**Distortion mask based on normalized eye**

- **Noise + waveform distortion**
- **Only waveform distortion**

- **$P_1$:** Amplitude of long (continuous) “1” (normalized to 1)
- **$P_0$:** Normalized amplitude of long (continuous) “0”
- **$W$:** Jitter window

**Legend:**
- $A$: normalized lowest “1” level
- $B$: Normalized highest “0” level

*R. Hui*
Normalized eye opening versus average power

Average power (can be measured by a slow power meter) is determined by long “1”s and long “0”s. Assume equal probability of “0”s and “1”s:

Use $P_{ave}$ as a normalization factor, we define, $A$ and $B$ as normalized eye opening parameters:

1) Ideally without waveform distortion, $A = 1$ and $B = 0$, (so that $2P_{ave}A = P_1$, $2P_{ave}B = P_0 = 0$)
2) With waveform distortion, $A < 1$ and $B > 0$, (so that $2P_{ave}A < P_1$, $2P_{ave}B > P_0$)

To calculate $Q$: $Q = \frac{v_1 - v_0}{\sigma_1 + \sigma_0}$ we need to use $v_1 = 2RP_{ave}A$ $v_0 = 2RP_{ave}B$

Note: this is in the electric domain

R. Hui
Three types of optical receivers

(1) Simple optical system without optical amplifier:

(2) System without inline optical amplifier, but with an optical pre-amplifier as part of the receiver

(3) Long haul optical system with multiple inline optical amplifiers

(1) and (2) can be in the same category by setting $G = 1$
**Receiver sensitivity:**

Minimum signal optical power $P_{in}$ required at the receiver to achieve a specified BER, such as $10^{-9}$, $10^{-12}$, or $10^{-15}$

Assume the received average optical signal power is $P_{ave}$ before the pre-amplifier

**Signal:** \[v_1 - v_0 = 2\Re G P_{ave} (A - B)\]

**Noise:**
\[
\begin{align*}
\sigma_1 &= \sqrt{\left(\sigma_{th}^2 + \sigma_{sh,1}^2 + \sigma_{dk}^2 + \sigma_{S-ASE,1}^2 + \sigma_{ASE-ASE}^2 + \sigma_{RIN,1}^2\right) B_e} \\
\sigma_0 &= \sqrt{\left(\sigma_{th}^2 + \sigma_{sh,0}^2 + \sigma_{dk}^2 + \sigma_{S-ASE,0}^2 + \sigma_{ASE-ASE}^2 + \sigma_{RIN,0}^2\right) B_e}
\end{align*}
\]

with
\[
\begin{align*}
\sigma_{th}^2 &= 4kT / R_L \\
\sigma_{dk}^2 &= 2qI_{dk} \\
\sigma_{ASE-ASE}^2 &= \Re^2 \rho_{ASE}^2 B_0 / 2
\end{align*}
\]

**Signal-independent noise**

**Signal-dependent noise** associated with signal $2P_{ave}A$

**Signal-dependent noise** associated with signal $2P_{ave}B$

\[
Q = \frac{v_1 - v_0}{\sigma_1 + \sigma_0}
\]

R. Hui
**Thermal noise limited receiver sensitivity:**
This is usually for simple optical receiver without optical pre-amplifier (or G = 1)

Signal: \( v_1 - v_0 = 2 \Re P_{\text{ave}}(A - B) \)

Noise (only consider thermal noise): \( \sigma_0 = \sigma_1 = \sqrt{4kT B_e / R_L} \)

\[
Q = \frac{v_1 - v_0}{\sigma_1 + \sigma_0} = \frac{2 \Re P_{\text{ave}}(A - B)}{2 \sqrt{4kT B_e / R_L}} = \frac{\Re P_{\text{ave}}(A - B)}{\sqrt{4kT B_e / R_L}}
\]

Or,
\[
P_{\text{ave}} = \frac{Q \sqrt{4kT B_e / R_L}}{\Re(A - B)} = P_{\text{sen,th}}
\]

If we set \( Q = 7 \) (BER = 10\(^{-12}\)) as the targeted system performance, the required signal average power (receiver sensitivity) will be:

\[
P_{\text{ave}} = \frac{7 \times \sqrt{4kT B_e / R_L}}{\Re(A - B)}
\]

Electric SNR is:
\[
SNR = \frac{(\Re P_{\text{ave}})^2}{4kT B_e / R_L} = \left( \frac{\Re P_{\text{ave}}}{\sqrt{4kT B_e / R_L}} \right)^2
\]

\(Q\)-value is related to \(SNR\) as:
\[
Q = \sqrt{SNR(A - B)}
\]
Thermal noise limited receiver sensitivity

Example:

Assume a 10Gb/s system with:
\( B_e = 7.5 \text{GHz}, \) load resistance \( R_L = 50\Omega, \) photodiode responsivity \( R = 0.85, \) \( T = 300^\circ\text{K} \)

Targeted system performance \( Q = 7. \)

(1) In the ideal case there is no waveform distortion: \( A = 1, B = 0 \)

\[
P_{\text{ave}} = \frac{7 \times \sqrt{4kB_e R}}{R(A-B)} = \frac{7 \times \sqrt{4 \times 1.28 \times 10^{-23} \times 300 \times 7.5 \times 10^9 / 50}}{0.85} = 1.25 \times 10^{-5} W = -19 \text{dBm}
\]

In this case, the required \( \text{SNR} \) can be found as: \( \text{SNR} = Q^2 = 49 = 16.9 dB \)

(2) Waveform distortion results in \( A = 0.8, B = 0.2, \) more signal power is required to reach \( Q = 7 \)

\[
P_{\text{ave}} = \frac{7 \times \sqrt{4kB_e R}}{R(A-B)} = \frac{7 \times \sqrt{4 \times 1.28 \times 10^{-23} \times 300 \times 7.5 \times 10^9 / 50}}{0.85 \times (0.8 - 0.2)} = 2.08 \times 10^{-5} W = -16.8 \text{dBm}
\]

In this case, the required \( \text{SNR} \) can be found as: \( \text{SNR} = \left( \frac{Q}{A - B} \right)^2 = \left( \frac{7}{0.6} \right)^2 = 136 = 21.3 dB \)

Due to waveform distortion \( (A = 0.8, B = 0.2), \) the receiver requires 
2.2dB higher optical signal power, or equivalently 4.4dB higher \( \text{SNR} \)
**Shot noise limited receiver sensitivity:**

This is for an ideal photodiode with only shot noise

Signal: \( v_1 - v_0 = 2\Re P_{ave}(A - B) \)

Noise (only consider shot noise):

\[
\begin{align*}
\sigma_1 &= \sqrt{4qB_e \Re P_{ave} A} \\
\sigma_0 &= \sqrt{4qB_e \Re P_{ave} B}
\end{align*}
\]

\[
Q = \frac{v_1 - v_0}{\sigma_1 + \sigma_0} = \frac{2\Re P_{ave}(A - B)}{\sqrt{4qB_e P_{ave} A + 4qB_e P_{ave} B}} = \sqrt{\frac{\Re P_{ave}}{qB_e} \left( \frac{\sqrt{A} - \sqrt{B}}{\sqrt{A} + \sqrt{B}} \right)}
\]

So that the quantum-limited receiver sensitivity is:

\[
P_{ave} = \frac{qB_e Q^2}{\Re \left( \frac{Q}{\sqrt{A} - \sqrt{B}} \right)} = P_{sen,q}
\]

If we assume that the time of each bit is \( T_0 = 1/B_e \), and the photodiode has 100% quantum efficiency, this quantum-limited sensitivity can be expressed as the required number of photons per data bit \( N_p \):

Without waveform distortion: \( P_{ave} = \frac{qB_e}{q/(hf)} Q^2 = \frac{hf}{T_0} Q^2 \rightarrow N_p = \frac{P_{ave}}{hf} T_0 = Q^2 \)

To achieve \( Q = 7 \), at least 49 photons are required to carry each data bit. This is known as *quantum limit*
Shot noise limited receiver sensitivity

Example:

Assume a 10Gb/s system with: 
\( B_e = 7.5 \text{GHz} \), load resistance \( R_L = 50 \Omega \), photodiode responsivity \( R = 0.85 \), \( T = 300^\circ \text{K} \)

Targeted system performance \( Q = 7 \).

(1) In the ideal case there is no waveform distortion: \( A = 1, B = 0 \)

\[
P_{\text{ave}} = \frac{qB_e}{R} Q^2 = \frac{1.6 \times 10^{-19} \times 7.5 \times 10^9}{0.85} \times 49 = 6.9 \times 10^{-8} W = -41.6 \text{dBm}
\]

(2) Waveform distortion results in \( A = 0.8, B = 0.2 \), more signal power is required to reach \( Q = 7 \)

\[
P_{\text{ave}} = \frac{qB_e}{R} \left( \frac{Q}{\sqrt{A} - \sqrt{B}} \right)^2 = \frac{1.6 \times 10^{-19} \times 7.5 \times 10^9}{0.85 \times (0.8944 - 0.4472)^2} \times 49 = 3.46 \times 10^{-7} W = -34.6 \text{dBm}
\]

- Due to waveform distortion \( (A = 0.8, B = 0.2) \), the receiver requires \( 3.5 \times 2 = 7 \text{dB} \) higher optical signal power.

- In comparison to thermal noise limit, quantum-noise limited receiver requires \( 22.6 \text{dB} \) less signal power (-19dBm versus -41.6dBm), but the penalty due to waveform distortion is increased \( (2.2 \text{dB} \text{ versus } 7 \text{dB}) \)

R. Hui
Sources contributing to receiver sensitivity degradation: simple photodiode receiver

Example with 10Gb/s binary system, no optical amplifier, $B_e = 7.5$ GHz, $R = 0.85 \text{W/A}$, $R_L = 50\Omega$, $I_d = 5\text{nA}$, and $T = 300K$. Note: for $Q = 7$ ($10\log_{10}(Q) = 8.45 \text{dB}$)
Sensitivity of receiver with optical pre-amplifier

Signal: \( v_1 - v_0 = 2\Re G P_{ave} (A - B) \)

Noise: \( \sigma_1 = \sqrt{(\sigma_{sh,1}^2 + \sigma_{S-ASE,1}^2)} B_e \quad \sigma_0 = \sqrt{(\sigma_{sh,0}^2 + \sigma_{S-ASE,0}^2)} B_e \)

- As optical signal is amplified by \( G \), shot noise is much bigger than thermal noise and dark current noise.
- Assume optical bandwidth \( B_0 \) is narrow, ASE-ASE beat noise is much smaller than signal-ASE beat noise.
- In optically amplified optical receiver, shot noise and signal-ASE beat noise are major noise sources.
- Because \( B_0 \) is narrow, shot noise generated by ASE (proportional to \( B_0 \rho_{ASE} \)) is also neglectable as well.

\[
Q = \frac{v_1 - v_0}{\sigma_1 + \sigma_0} = \frac{2\Re G P_{ave} (A - B)}{\sqrt{4(q\Re + \Re^2 \rho_{ASE}) G P_{ave} B_e A + \sqrt{4(q\Re + \Re^2 \rho_{ASE}) G P_{ave} B_e B}}}
\]

To further simplify, let’s compare the strengths of signal-ASE beat noise and shot noise:

\[
\frac{\Re^2 \rho_{ASE}}{q\Re} = \frac{\Re \rho_{ASE}}{q} = \frac{\rho_{ASE}}{hf} = \frac{2n_{sp} hf (G - 1)}{hf} = 2n_{sp} (G - 1)
\]

For \( G >> 1 \), and \( n_{sp} > 1 \), signal-ASE beat noise is much bigger than shot noise, so that signal-ASE beat noise is the dominate noise source.

R. Hui
Sensitivity of receiver with optical pre-amplifier (only consider signal-ASE beat noise):

\[ Q \approx \frac{2\Re GP_{ave}(A - B)}{\sqrt{4R^2 \rho_{ASE} GP_{ave} B_e A + 4R^2 \rho_{ASE} GP_{ave} B_e B}} = \sqrt{\frac{GP_{ave}}{\rho_{ASE} B_e}}(\sqrt{A} - \sqrt{B}) \]

Further consider \( G \gg 1 \), \( Q \approx \sqrt{\frac{GP_{ave}}{2n_{sp} hf (G - 1) B_e}}(\sqrt{A} - \sqrt{B}) \approx \sqrt{\frac{P_{ave}}{2n_{sp} hf B_e}}(\sqrt{A} - \sqrt{B}) \)

Receiver sensitivity:

\[ P_{ave} = 2n_{sp} hf B_e \left( \frac{Q}{\sqrt{A - B}} \right)^2 = P_{sen, S-ASE} \]

Recall, shot noise limited receiver sensitivity is:

\[ P_{sen,q} = \frac{qB_e}{\Re} \left( \frac{Q}{\sqrt{A - B}} \right)^2 \]

\[ \frac{P_{sen,S-ASE}}{P_{sen,q}} = 2n_{sp} hf \frac{\Re}{q} = 2n_{sp} hf \frac{\eta_q q/(hf)}{q} = 2n_{sp} \eta_q \]

where \( \Re = \eta_q \frac{q}{hf} \)

For an ideal photodiode \( \eta_q = 1 \), and ideal optical amplifier \( n_{sp} = 1 \), optically pre-amplified receiver only needs 3dB (a factor of 2) more signal power optical compared to quantum limit.
In this type of optical system with multiple in-line optical amplifiers, signal optical power $P_{ave}$ that reaches the photodiode can be set as high as one wants, simply by increasing the gain of optical amplifiers, and thus the concept of receiver sensitivity is no longer valid.

But with the increase of amplifier optical gain, ASE noise power spectral density accumulated along the system and arrived at photodiode $\rho_{ASE}$ will be increased.

Receiver electric SNR and Q after photodiode are limited by the optical signal to noise ratio (OSNR) before photodiode.

Required-OSNR (R-OSNR) to achieve the targeted Q-value is the parameter to replace receiver sensitivity.
System performance versus OSNR
(only consider signal-ASE beat noise)

\[
Q = \frac{v_1 - v_0}{\sigma_1 + \sigma_0} = \frac{2\Re P_{ave}(A - B)}{\sqrt{4\Re^2 \rho_{ASE} P_{ave} B e A} + \sqrt{4\Re^2 \rho_{ASE} P_{ave} B e B}} = \sqrt{\frac{P_{ave}}{\rho_{ASE} B e}} \left(\sqrt{A} - \sqrt{B}\right)
\]

That is: \( Q = \sqrt{\frac{OSNR_{1Hz}}{B e}} \left(\sqrt{A} - \sqrt{B}\right) \)

- Practically, optical signal spectra are measured by optical spectrum analyzers, and the spectral resolution.
- 0.1nm spectral resolution (= 125GHz in the 1550nm wavelength window) is commonly used practical in the industry

\[
OSNR_{0.1nm} = \frac{OSNR_{1Hz}}{125 \times 10^9} \quad \Longrightarrow \quad Q = \sqrt{OSNR_{0.1nm}} \frac{12.5 \times 10^9}{B e} \left(\sqrt{A} - \sqrt{B}\right)
\]

For a targeted Q-value, the required-OSNR is

\[
OSNR_{0.1nm} = \frac{B e}{12.5 \times 10^9} \left(\frac{Q}{\sqrt{A} - \sqrt{B}}\right)^2 = ROSNR
\]

This ROSNR is defined with 0.1nm (12.5GHz) resolution bandwidth to measure optical noise power spectral density \( \rho_{ASE} \)

R. Hui
**System performance versus OSNR example**

For a system with: $B_e = 10\text{GHz}$, and targeted system $Q = 7$:

1. In the ideal case there is no waveform distortion: $A = 1$, $B = 0$

$$ROSNR_{0.1\text{nm}} = \frac{B_e Q^2}{12.5 \times 10^9} = \frac{10 \times 10^9 \times 49}{12.5 \times 10^9} = 39.2 = 15.9\text{dB}$$

2. Waveform distortion results in $A = 0.8$, $B = 0.2$, more signal power is required to reach $Q = 7$

$$ROSNR_{0.1\text{nm}} = \frac{10 \times 10^9}{12.5 \times 10^9} \left(\frac{7}{\sqrt{0.8} - \sqrt{0.2}}\right)^2 = 196 = 22.9\text{dB}$$

The 7dB ROSNR increase is due to $10\log_{10}\left(\sqrt{0.8} - \sqrt{0.2}\right) = 10\log_{10}(0.447) = 7\text{dB}$

*Note: this only takes into account the impact of signal-ASE beat noise*
System performance versus OSNR example

Example with $B_e = 10 \text{ GHz}$, $B_o = 25 \text{ GHz}$, $R = 0.85 W/A$, $R_L = 50 \Omega$, $I_d = 5 \text{nA}$, $P_{ave} = 0 \text{ dBm}$, $\lambda = 1550 \text{ nm}$, and $T = 300K$. No waveform distortion ($A = 1$, $B = 0$)
**Required optical signal to noise ratio (R-OSNR)**

$L_1, L_2, L_3, \ldots, L_N$: Attenuation loss (in [dB]) of each fiber span

$G_1, G_2, G_3, \ldots, G_N$: Gain (in [dB]) of each optical amplifier

\[ \rho_1 = \rho_{ASE,1} \times 10^{\left( \frac{\sum_{k=2}^{N} G_k - \sum_{k=2}^{N} L_k}{10} \right)} \]

\[ \rho_2 = \rho_{ASE,2} \times 10^{\left( \frac{\sum_{k=3}^{N} G_k - \sum_{k=3}^{N} L_k}{10} \right)} \]  

If the gain exactly compensates the loss in each span: \( L_k = -G_k \) and assume \( G_1 = G_2 = \ldots = G_N \)

\[ \rho_{ASE} = \sum_{i=1}^{N} \rho_i \]

in [W/Hz]

\[ \rho_{ASE} = \sum_{i=1}^{N} \rho_{ASE,i} = 2n_{sp} hf \sum_{i=1}^{N} (G_i - 1) \approx 2n_{sp} hf \cdot N \cdot G \]

Optical signal to noise ratio (with 1Hz resolution bandwidth):

\[ OSNR_{1Hz} = \frac{P_{ave}}{\rho_{ASE}} = \frac{P_{ave}}{2n_{sp} hf \cdot N \cdot G} \]  
in [W * Hz]
ASE noise accumulation along a system

\[
\text{OSNR}_{1\text{Hz}} = \frac{P_{\text{ave}}}{\rho_{\text{ASE}}} = \frac{P_{\text{ave}}}{2n_{\text{sp}}hf \cdot N \cdot G}
\]

Assume the total loss of the fiber system is \( L_{\text{total}} \) in [dB] equally divided into \( N\)-span, optical gain of each amplifier is \( G = L_{\text{total}}/N \) in [dB]. Converting into linear scale this gain is

\[
G = 10^{\frac{L_{\text{total}}}{10N}} \quad \text{Then,} \quad P_t \text{ is the Tx power (}= P_{\text{ave}} \text{ in this case)}
\]

\[
\text{OSNR}_{0.1\text{nm}} = \left( \frac{1}{12.5 \times 10^9} \right) \frac{P_t}{2hvn_{\text{sp}}N \times 10^{10N}}
\]

For the same total system distance, shorter fiber spans and larger number of amplifiers help improving OSNR, but would increase cost

Example: EDFA noise figure \( F = 5\text{dB} \) \((n_{\text{sp}} = 1.58)\), fiber loss \( \alpha = 0.25\text{dB/km} \), and \( P_t = P_{\text{ave}} = 1\text{mW} \)