Part 2 (solution)


The nonlinear phase shift in Equation (1) should be (here fiber loss is neglected)

\[ \varphi = \gamma |E|^2 L = \frac{2\pi n_2 L}{A_{\text{eff}} \lambda} |E|^2 \]

Where \( E \) is the optical field, \( n_2 \) is the nonlinear refractive index, \( A_{\text{eff}} \) is the effective cross section area of the fiber (so the \( |E|^2 / A_{\text{eff}} \) is the power density), \( \lambda \) is the signal wavelength, and \( L \) is the loop length.

(1) (20 pts) Assume the input optical field is \( E_{01}^+ = E_{IN} \) at the 1st port, and there is no signal entering the 2nd port (\( E_{02}^+ = 0 \)). The fiber in the loop has an attenuation parameter \( \alpha_{Np} \) in \([\text{Neper/m}]\). Please derive the equations of nonlinear transmission

\[ T = \left| \frac{E_2^+}{E_1^+} \right|^2 \]

and nonlinear reflection \( R = \left| \frac{E_1^+}{E_1^+} \right|^2 \)

(2) (30 pts) The loop is made of standard single mode fiber which has

\( n_2 = 3.2 \times 10^{-16} \text{ cm}^2 / \text{W} \), \( A_{\text{eff}} = 80 \mu \text{m}^2 \) and attenuation parameter \( \alpha_{Np} = 0.02 \text{Neper/m} \).

Loop length is \( L = 5 \text{m} \) and assume that there is no chromatic dispersion.

(a) Plot power reflectivity \( R \) and transmission \( T \) as the function of splitting ratio \( \alpha \) for the signal power levels of 0W, 200W, 400W and 600W, respectively.

(b) Assume \( \alpha = 0.45 \) and the input optical pulse is Gaussian with 500fs FWHM width and 200W peak power, please (use Matlab or Mathematica) plot input pulse, reflected pulse and transmitted pulse (in the same figure for comparison of their shapes), and find their FWHM width.

(c) Design an optical sampling system based on the nonlinear loop-mirror and using a mode-locked laser as the sampling pulse source. Draw necessary block diagram and explanation of how it works. If the maximum pulse peak power is 500W, what is the best splitting ratio \( \alpha \) you would like to use? (explain the reason).

Solution:

(1) Derive equations:

\[
\begin{bmatrix}
    b_1 \\
    b_2
\end{bmatrix} = \begin{bmatrix}
    \sqrt{1-\alpha} & j \sqrt{\alpha} \\
    j \sqrt{\alpha} & \sqrt{1-\alpha}
\end{bmatrix} \begin{bmatrix}
    E_1^+ \\
    0
\end{bmatrix}
\]

that is,

\( b_1 = E_1^+ \sqrt{1-\alpha} \), \( b_2 = E_1^+ j \sqrt{\alpha} \)

For the loop delay,

\( c_1 = b_2 e^{-\alpha \lambda t} e^{i \phi} \), \( c_2 = b_1 e^{-\alpha \lambda t} e^{i \phi} \)
Where \( \phi_1 = \gamma \int_0^L |b_2|^2 e^{-a_z} dz = \gamma |b_1|^2 \frac{1-e^{-a_L}}{a_L} \frac{2\pi n_2}{A_{\text{eff}} \lambda} \alpha |E_1^r|^2 \frac{1-e^{-a_L}}{a_L} \)

\( \phi_2 = \gamma \int_0^L |b_1|^2 e^{-a_z} dz = \gamma |b_1|^2 \frac{1-e^{-a_L}}{a_L} \frac{2\pi n_2}{A_{\text{eff}} \lambda} (1-\alpha) |E_1^r|^2 \frac{1-e^{-a_L}}{a_L} \)

\( \Delta \phi = |\phi_1 - \phi_2| = \frac{2\pi n_2}{A_{\text{eff}} \lambda} (1-2\alpha) |E_1^r|^2 \frac{1-e^{-a_L}}{a_L} \)

\[
\begin{bmatrix}
E_1^- \\
E_2^+
\end{bmatrix} = \begin{bmatrix}
\sqrt{1-\alpha} & j\sqrt{\alpha} \\
j\sqrt{\alpha} & \sqrt{1-\alpha}
\end{bmatrix}
\begin{bmatrix}
c_1 \\
c_2
\end{bmatrix} = \begin{bmatrix}
\sqrt{1-\alpha} & j\sqrt{\alpha} \sqrt{1-\alpha} \\
j\sqrt{\alpha} & \sqrt{1-\alpha}
\end{bmatrix}
\begin{bmatrix}
E_1^+ j\sqrt{\alpha} e^{-a_L e^{j\delta}} e^{j\theta_1} \\
E_1^+ \sqrt{1-\alpha} e^{-a_L e^{j\delta}} e^{j\theta_1}
\end{bmatrix}
\]

So that,

\[
E_1^- = E_1^+ \sqrt{1-\alpha} j\sqrt{\alpha} e^{-a_L e^{j\delta}} e^{j\theta_1} + jE_1^+ \sqrt{1-\alpha} \sqrt{1-\alpha} e^{-a_L e^{j\delta}} e^{j\theta_1} = jE_1^+ \sqrt{\alpha} (1-\alpha) e^{-a_L} \left( e^{j\theta_1} + e^{j\theta_2} \right)
\]

\[
|E_1^-|^2 = |E_1^+|^2 4\alpha (1-\alpha) e^{-2a_L \cos^2(\Delta \phi/2)} = |E_1^+|^2 2\alpha (1-\alpha) e^{-2a_L [1 + \cos(\Delta \phi)]}
\]

Thus,

\[
R = \frac{|E_1^-|^2}{|E_1^+|^2} = 2\alpha (1-\alpha) e^{-2a_L [1 + \cos(\Delta \phi)]}
\]

\[
E_2^+ = -E_1^+ \alpha e^{-a_L e^{j\delta}} e^{j\theta_1} + E_1^+ (1-\alpha) e^{-a_L e^{j\delta}} e^{j\theta_1} = E_1^+ e^{-a_L e^{j\delta}} \left( (1-\alpha) e^{j\theta_1} - \alpha e^{j\theta_1} \right)
\]

\[
|E_2^+|^2 = |E_1^+|^2 e^{-2a_L \alpha e^{j\theta_1/2} - \alpha e^{-j\theta_1/2}} - e^{j\theta_1/2} \right|^2
\]

\[
|E_2^+|^2 = |E_1^+|^2 4\alpha \cos^2\left(\frac{\Delta \phi}{2}\right) - e^{j\theta_1/2} \right|^2 = |E_1^+|^2 4\alpha \cos^2\left(\frac{\Delta \phi}{2}\right) - 2\alpha \cos^2\left(\frac{\Delta \phi}{2}\right) (e^{j\theta_1/2} + e^{-j\theta_1/2}) + 1
\]

\[
|E_2^+|^2 = |E_1^+|^2 4\alpha \cos^2\left(\frac{\Delta \phi}{2}\right) - 4\alpha \cos^2\left(\frac{\Delta \phi}{2}\right) + 1 = |E_1^+|^2 e^{-2a_L \alpha} \left[ 1 - 4\alpha (1-\alpha) \cos^2\left(\frac{\Delta \phi}{2}\right) \right]
\]

Thus,

\[
T = \frac{|E_2^+|^2}{|E_1^+|^2} = e^{-2a_L \alpha} \left[ 1 - 2\alpha (1-\alpha) \right][1 + \cos(\Delta \phi)]
\]

To check energy conservation, the total output (reflection + transmission) is:

\[
R + T = 2\alpha (1-\alpha) e^{-2a_L \alpha} [1 + \cos(\Delta \phi)] + e^{-2a_L \alpha} [1 - 2\alpha (1-\alpha) \cos(\Delta \phi)] = e^{-2a_L \alpha}
\]

which is equal to the loss of the fiber ring.
(2),

(a) For loop length $L = 5\text{m}$, Figure 1 shows the transmission ($T$) and reflection ($R$) at optical power levels of 0W (linear case), 200W, 400W and 600W, respectively, as the function of coupler splitting ratio.

![Figure 1](image1.png)

Figure 1, Transmission ($T$) and reflection ($R$) at optical power levels of 0W (linear case), 200W, 400W and 600W, respectively, as the function of coupler splitting ratio

(b) For $\alpha = 0.45$, a Gaussian pulse with 500fs FWHM width and 200W peak power will have transmitted and reflected pulse shapes shown in top row of Figure 2. The transmitted pulse has much lower peak power because of the high loss due to reflection, but the pulse width is slightly narrower than the initial pulse. When the pulse peak power is higher, for example at 600W, the transmitted pulse is more compressed as shown in the bottom row of Figure 2.
Figure 2, Left side: shapes of input $P_{in}(t)$, transmitted pulse $T(t)$, and reflected pulse $R(t)$ with input pulse peak power of 200W (top) and 600W (bottom). Right side: shapes of normalized transmission $[T(t)/T_{max}(t)]$, and normalized reflection $[R(t)/R_{max}(t)]$ with input pulse peak power of 200W (top) and 600W (bottom).

(c) The optical sampling system can be constructed with a nonlinear fiber loop mirror, and the configuration is shown in Figure 3.

Figure 3, Block diagram of an optical sampling system based on a nonlinear loop mirror.
Figure 4 shows the nonlinear transmission of a loop mirror with coupler splitting ratio of $\alpha = 0.45$, where the pulse peak transmission $T$ is a function of power. The right column of Figure 4 shows the ratio between nonlinear and linear transmission in dB. When the splitting ratio $\alpha$ approaches 0.5, the extinction ratio is approximately 6.8 dB for the peak power of 500 W for the nonlinear switch. But at this point ($\alpha = 0.5$) the transmission loss is too high (close to infinite). We can choose $\alpha = 0.4$ where we still have an extinction ratio of about 6.5 dB, but much lower loss.

If the maximum pulse power is increased to 1 kW, the extinction ratio can be increased to 11 dB at $\alpha = 0.4$, and the power transmission can reach to about 40% at the pulse peaks.