Homework #4

1. When the fiber length is much longer than the effective length ($L >> L_{\text{eff}}$), normalized FWM efficiency $\eta_{\text{FWM}}$ is only related to fiber loss, dispersion and channel spacing. Consider a degenerate FWM caused by two frequency components in the 1550nm wavelength window separated by 25GHz, and assume the fiber loss is $\alpha_{\text{dB}} = 0.2\text{dB/km}$. Plot FWM efficiency normalized FWM efficiency $\eta_{\text{FWM}}$ as the function of dispersion parameter $D$ (from 0 to 20ps/(nm-km)). At which dispersion value, $\eta_{\text{FWM}}$ is 1%?

2. Consider an LED operating in 800nm wavelength window with 3% external efficiency. The radiative and nonradiative recombination carrier lifetimes are 2ns and 8ns, respectively.
   (a) In order to have 2mW output optical power, what is the required electrical current?
   (b) What is the 3dB optical modulation bandwidth of this LED?
   (c) What is the 3dB modulation bandwidth in electrical domain?

3. An LED operating in 1310nm wavelength window has 5% external efficiency and 80% internal efficiency. An electrical signal used to modulate this LED is $I(t) = I_B + I_m \cos(\omega t)$, where $I_m = 20\text{mA}$. What is the minimum bias current $I_B$ so that the output optical power waveform has no distortion? What is the average optical power, and what is the peak optical power?

4. Assume the gain parameter of a semiconductor material has a parabolic shape $g(\lambda) = g_0 \left(1 - \left(\frac{\lambda - \lambda_0}{\Delta \lambda_g}\right)^2\right)$ where $\lambda_0 = 1550\text{nm}$, and $\Delta \lambda_g = 15\text{nm}$. The length of the laser cavity is $L = 400\mu\text{m}$, and refractive index of the material is $n = 3.6$. Please find the number wavelengths which satisfy the phase condition and with $g \geq 0.5g_0$.

5. A laser cavity is formed by Fresnel reflections between two cleaved facets. The refractive index of the laser cavity is $n = 3.5$, and the absorption parameter is $\alpha = 30\text{cm}^{-1}$. At which cavity length, $L$, the mirror loss is equal to the absorption loss in the laser cavity?
Solutions

1. When the fiber length is much longer than the effective length \(L \gg L_{\text{eff}}\), normalized FWM efficiency \(\eta_{\text{FWM}}\) is only related to fiber loss, dispersion and channel spacing. Consider a degenerate FWM caused by two frequency components in the 1550nm wavelength window separated by 25GHz, and assume the fiber loss is \(\alpha_{\text{dB}} = 0.2\text{dB/km}\). Plot FWM efficiency normalized FWM efficiency \(\eta_{\text{FWM}}\) as the function of dispersion parameter \(D\) (from 0 to 20ps/(nm-km)). At which dispersion value, \(\eta_{\text{FWM}}\) is 1%?

Solution,

Based on equations (2.6.24) and (2.6.28),

\[
\Delta \beta_{jkl} = \frac{2\pi D}{\lambda^2} (\lambda_j - \lambda_i)(\lambda_k - \lambda_l), \quad \text{and} \quad \eta_{\text{FWM}} \approx \frac{\alpha^2}{\Delta \beta_{jkl}^2 + \alpha^2}
\]

For degenerate FWM, \(\lambda_j = \lambda_k\), so that \(\Delta \beta_{jkl} = \frac{2\pi D(\lambda_k - \lambda_i)}{\lambda^2} = 125.5D\)

Where \((\lambda_k - \lambda_i) = 0.4\text{nm}\) was used for 25GHz channel spacing.

\[
\alpha = \alpha_{\text{dB}} / 4.343 = 0.0461 Np / \text{km} = 4.61 \times 10^{-5} Np / \text{m}
\]

To match the unit, we also need to convert \(D\) from [s/(m-m)] to [ps/(nm-km)], so that \(\Delta \beta_{jkl} = \frac{2\pi D(\lambda_k - \lambda_i)}{\lambda^2} = \frac{2\pi D}{125.5D} = 125.5D \times 10^{-6}\)

To find the dispersion value where \(\eta_{\text{FWM}} = 0.01\), \((\Delta \beta_{jkl} / \alpha)^2 = 99\), that is, \(\Delta \beta_{jkl} \approx 10\alpha\), or,

\[
D = \frac{10\alpha}{125.5 \times 10^{-6}} = \frac{10 \times 4.61 \times 10^{-5}}{125.5 \times 10^{-6}} = 3.67 \left(\frac{ps}{nm \cdot km}\right)
\]

2. Consider an LED operating in 800nm wavelength window with 3% external efficiency. The radiative and nonradiative recombination carrier lifetimes are 2ns and 8ns, respectively.

(a) In order to have 2mW output optical power, what is the required electrical current?
(b) What is the 3dB optical modulation bandwidth of this LED?
(c) What is the 3dB modulation bandwidth in electrical domain?

Solution,
(a) The quantum efficiency is \[ \eta_q = \frac{R_r}{R_r + R_{nr}} = \frac{\tau_{nr}}{\tau_r + \tau_{nr}} = \frac{8\text{ns}}{8\text{ns} + 2\text{ns}} = 0.8 \]

The output optical power can be calculated by \[ P_{opt} = \eta_q \eta_{ext} \frac{hc}{\lambda q} I \], so that the injection current is, \[ I = \frac{P_{opt} \lambda q}{\eta_q \eta_{ext} hc} = \frac{2 \times 10^{-3} \times 800 \times 10^{-9} \times 1.6 \times 10^{-19}}{0.8 \times 0.03 \times 6.63 \times 10^{-34} \times 3 \times 10^8} = 53.6mA \]

(b) The carrier lifetime is \[ \tau = \left(\frac{1}{\tau_r} + \frac{1}{\tau_{nr}}\right)^{-1} = \left(\frac{1}{2} + \frac{1}{8}\right)^{-1} = 1.6\text{ns} \], so that the optical bandwidth is \[ f_{3dB, opt} = \sqrt{3/(2\pi\tau)} \approx 173MHz \].

(c) The electrical bandwidth is \[ f_{3dB, ele} = 1/(2\pi\tau) \approx 99.5MHz \]

3. An LED operating in 1310nm wavelength window has 5% external efficiency and 80% internal efficiency. An electrical signal used to modulate this LED is \[ I(t) = I_n + I_m \cos(\omega t) \], where \( I_m = 20\text{mA} \). What is the minimum bias current \( I_B \) so that the output optical power waveform has no distortion? What is the average optical power, and what is the peak optical power?

Solution,
With no waveform distortion, the minimum bias current \( I_B \) has to be equal to \( I_m \), that is \( I_B = 20\text{mA} \).

The average optical power is \[ P_{ave} = \eta_q \eta_{ext} \frac{hc}{\lambda q} I_B = 0.76mW \]

The peak power is \[ P_{peak} = 2P_{peak} = 1.52mW \]

4. Assume the gain parameter of a semiconductor material has a parabolic shape \[ g(\lambda) = g_0 \left[ 1 - \left(\lambda - \lambda_0\right)/\Delta\lambda_g \right]^2 \] where \( \lambda_0 = 1550\text{nm} \), and \( \Delta\lambda_g = 15\text{nm} \). The length of the laser cavity is \( L = 400\mu\text{m} \), and refractive index of the material is \( n = 3.6 \). Please find the number wavelengths which satisfy the phase condition and with \( g \geq 0.5g_0 \).

Solution:
Based on the phase condition $\lambda_m = 2nL / m$, mode spacing can be found by 
$\delta m = (2nL / \lambda_m^2) \delta \lambda$, for adjacent modes $\delta m = 1$, and thus mode spacing is 
$\delta \lambda = \frac{\lambda_m^2}{2nL} \approx \frac{\lambda_0^2}{2nL} = \frac{(1550 \times 10^{-9})^2}{2 \times 3.6 \times 4 \times 10^{-4}} = 0.8342 \text{nm}$

For $g = 0.5g_0$, $[(\lambda - \lambda_0) / \Delta \lambda_g]^2 = 0.5$, $\lambda - \lambda_0 = \pm \sqrt{0.5} \Delta \lambda_g$, so that the full bandwidth is
$\Delta \lambda = 2\sqrt{0.5} \Delta \lambda_g = 21.2 \text{nm}$

The number of modes is $N = \Delta \lambda / \delta \lambda = 21.2 \text{nm} / 0.8342 \text{nm} \approx 25$

5. A laser cavity is formed by Fresnel reflections between two cleaved facets. The refractive index of the laser cavity is $n = 3.5$, and the absorption parameter is $\alpha = 30 \text{cm}^{-1}$. At which cavity length, $L$, the mirror loss is equal to the absorption loss in the laser cavity?

Solution

Power reflectivity on each facet is $R = \left( \frac{n-1}{n+1} \right)^2 = \left( \frac{3.5-1}{3.5+1} \right)^2 = 0.3086$

Based on equation (3.3.14), the mirror loss $\alpha_m = -\frac{\ln(R_1R_2)}{4L}$. When it is equal to the absorption loss, $L = -\frac{\ln(R_1R_2)}{4\alpha} = -\frac{2\ln(0.3086)}{4 \times 30 \text{cm}^{-1}} = 0.0196 \text{cm} = 196 \mu\text{m}$