Homework #9

1. Consider an optical transmission system with NRZ intensity modulation at a data rate of 5Gb/s with the receiver electric bandwidth $B_e = 5$GHz. As shown in the following figure, the average optical signal optical power emitted from the transmitter is $P_{tx} = 0$dBm, the loss coefficient of the optical fiber is $\alpha = 0.25$dB/km, and neglect the waveform distortion due for simplicity. Signal wavelength $\lambda = 1550$nm, Load resistance is $R_L = 50\Omega$, temperature $T = 300$K, and photodiode responsivity $\mathcal{R} = 1A/W$.

(a) If there is no optical amplifier in the receiver (set the gain to be $G = 0$dB), thermal noise in the receiver is the dominant noise. In this case, only consider thermal noise, in order to ensure $BER \leq 10^{-12}$, what is the maximum allowable fiber length $L_{max}$? (Load resistance is $R_L = 50\Omega$, temperature $T = 300$K,

(b) An optical pre-amplifier is used then added in front of the photodiode with 6dB noise figure and very high optical gain ($G >> 1$). Signal-ASE beat noise is considered as the major noise in the receiver (neglect other noise sources). What is the maximum allowable fiber length $L_{max}$ in this case?

2. A long distance fiber-optic system operating at 1550nm wavelength has $N$ in-line optical amplifiers each with 6dB noise figure. Each fiber span has the same fiber length $L = 80$km and loss parameter $\alpha = 0.25$dB/km. The gain $G$ of each optical amplifier exactly compensates the loss of the optical fiber span immediately before it. The signal optical power that enters the first fiber span is $P_{tx} = 1$mW.

(a) If optical signal to noise ratio (OSNR) is defined as the signal optical power divided by the optical noise power measured within 0.1nm optical bandwidth, and assume $N = 10$, what is the OSNR at the input of the photodiode.

(b) Assume the system carries 10Gb/s NRZ binary data signal, the photodiode has responsivity $\mathcal{R} = 1A/W$ and electric bandwidth $B_e = 10$GHz. Only consider signal-ASE beat noise at the receiver, in order to satisfy $BER < 10^{-12}$, what is the maximum number of fiber spans, $N$, this transmission system can allow?
Homework #9 solution:

1. Consider an optical transmission system with NRZ intensity modulation at a data rate of 5Gb/s with the receiver electric bandwidth $B_e = 5$GHz. As shown in the following figure, the average optical signal optical power emitted from the transmitter is $P_{tx} = 0$dBm, the loss coefficient of the optical fiber is $\alpha = 0.25$dB/km, and neglect the waveform distortion due for simplicity. Signal wavelength $\lambda = 1550$nm, Load resistance is $R_L = 50\Omega$, temperature $T = 300$K, and photodiode responsivity $\Re = 1A/W$.

![Diagram](image)

(a) If there is no optical amplifier in the receiver (set the gain to be $G = 0$dB), thermal noise in the receiver is the dominant noise. In this case, only consider thermal noise, in order to ensure $BER \leq 10^{-12}$, what is the maximum allowable fiber length $L_{max}$? (Load resistance is $R_L = 50\Omega$, temperature $T = 300$K,

(b) An optical pre-amplifier is used then added in front of the photodiode with 6dB noise figure and very high optical gain ($G >>1$). Signal-ASE beat noise is considered as the major noise in the receiver (neglect other noise sources). What is the maximum allowable fiber length $L_{max}$ in this case?

Solution:

(a) At the optical receiver, thermal noise is constant which is

$$i_{th}^2 = \frac{4kTB_e}{R_L} = \frac{4 \times 1.38 \times 10^{-23} \times 300}{50} \times 5 \times 10^9 = 1.66 \times 10^{-12} W / Hz$$

$$\sigma_0 = \sigma_i = \sqrt{1.66 \times 10^{-12}} = 1.29 \times 10^{-6} \sqrt{W / Hz}$$

Normalized signal level at digital "1" and "0" are

$v_i = 2\Re P_{in}$ and $v_0 = 0$

For a receiver $Q = 7$ (corresponding to $BER = 10^{-12}$),

$$Q = \frac{v_i - v_0}{\sigma_0 + \sigma_i} = \frac{2\Re P_{in}}{2 \times 1.29 \times 10^{-6}} = 7$$

$$P_{ave} = 7 \times 1.29 \times 10^{-6} = 9 \times 10^{-6} W = 9 \times 10^{-3} mW = -20.5 dBm$$

Since the transmitter power is 6dBm, and the fiber loss is 0.25dB/km, the maximum fiber length is,

$$L = \frac{20.5}{0.25} = 82 km$$

(b) Optical amplifier noise figure of 6dB is equivalent to $n_{sp} = 2$.
At signal "1", noise power is \( i_{S-ASE,1}^2 = 4n_sp \nu R^2 P_1 G(G-1)B_e \), so that noise standard deviation is 
\[
\sigma_{S-ASE,1} = \sqrt{4n_sp \nu R^2 \nu P_1 G(G-1)B_e} = \sqrt{8n_sp \nu R^2 P_{ave} G(G-1)B_e}
\]
At signal "0", \( i_{S-ASE,0}^2 = 0 \), and \( \sigma_{S-ASE,0} = 0 \)
\[
Q = \frac{v_1 - v_0}{\sigma_{S-ASE,1} + \sigma_{S-ASE,0}} = \frac{9RG\nu}{\sigma_{S-ASE,1}} = \frac{29RG_{ave}}{\sqrt{8n_sp \nu R^2 P_{ave} G(G-1)B_e}} \approx \sqrt{\frac{P_{ave}}{2n_sp \nu B_e}} = 7
\]
\[
P_{ave} = 49 \times 2n_sp \nu B_e = 49 \times 2 \times 2 \times 1.28 \times 10^{-19} \times 5 \times 10^9 = 1.25 \times 10^{-7} W = 1.25 \times 10^{-4} mW
\]
This is \( P_{ave} = -39 dBm \)

The maximum allowable fiber length is, \( L = \frac{39}{0.25} = 156 km \)

2. A long distance fiber-optic system operating at 1550nm wavelength has \( N \) in-line optical amplifiers each with 6dB noise figure. Each fiber span has the same fiber length \( L = 80 km \) and loss parameter \( \alpha = 0.25 dB/km \). The gain \( G \) of each optical amplifier exactly compensates the loss of the optical fiber span immediately before it. The signal optical power that enters the first fiber span is \( P_{tx} = 1mW \).

(a) If optical signal to noise ratio (OSNR) is defined as the signal optical power divided by the optical noise power measured within 0.1nm optical bandwidth, and assume \( N = 10 \), what is the OSNR at the input of the photodiode.

(b) Assume the system carries 10Gb/s NRZ binary data signal, the photodiode has responsivity \( R = 1A/W \) and electric bandwidth \( B_e = 10GHz \). Only consider signal-ASE beat noise at the receiver, in order to satisfy BER < 10\(^{-12}\), what is the maximum number of fiber spans, \( N \), this transmission system can allow?

Solution

(a) Since the loss of each fiber span is compensated by the gain of optical amplifier, the accumulated optical noise PSD at the receiver is 
\[
\rho_{ASE} = \sum 2n_sp \nu (G-1) = 2Nn_sp \nu (G-1) = 20n_sp \nu (G-1) \text{ in } [W/Hz]
\]
where \( G = \alpha L = 0.25 \times 80 = 20 dB \)
\[
\rho_{ASE} = 20 \times 2 \times 1.28 \times 10^{-19} \times 99 = 5.07 \times 10^{-16} \text{ [W/Hz]}
\]
0.1nm is equivalent to \( B_o = \frac{c}{\lambda^2} \times 0.1 \times 10^{-9} = \frac{3 \times 10^8}{(1550 \times 10^{-9})^2} \times 0.1 \times 10^{-9} = 12.5 GHz \)
\[
P_{ASE} = \rho_{ASE} B_o = 5.07 \times 10^{-16} \times 12.5 \times 10^9 = 6.33 \times 10^{-8} W
\]
As the signal optical power arriving at the receiver is 1mW, the OSNR is
\[
\frac{P_{\text{ASE}}}{P_{\text{ASE}}} = \frac{10^{-3}}{6.33 \times 10^{-6}} = 158 \approx 22\text{dB}
\]

(b) For the receiver

At signal "1", noise power is \(i_{S-\text{ASE},1}^2 = 29\Re P_1 \rho_{\text{ASE}} B_e \), so that noise standard deviation is

\[
\sigma_{S-\text{ASE},1} = 9\Re \sqrt{2 P_1 \rho_{\text{ASE}} B_e} = 29 \Re \sqrt{P_{\text{ave}} \rho_{\text{ASE}} B_e},
\]

where \(P_1 = 2P_{\text{ave}}\) without distortion

At signal "0", \(i_{S-\text{ASE},0}^2 = 0\), and \(\sigma_{S-\text{ASE},0} = 0\)

\[
Q = \frac{v_1 - v_0}{\sigma_{S-\text{ASE},1} + \sigma_{S-\text{ASE},0}} = \frac{9\Re P_1}{\sigma_{S-\text{ASE},1}} = \frac{29 \Re P_{\text{ave}}}{\sigma_{S-\text{ASE},1}} \approx \frac{P_{\text{ave}}}{\rho_{\text{ASE}} B_e} = 7
\]

\[
\rho_{\text{ASE}} = \frac{P_{\text{ave}}}{49 B_e} = \frac{10^{-3}}{49 \times 10^{10}} = 2.04 \times 10^{-15} \text{ [W/Hz]}
\]

Since each EDFA contributes

\[
\rho_{\text{ASE},i} = 2n_e \hbar (G - 1) = 2 \times 2 \times 1.28 \times 10^{-19} \times 99 = 5.07 \times 10^{-17} \text{ in [W/Hz]}
\]

The number of EDFA should be no more than

\[
N \leq \frac{P_{\text{ASE}}}{\rho_{\text{ASE},i}} = \frac{2.04 \times 10^{-15}}{5.07 \times 10^{-17}} = 40
\]