
\Delta \Phi_\perp = -2 \tan^{-1}\left( \frac{n_0 \sqrt{(n_1^2 / n_0^2) \sin^2 \theta - 1}}{n_1 \cos \theta} \right) = -1.671 \, \text{rad.}
\]

and the phase delay due to roundtrip propagation is,
\[
\Delta \Phi_\parallel = \Delta \Phi_\perp + \frac{4\pi d}{\lambda \cos \theta} = 116.34 \, \text{rad.}
\]

Total phase shift is then, \( \Delta \Phi = \Delta \Phi_\perp + \frac{4\pi d}{\lambda \cos \theta} = 116.34 - 1.671 \times 2 = 113\, \text{rad.} \approx 36\pi \)

The penetration depth can be found as,
\[
z_e = \frac{\lambda}{2\pi n_1^2 \sin^2 \theta_1 - n_2^2} = \frac{0.63}{5.21} = 0.121 \, \mu m
\]