Problem 10 solution

1. There is an optical pre-amplifier in an optical receiver before the photodiode. The input optical power to the optical amplifier is \( P_{\text{in}} = -20 \text{dBm} \), and the optical amplifier has a 6dB noise figure. Other parameters are, operation temperature \( T = 300k \), load resistance \( R_L = 50 \Omega \), photodiode responsivity \( \mathcal{R} = 0.9 A/W \), and the operation wavelength \( \lambda = 1550 \text{nm} \).

(a) if the gain of the optical amplifier is \( G = 30 \text{dB} \), please find the noise power spectral densities of thermal noise, shot noise and signal-ASE beat noise.

(b) What is the required optical gain of the amplifier so that signal-ASE beat noise is 10dB higher than the thermal noise after photo-detection?

Solution:

(a) thermal noise: \( \sigma_{th}^2 = \frac{4kT}{R_L} = \frac{4 \times 1.38 \times 10^{-23} \times 300}{50} = 3.312 \times 10^{-22} W/Hz \)

shot noise: \( \sigma_{sh}^2 = 2q\mathcal{R}GP_{\text{in}} = 2 \times 1.6 \times 10^{-19} \times 0.9 \times 10^3 \times 10^{-5} = 2.9 \times 10^{-21} W/Hz \)

Signal-ASE beat noise:
\[
\sigma_{S-ASE}^2 = 4R^2P_{\text{in}}Ghfn_{sp}(G-1)
\]
\[
= 4 \times 0.81 \times 10^{-5} \times 10^3 \times 1.283 \times 10^{-19} \times 2 \times 999 = 8.3 \times 10^{-18} W/Hz
\]

(b) When signal-ASE beat noise has to be 10 times higher than thermal noise, \( 4R^2P_{\text{in}}hfn_{sp}G(G-1) = 10 \times 4kT / R_L \), that is,
\[
G \approx \sqrt{\frac{10kT}{R_L}} = \sqrt{\frac{10 \times 1.38 \times 10^{-23} \times 300}{50 \times 0.81 \times 10^{-5} \times 1.283 \times 10^{-19} \times 2}} = 20 = 13 \text{dB}
\]

Therefore, if the optical amplifier gain is higher than 13dB, thermal noise contribution in the receiver can be negligible.

2. For a semiconductor optical amplifier (SOA) with 25dB peak optical gain, if both end surface have the same power reflectivity \( R \), what is the maximum \( R \) allowed so that the maximum gain ripple is less than 1dB near the peak gain wavelength?

Solution: The power transfer function of a SOA is,
\[
T = \frac{(1-R)^2 G_s}{(1-RG_s)^2 + 4RG_s \sin^2(2\pi f \tau)}, \text{ so that the maximum and the minimum gain are,}
\]

\[
\quad
\]
$$T_{max} = \frac{(1 - R)^3 G_s}{(1 - RG_s)^3} \quad \text{and} \quad T_{min} = \frac{(1 - R)^3 G_s}{(1 - RG_s)^3 + 4RG_s},$$

so that

$$\frac{T_{max}}{T_{max}} = \frac{(1 - RG_s)^3 + 4RG_s}{(1 - RG_s)^3} \leq 10^{0.1} = 1.259,$$

that is,

$$(1 - RG_s)^3 + 4RG_s = 1.259(1 - RG_s)^3, \quad (1 - RG_s)^3 - 15.44RG_s = 0$$

That is, $(RG_s)^3 - 17.44RG_s + 1 = 0$, the solution is $RG_s = 0.058$.

Since $G_s \approx 25 dB = 10^{25/10} = 316$

$$R \leq \frac{0.058}{G_s} = \frac{0.058}{316} = 1.84 \times 10^{-4} = -37.4 dB$$

Note: there is another solution:

$$RG_s = 17.38 > 1,$$

which would have the roundtrip gain bigger than one, and thus not physically possible.

3. At $\lambda = 1548$nm wavelength, the emission and absorption cross-sections of an erbium-doped fiber (EDF) are $\sigma_e = 1 \times 10^{-24} m^2$ and $\sigma_a = 0.7 \times 10^{-24} m^2$, respectively. Erbium doping density of this EDF is $N_T = 7.5 \times 10^{24} m^{-3}$, and the confinement factor is $\Gamma = 0.1$.

(a) Please find the emission and absorption rate in dB/m for this EDF.

(b) If the length of the EDF is $L = 2$m, and the pump power is strong enough so that the carrier inversion is complete (that is: $N_2 = N_T, N_1 = 0$), what is the small-signal optical gain at 1548nm wavelength?

Solution:

(a) Emission rate:

$$\exp(\Gamma \sigma_e(\lambda) N_T) = \exp(0.1 \times 1 \times 10^{-24} \times 7.5 \times 10^{24}) = \exp(0.75) = 2.12$$

$$10 \log(2.12) = 3.26 dB/m$$

Absorption rate:

$$\exp(\Gamma \sigma_a(\lambda) N_T) = \exp(0.1 \times 0.7 \times 10^{-24} \times 7.5 \times 10^{24}) = \exp(0.525) = 1.69$$

$$10 \log(1.69) = 2.28 dB/m$$

(b) the optical gain at 1548nm wavelength is

$$G(\lambda_e) = \exp\left[\Gamma \left[\left(\sigma_e N_2 - \sigma_a N_1\right) L\right]\right] = \exp\left[0.1 \times 1 \times 10^{-24} \times 7.5 \times 10^{24} - 0 \times 2\right] = 4.48$$

which is $6.514 dB$

4. An EDFA originally has an optical gain of $G = 25 dB$ and a noise figure $F = 5 dB$ at a certain wavelength. Neglect gain saturation effect.

Now there is an addition 3dB loss associated with the EDFA. This forms an "extended EDFA" as illustrated below.
(a) If the 3dB loss at the input side of the original EDFA, what is the noise figure of the extended EDFA? *(Hint: use noise figure definition)*

(b) If that 3dB loss is at the output side of the EDFA, then what is the noise figure of the extended EDFA? Please explain.

Solution:

(a) By definition noise figure is, \[ F = \frac{SNR_{in}}{SNR_{out}} \]

If the input has 3dB loss, \( SNR_{in} \) does not change. Signal optical power at output will be reduced by 3dB, but optical noise power spectral density will not change (because the gain \( G \) does not change). Since \( SNR_{out} \) is determined by the signal-ASE beat noise which is linearly proportional to the signal optical power, it will be reduced by 3dB. Therefore, the noise figure will be increased by 3dB (from 5dB to 8dB).

(b) If the output has 3dB loss, \( SNR_{in} \) does not change. Both signal optical power and the optical noise power spectral density at output will be reduced by 3dB. Thus \( SNR_{out} \) will not change. Therefore, the noise figure will not change.