Problem 8 solution

1. Silicon has a bandgap of 1.13 eV. For a silicon photodiode with a quantum efficiency of 0.85, what are the responsivities of this photodiode at signal wavelengths of 700 nm, 1000 nm, and 1500 nm?

Solution: Responsivity is $\mathcal{R} = \eta \frac{q \lambda}{hc}$

at $\lambda = 700$ nm, $\mathcal{R} = 0.85 \times \frac{1.6 \times 10^{-19} \times 700 \times 10^{-9}}{6.626 \times 10^{-34} \times 3 \times 10^8} = 0.48 A/W$

at $\lambda = 1000$ nm, $\mathcal{R} = 0.85 \times \frac{1.6 \times 10^{-19} \times 1000 \times 10^{-9}}{6.626 \times 10^{-34} \times 3 \times 10^8} = 0.684 A/W$

Since $\lambda = 1550$ nm is longer than the cutoff wavelength, there is no response: $\mathcal{R} = 0$

2. For a PIN photodiode shown in the following figure, the photon absorption coefficient rates are $\alpha_p = 400 \text{ cm}^{-1}$ for the $p$- and $n$-doped regions, and $\alpha_i = 1000 \text{ cm}^{-1}$ for the intrinsic region. Assume both $p$- and $n$-doped regions are highly conductive so that the depletion region is only restricted within the intrinsic layer. Layer thicknesses are $W_p = 2 \mu m$, $W_i = 10 \mu m$, and $W_n = 4 \mu m$. Neglect all surface reflections.

(a) Find the quantum efficiency of this photodiode

(b) Assume the cross section area of this photodiode is $A = 10^{-4} \text{ cm}^2$, the relative dielectric constant is $\varepsilon_r = 11.7$, dielectric constant of vacuum is $\varepsilon_0 = 8.85 \times 10^{-14} F/cm$, the load resistance is $50 \Omega$, and the carrier velocity inside the intrinsic region is $v_n = 2 \times 10^7 \text{ cm/s}$. If the thickness of the intrinsic region $W_i$ can be changed, what is the optimum value so that the cutoff frequency determined by the RC constant and by the carrier transient is the same?

Solution:

(a) $\eta = (1 - R) \exp(-\alpha_p W_p) [1 - \exp(-\alpha_i W_i)]$

$= \exp(-400 \times 2 \times 10^{-4}) [1 - \exp(-1000 \times 1 \times 10^{-3})] = 0.584$
(b) use equation, \( \frac{1}{R_c C_j} = 2.8 \frac{V_a}{W_I} \)

Since \( C_j = \frac{\varepsilon_\text{in} A}{W_I} \), this relation becomes \( \frac{1}{R_c} \frac{W_I}{\varepsilon_\text{in} A} = 2.8 \frac{V_a}{W_I} \), that is

\[
W_I^2 = 2.8 \varepsilon_\text{in} \varepsilon_0 A R_L = 2.8 \times 2 \times 10^{-7} \times 11.7 \times 8.85 \times 10^{-14} \times 10^{-6} \times 50 = 2.9 \times 10^{-7} \text{cm}^2
\]

\[
W_I = \sqrt{2.9 \times 10^{-7}} = 5.39 \times 10^{-4} \text{cm} = 5.39 \mu\text{m}
\]

3. An optical signal \( P(t) = P_{\text{ave}} [1 + \cos(2\pi f_0 t)] \) is detected by a photodiode with the bandwidth much higher than \( f_0 \). The photodiode has a responsivity \( \mathcal{R} = 0.9 A/W \), and there is a DC block (which removes the DC component from the photocurrent) at the photodiode output circuit.

(a) For \( P_{\text{ave}} = -20 \text{dBm} \), what is the mean-square of the photocurrent at the photodiode output?
(b) For every dB increase of the \( P_{\text{ave}} \), what is the corresponding increase (in dB) of the electrical signal power at the photodiode output?

Solution:
The photocurrent is, \( I(t) = \mathcal{R} P(t) = \mathcal{R} P_{\text{ave}} \cos(2\pi f_0 t) \) where DC term is removed

(a) \( P_{\text{ave}} = -20 \text{dBm} = 10^{-5} \text{W} \).

Mean-square photocurrent is \( \langle I(t)^2 \rangle = (0.9 \times 10^{-5})^2 / 2 = 4.05 \times 10^{-11} A^2 \)

(b) mean-square photocurrent is proportional to the electrical power, so that a 1dB increase of the optical power is equivalent to a 2dB increase of the signal electric power.

4. A PIN photodiode operates in the 1550nm wavelength window with responsivity \( \mathcal{R} = 0.5 A/W \). The operating temperature is \( T = 300 \text{K} \), load resistance is \( R_L = 50 \Omega \), and the dark current is 1nA.

(a) Please find the total power of thermal noise, shot noise and dark current noise within 10GHz electric bandwidth for each of the following input signal optical power levels: -20dBm, -10dBm and 0dBm,

(b) At which input signal optical power shot noise is equal to thermal noise?

Solution:
(a) Thermal noise is independent of the signal optical power

\[
\langle i_{\text{th}}^2 \rangle = \frac{4kT}{R_e} B_e = \frac{4 \times 1.38 \times 10^{-23} \times 300}{50} \times 10^{10} = 3.312 \times 10^{-12} \text{W}
\]

Dark current noise is also signal-independent,
\[ \langle i_{sh}^2 \rangle = 2qI_{dk}B_e = 2 \times 1.6 \times 10^{-19} \times 10^{-9} \times 10^{10} = 3.2 \times 10^{-18} W \]

Shot noise is signal-dependent, for \( P_{opt} = -20 \text{dBm} = 10^{-5} \text{W} \),
\[ \langle i_{sh}^2 \rangle = 2q\Re P_{opt}B_e = 2 \times 1.6 \times 10^{-19} \times 0.5 \times 10^{-5} \times 10^{10} = 1.6 \times 10^{-14} W \]
\[ P_{opt} = -10 \text{dBm} = 10^{-4} \text{W}, \ \langle i_{sh}^2 \rangle = 1.6 \times 10^{-13} W \]
\[ P_{opt} = 0 \text{dBm} = 10^{-3} \text{W}, \ \langle i_{sh}^2 \rangle = 1.6 \times 10^{-12} W \]

(b) In order for the shot noise to be equal to the thermal noise,
\[ 2q\Re P_{opt} = \frac{4kT}{R_L}, \text{ that is,} \]
\[ P_{opt} = \frac{4kT}{2q\Re R_L} = \frac{4 \times 1.38 \times 10^{-23} \times 300}{2 \times 0.5 \times 1.6 \times 10^{-19} \times 50} = 2.1 mW = 3.22 dBm \]