Problem 11

1. For a binary modulated system without waveform distortion (that is: \(v_0=0\) and \(v_1=1\)), if the normalized noise standard deviations associated with signal "0" and "1" levels are \(\sigma_0=0.05\) and \(\sigma_1=0.15\) for the normalized eye diagram,

(a) Please find the normalized optimum decision threshold \(v_{th}\)

(b) Please find the \(Q\)-value and BER

(c) If the normalized decision threshold is chosen to be \(v_{th}=0.5\), what is the BER?

Solution:

(a) Based on Equation 8.1.12, the optimum decision threshold can be obtained by equalizing errors from 0 and 1 levels, so that
\[
v_{th} = \frac{v_0\sigma_1 + v_1\sigma_0}{\sigma_0 + \sigma_1} = \frac{0.05 \times 1 + 0}{0.15 + 0.05} = \frac{0.05}{0.2} = 0.25
\]

(b) Under this “optimum” decision threshold, the Q value is,
\[
Q = \frac{v_1 - v_0}{\sigma_1 + \sigma_0} = \frac{0.2}{0.2} = 5, \text{ and } BER = \frac{1}{2} \text{erfc}\left(\frac{5}{\sqrt{2}}\right) = 2.87 \times 10^{-7}
\]

(c) If the normalized decision threshold is chosen at \(v_{th} = 0.5\),
\[
Q_0 = \frac{v_{th} - v_0}{\sigma_0} = \frac{0.5 - 0}{0.05} = 10, \quad Q_1 = \frac{v_1 - v_{th}}{\sigma_1} = \frac{1 - 0.5}{0.15} = 3.3333
\]
\[
BER = \frac{1}{4} \left[ \text{erfc}\left(\frac{Q_0}{\sqrt{2}}\right) + \text{erfc}\left(\frac{Q_1}{\sqrt{2}}\right) \right] = \frac{1}{4} \left[ \text{erfc}\left(\frac{10}{\sqrt{2}}\right) + \text{erfc}\left(\frac{3.3333}{\sqrt{2}}\right) \right]
\]
\[
= \frac{1}{4} \left(1.54 \times 10^{-23} + 8.58 \times 10^{-4}\right) = 2.15 \times 10^{-4}
\]

2. Consider an optical transmission system with binary intensity modulation at a data rate of 5Gb/s. The average optical signal optical power emitted from the transmitter is \(P_{tx} = 6\)dBm, and the loss coefficient of the optical fiber is \(\alpha = 0.25\)dB/km. In the direct detection receiver a PIN photodiode is used which has a responsivity \(R = 1A/W\), a load resistance \(R_L = 50\Omega\), and an electrical bandwidth \(B_e = 5GHz\). Thermal noise is considered as the major noise in the receiver (only consider thermal noise and neglect other noise sources).
(a) If there is no waveform distortion (normalized eye opening $A = 1$, $B = 0$), in order to achieve $Q = 7$, please find the receiver sensitivity (in [dBm]) and the maximum allowable transmission fiber length (in [km]).

(b) Now considering waveform distortion caused by chromatic dispersion so that $A = 1 - 2 \times 10^{-3} L$ and $B = 2 \times 10^{-3} L$, where $L$ in [km] is the fiber length. In order to achieve $Q = 7$, what is the maximum allowable transmission fiber length? (Numerical solution is needed in this case)

Solution:
(a) Without waveform distortion

At the optical receiver, thermal noise is constant which is

$$\sigma_{th}^2 = \frac{4kTB_e}{R_L} = \frac{4 \times 1.38 \times 10^{-23} \times 300}{50} \times 5 \times 10^9 = 1.66 \times 10^{-12} W$$

$$\sigma_0 = \sigma_1 = \sqrt{1.66 \times 10^{-12}} = 1.29 \times 10^{-6}$$

For a receiver $Q = 7$ (corresponding to $BER = 10^{-12}$),

$$Q = \frac{v_1 - v_0}{\sigma_0 + \sigma_1} = \frac{2\gamma P_{ave}}{2 \times 1.29 \times 10^{-6}} = \frac{2}{2 \times 1.29 \times 10^{-6}} \times 10^{6-0.25L/10} \times 10^{-3} = 7$$

$$P_{ave} = 7 \times \frac{2 \times 1.29 \times 10^{-6}}{2\gamma} = 7 \times 1.29 \times 10^{-6} = 9.03 \times 10^{-6} W = -20.4 dBm \text{ (this is the sensitivity)}$$

$$L = \frac{6 - (-20.4)}{0.25} = 105.6 km$$

(b) Consider the waveform distortion,

The received optical power is

$$P_{ave} = P_{tx} - 0.25L \text{ in [dBm]}, \text{ so that } P_{ave} = 10^{10(6-0.25L)/10} \times 10^{-3} \text{ in } [W]$$

$$Q = \frac{v_1 - v_0}{\sigma_0 + \sigma_1} = \frac{1 - 4 \times 10^{-3} L}{2 \times 1.29 \times 10^{-6}} \times 2\gamma P_{ave} = \frac{1 - 4 \times 10^{-3} L}{2 \times 1.29 \times 10^{-6}} \times 2 \times 10^{6-0.25L/10} \times 10^{-3} = 7 ,$$

where, $L$ is in [km]

$$(1 - 4 \times 10^{-3} L) \times 10^{6-0.25L/10} = 9 \times 10^{-3}$$

Use numerical solution: $L = 97 km$

Apparently in this particular case, chromatic dispersion is not a very important limiting factor.

3. Consider the same optical system as in problem 2, except that an optical pre-amplifier is added in front of the photodiode with 6dB noise figure and very high optical gain ($G >> 1$). Signal-
ASE beat noise is considered as the major noise in the receiver (only consider signal-ASE beat noise and neglect other noise sources). Operating wavelength is 1550nm.

(a) If there is no waveform distortion (normalized eye opening $A = 1$, $B = 0$), in order to achieve $Q = 7$, please find the receiver sensitivity (in [dBm]) and the maximum allowable transmission fiber length (in [km]).

(b) Now considering waveform distortion caused by chromatic dispersion so that $A = 1 - 2 \times 10^{-3} L$ and $B = 2 \times 10^{-3} L$, where $L$ in [km] is the fiber length. In order to achieve $Q = 7$, what is the maximum allowable transmission fiber length? (Numerical solution is needed in this case)

Solution:
Based on Equation 8.3.7,

$$Q = \sqrt{\frac{P_{ave}}{2n_{sp}(hc/\lambda)B_e}}(\sqrt{A} - \sqrt{B}), \quad P_{ave} = 2n_{sp}(hc/\lambda)B_eQ^2(\sqrt{A} - \sqrt{B})^2$$

For noise figure $F=6dB=4$, so that $n_{sp} = 2$. The target $Q$ value is 7.

that is:

$$P_{ave} = 4Q^2(hc/\lambda)B_e(\sqrt{A} - \sqrt{B})^2 = 4 \times 49 \times \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{1550 \times 10^{-9}} \times 5 \times 10^8(\sqrt{A} - \sqrt{B})^2$$

$$= 1.26 \times 10^{-7}(\sqrt{A} - \sqrt{B})^2$$

(a) Without waveform distortion,

$$Q = \sqrt{\frac{P_{ave}}{2n_{sp}(hc/\lambda)B_e}} = 7,$$

The sensitivity is

$$P_{ave} = 49 \times 4(hc/\lambda)B_e = 49 \times 4(hc/\lambda)B_e = 1.26 \times 10^{-7}W = -39dBm$$

$$L = \frac{6 - (-39)}{0.25} = 180km$$

(b) Consider waveform distortion with $A = 1 - 2 \times 10^{-3} L$, $B = 2 \times 10^{-3} L$ with $L$ the fiber length in km, and

$$P_{ave} = 10^{\frac{6-0.25L}{10}} \times 10^{-3} \times 10^{-3} = 1.26 \times 10^{-7}(\sqrt{1-2 \times 10^{-3} L} - \sqrt{2 \times 10^{-3} L})^2$$

Use numerical solution: $L = 140km$

In this particular case, chromatic dispersion has a much bigger impact.
4. A 1550nm wavelength optical system shown below has 3 spans of single mode optical fiber, and 3 optical amplifiers each with 6dB noise figure. Each amplifier has 20dB optical gain but the length of each fiber span is different, they are 80km, 120km and 50km, respectively. The fiber attenuation is 0.25dB/km. The signal optical power emitted from the transmitter is 10mW. What is the optical noise power spectral density at the output of the laser EDFA (in the unit of [dBm/nm])?

Solution:
At 1550nm wavelength, photon energy is $h\nu = 1.28 \times 10^{-19} W$
Since the optical gain of each EDFA is 20dB, the optical noise spectral density generated by each EDFA is (here $F = 6dB = 4$ is equivalent to $n_{sp} = 2$)
$$\rho_{ASE,1} = \rho_{ASE,2} = \rho_{ASE,3} = 2h\nu n_{sp}(G - 1) = 1.28 \times 10^{-19} \times 4 \times 99 = 5.06 \times 10^{-17} W/Hz$$
The losses of three fiber spans are, $A_1 = 80 \times 0.25 = 20dB$, $A_2 = 120 \times 0.25 = 30dB$, and $A_3 = 50 \times 0.25 = 12.5dB$
There is 40 - 42.5 = -2.5dB net gain from EDFA1 to the receiver and 20 - 12.5 = 7.5dB net gain from EDFA2 to the receiver.
At the receiver, the accumulated optical noise spectral density is
$$\rho_{total} = \rho_{ASE,1} \times 10^{-0.25} + \rho_{ASE,2} \times 10^{0.75} + \rho_{ASE,3} = \rho_{ASE,1}(0.5623 + 5.6234 + 1) = 7.18\rho_{ASE,1}$$
That is, $\rho_{total} = 7.18 \times 2h\nu n_{sp}(G - 1) = 3.64 \times 10^{-16} W/Hz$
As 1nm is equivalent to 125GHz at 1550nm wavelength window, this noise power spectral density is equivalent to,
$$\rho_{total} = 3.64 \times 10^{-16} \times 125 \times 10^9 \times 10^3 mW/nm = -13.4 dBm/nm$$