Improved Rate Equations for External Cavity Semiconductor Lasers

HUI RONG-QING AND TAO SHANG-PING

Abstract—**A set of improved Langevin rate equations for external cavity semiconductor lasers is derived analytically, which can be used to solve transient as well as steady-state problems. A simple relationship among chirp reduction, line narrowing, and dynamic stability is given. An experiment on an external coupled cavity semiconductor laser consisting of a 1.3 μm BH laser diode with an AR-coated facet and a 4 mm GRINROD is reported. Because of its strong optical feedback, it provides a stable single mode, which is not sensitive to the environmental perturbations. This agrees with the theoretical results. No mode jumping occurred during 12 h continuous observation.**

I. Introduction

At present, the external cavity semiconductor laser is a feasible means to realize a single-frequency semiconductor laser. In order to develop its theory, an improved form of differential equations which describe the behavior of the external cavity semiconductor laser (ECSL) is needed. This problem has been treated by many authors [1]–[4]. Recently, the works of Sato and Ohya [5], Hjelme and Mickelson [6], and Kazarinov and Henry [7] have made noticeable progress, especially the last authors, whose formulas can be used for semiconductor lasers with strong and weak feedback. However, their equations are restricted to solve the transient problems and cease to be effective in the steady state. We have also obtained a set of improved rate equations [8] in this paper. The derivation in [7] is mainly from the physical property of the slope of the phase curve and loss curve of ECSL, but ours are derived directly from the three-mirror cavity theory and analytical calculation of effective reflectivity. The result can be used in the transient as well as steady state of ECSL, which enables us to analyze these problems systematically in this paper. The stability of ECSL is also studied with the improved rate equations.

II. Effective Reflection Coefficient

Our ECSL model is a three-mirror cavity laser as in [5], shown in Fig. 1. In order to improve the differential equations of ECSL such as [4, eq. (40)], let us derive the optical feedback term from the external mirror analytically. Following the idea of Osmundsen [2], the reflective coefficient of the right facet of the equivalent single cavity laser is $r_e = r_2 f$; here, we call f the complex feedback parameter. Considering multiple reflections in the external cavity, r_e can be calculated as follows:

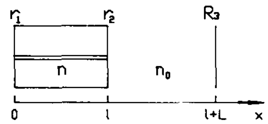

Fig. 1. External cavity semiconductor laser configuration.

$$r_e = r_2 + R_3(1 - r_2^2) \sum_{p=1}^{\infty} \sqrt{\frac{I(t - p\tau)}{I(t)}} (-r_2 R_3)^{p-1} \cdot \exp\left\{i\left[p\omega\tau + \phi_n(t) - \phi_n(t - p\tau)\right]\right\} \tag{1}$$

p is an integer (pth roundtrip)

where $I(t)$ is light intensity measured in photons, $\phi_n(t)$ is a random phase deviation, τ is a roundtrip time delay in the external cavity, $R_3 = Ar_3$, A represents coupling and absorption losses in the external cavity, ω is angular frequency, and

$$\beta(t) = \sqrt{I(t)} \exp\left\{-i\left[\omega t + \phi_n(t)\right]\right\}. \tag{2}$$

As $I(t)$ and $\phi_n(t)$ are random variables in stationary stochastic processes, denoting them by $X(t)$, they have the following property:

$$X[t - (n+1)\tau] - X(t - n\tau) = X(t - \tau) - X(t). \tag{3}$$

A simple expression for f can then be obtained in the following form:

$$f = \frac{1 + \dfrac{R_3}{r_2}\sqrt{\dfrac{I(t-\tau)}{I(t)}} \exp\left\{i\left[\varphi + \phi_n(t-\tau) - \phi_n(t)\right]\right\}}{1 + r_2 R_3 \sqrt{\dfrac{I(t-\tau)}{I(t)}} \exp\left\{i\left[\varphi + \phi_n(t-\tau) - \phi_n(t)\right]\right\}} \tag{4}$$

where $\varphi = \omega\tau$.

Usually, the length of the external cavity is much shorter than the coherent length of the laser; by linear ap-

Manuscript received September 13, 1988; revised January 11, 1989.
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IEEE Log Number 8927348.

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proximation, the following expression can be obtained:

$$Lgf = (H_1 + iP_1) + \tau(H_2 + iP_2) \cdot [\dot{I}(t)/2I(t) - i\dot{\phi}_n(t)] \quad (5)$$

where

$$H_1 = \frac{1}{2}\ln\left[\frac{1 + 2(R_3/r_2)\cos\varphi + (R_3/r_2)^2}{1 + 2r_2R_3\cos\varphi + (r_2R_3)^2}\right] \quad (5a)$$

$$P_1 = \tan^{-1}\left[\frac{R_3(1 - r_2^2)\sin\varphi}{r_2(1 + R_3^2) + R_3(1 + r_2^2)\cos\varphi}\right] \quad (5b)$$

$$H_2 = -\frac{1}{\tau}\, dP_1/d\omega \quad (5c)$$

$$P_2 = -\frac{1}{\tau}\, dH_1/d\omega. \quad (5d)$$

III. Improved Langevin Rate Equations for ECSL

The conventional form of a differential equation of complex field $\beta(t)$ in ECSL is [4]

$$\dot{\beta}(t) = [-i\Omega + \tfrac{1}{2}\Delta G(1 - i\alpha)]\beta(t) + k\beta(t - \tau) + F_\beta(t) \quad (6)$$

in which a time delayed term represents optical feedback from the external mirror; however, the multiple reflections cannot be included, so (6) can only be used in weak feedback. In order to overcome this difficulty and to modify (6), we take the following procedure. In the equation of $\beta(t)$ of a solitary laser, a term representing loss of external cavity is added; this loss is involved with $\ln(r_1r_2f)$; as $\ln(r_1r_2f) = \ln(r_1r_2) + \ln(f)$ where the first term is already contained in the solitary laser's equation, so that $\ln f$ is only needed to be added to it; this is

$$\dot{\beta}(t) = \left[-i\Omega + \tfrac{1}{2}\Delta G(1 - i\alpha) + \frac{1}{\tau_i}\ln f\right]\beta(t) + F_\beta(t) \quad (7)$$

where τ_i is the roundtrip time delay in the diode cavity.

For easily investigating the properties of light intensity and phase deviation of ECSL, by using (4), we can transfer the complex variables differential equation (7) into two coupled real variable differential equations of $I(t)$ and $\phi_n(t)$ as follows:

$$[1 + (\tau/\tau_i)H_2]\dot{I}(t) = I(t)\left[\Delta G + \frac{2}{\tau_i}H_1 - \frac{2\tau}{\tau_i}P_2\dot{\phi}_n(t)\right] + R_{sp} + F_I(t) \quad (8a)$$

$$[1 + (\tau/\tau_i)H_2]\dot{\phi}_n(t) = (\Omega - \omega) + \frac{\alpha}{2}\Delta G - \frac{1}{\tau_i}P_1 + \frac{\tau\dot{I}(t)}{2\tau_iI(t)}P_2 + F_\phi(t) \quad (8b)$$

where R_{sp} is the factor of spontaneous emission.

Equations (8a) and (8b) are a set of nonlinear Langevin rate equations for semiconductor lasers with any amount of external feedback. They are effective in dealing with problems of both the transient as well as steady state systematically, and the derivation is more clear and straightforward.

IV. Discussion

A. Steady-State Condition

The steady-state conditions are important in determining the longitudinal mode behavior of ECSL. However, [7, eq. (21), (22)] cease to be effective in the steady state. Here, the steady-state conditions can be found from the basic equation (8) by setting all the derivatives to time t to be zero. This results in the following equations determining ΔG and ω:

$$\Delta G = -\frac{2}{\tau_i}H_1 \qquad \text{gain condition} \quad (9a)$$

$$\omega = \Omega + \frac{1}{\tau_i}(\alpha H_1 - P_1) \qquad \text{phase condition.} \quad (9b)$$

The steady-state problems have been studied in detail in [2] where the enhancement factor α was ignored. Here, with α being taken into account, kinks appeared in strong optical feedback in the phase curve as shown in Fig. 2 (case 3). This is because of the coupling between the gain and the phase fluctuations which has a mean effect on the steady-state conditions. Moreover, the undulation amplitude of the phase condition curve decreased in the strong feedback; this facilitates the single-mode oscillation. In a weak feedback situation, (9) reduces to [9, eq. (4-5)].

B. Chirp Reduction

In semiconductor lasers, the central frequency of oscillation depends on the carrier density in the active region, which is a function of injection current; its variation causes chirp. In high-bit-rate fiber optical communication systems, low chirp is required; however, for coherent FSK or PSK systems, the situation is quite different. Chirp can be reduced by external optical feedback. In small-signal modulation, the laser's steady-state condition does not change very much; then the chirp reduction caused by the external optical feedback can be simply evaluated by differentiating (9b) with respect to ω and using (5c)–(5d):

$$F = \frac{d\Omega}{d\omega} = 1 + \frac{\tau}{\tau_i}(H_2 - \alpha P_2) \quad (10)$$

where $d\Omega$ and $d\omega$ are small deviations of central frequency in the solitary laser diode and ECSL, respectively.

C. Linewidth Narrowing

In studying the Lorentzian linewidth of ECSL, $dI(t)/dt = 0$ can be assumed. This is correct when the satellite peaks caused by intensity noise are greatly suppressed [10]. Starting from (8), set $I(t) = I_0$ and using (9b),

$$\dot{\phi}_n(t) = \left[F_\phi(t) - \frac{\alpha}{2I_0}F_I(t)\right]/F.$$

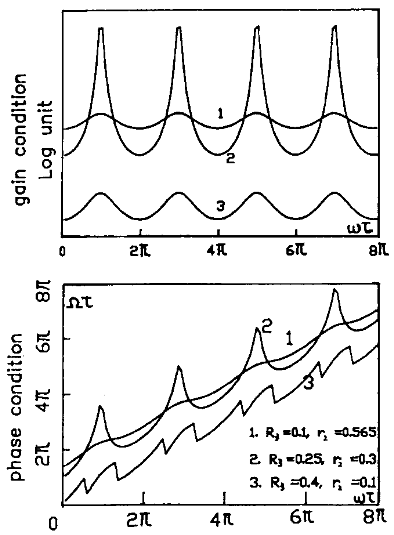

Fig. 2. Steady-state conditions of semiconductor laser with various amounts of optical feedback.

Following the standard treatment as in [3], the linewidth narrowing ratio can be directly obtained:

$$\Delta\nu_0/\Delta\nu = F^2 \tag{11}$$

where $\Delta\nu_0 = R_{sp}(1 + \alpha^2)/4\pi I_0$ is the linewidth in a solitary laser diode. Equation (11) can be verified by the experiment in [17] where the field amplitude reflectivities are $R_3 \sim 0.45$, $r_2 \sim 0.1$, and the length of the external cavity is around 10 mm (a three-quarter-pitch GRANROD). The measured linewidth of the lasers is 500 kHz at 2 mW laser output, while the linewidth-power production of a conventional solitary laser diode is about 150 MHz · mW. This means the line narrowing ratio in [17] is about 150, which roughly agrees with the calculated value 200 from (11). If the weak feedback approximation was used, e.g., [3, eq. (28)], the calculated line narrowing ratio would be 5000; hence, the error would be obvious.

D. Dynamic Stability

Based on the fundamental equation (8) and using the Routh–Hurwitz criterion for nonlinear systems [11], the characteristic equation can be derived as in [12]; in order that its roots are laid on the left-hand side of the complex plane, the following relation should be satisfied [8]:

$$F > 0. \tag{12}$$

From this condition, we can see that the stable region corresponds to the upward side of the phase curve in Fig. 2, which has been pointed out in [13] in weak feedback; here, we extend it to an arbitrary amount of feedback.

E. Stable Regions

From (10)–(12), we can see that chirp reduction and line narrowing are inherently related, and they have a simple relationship with the dynamic stability condition.

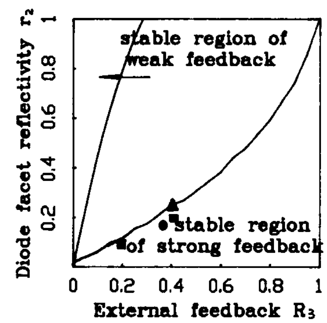

Fig. 3. Different regions of stability. The weak feedback stable region is defined with $L = 4$ mm, while the strong feedback stable region is independent of the external cavity length L. ▲ is the experimental data in [15] to indicate the transition from the unstable to the stable region of strong feedback. ● is the data of a stably operated ECSL described in this paper and ■ are that in [16].

Generally speaking, F depends on r_2, R_3, and φ. In a weak feedback situation,

$$F = 1 + k\tau\sqrt{1 + \alpha^2}\cos(\varphi + \varphi_R) \tag{13}$$

where $\varphi_R = \tan^{-1}\alpha$, $k = R_3(1 - r_2^2)/r_2\tau_i$.

When $k\tau\sqrt{1 + \alpha^2} < 1$, the laser is stable for all feedback phase φ; this is because the feedback is too weak to destroy the laser's stability. The most striking feature is that in very strong feedback, the laser will also operate stably ($F > 0$) for all the phases of feedback, with lower chirp and narrower linewidth ($F > 1$). Under weak optical feedback, the stable region depends on the length of the external cavity L; however, the strong feedback induced stable operating region is independent of the external cavity length. The physical explanation is that when the external feedback is strong enough and the laser's facet reflectivity is reduced to some extent, the boundary between the active and passive cavities becomes less significant; this leads to a quasi-single-cavity laser with a passive and an active section. Thus, the interference between the fields of the two sections becomes weaker; hence, the relative length of the external cavity is no longer important as far as the laser's stability is concerned. The dependence of the stable region on the diode facet reflectivity r_2 and the effective feedback R_3 is shown in Fig. 3. In the strong feedback stable region, the ECSL is immune to external perturbations; this occurs because of the external cavity mode becoming the dominant mode. It has been verified in our experiment of a semiconductor laser with strong optical feedback and some other published works [15]–[16].

V. Experiment

In our experiment, a 4 mm (0.23 pitch) GRINROD with 0.45 NA is used as the external cavity. The front facet of the GRINROD is anti-reflection coated; the rear facet is coated with gold. The laser diode used here is InGaAsP BH type; its facet facing the GRINROD is AR coated in order to get strong optical feedback. The reflectivity of the uncoated laser facet is about 0.565. By measuring the external differentiation quantum efficiency of

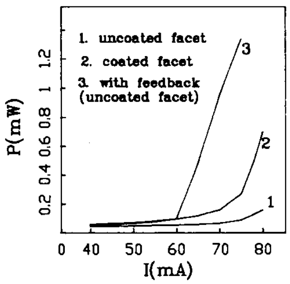

Fig. 4. P-I curve of the laser without optical feedback (1, 2) and with optical feedback (3).

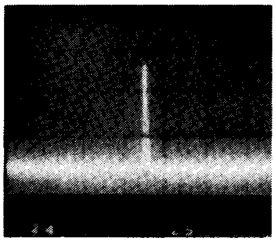

Fig. 5. The spectrum measured by a confocal scanning F-P interferometer 100 MHz/div.

the two facets of the diode [14], the amplitude reflectivity of the coated facet can be estimated to be about 0.16. After adding the external cavity, the threshold current of the diode is reduced from 75 to 62 mA as shown in Fig. 4. The effective external reflectivity can be evaluated as follows:

$$\Delta I_{th}/I_{tho} = \ln\left[1 + (1 - r_2^2)R_3^2/r_2^2\right]/2(\tau l - \ln r_1 r_2) \tag{14}$$

where I_{tho} is the threshold current of the solitary laser diode, ΔI_{th} is the change in threshold current due to external feedback, $r_1 = 0.565$, $r_2 = 0.16$ are the facet amplitude reflectivities of the diode, γ is the effective absorbtion coefficient, $l = 0.25$ mm is the diode cavity length, and $\gamma l = 0.8$ is assumed. From (14), we can estimate R_3 in our configuration to be 0.38. From (11), the linewidth reduction ratio F^2 can be estimated approximately to be 100. The linewidth of the solitary laser diode measured is about 75 MHz at 2 mW, so the calculated linewidth of the ECSL is about 750 kHz. Fig. 3 shows the spectrum of the ECSL with an operating current I = 65 mA and temperature $T = 30°$C measured by a confocal scanning F-P interferometer. The main-side mode ratio here is large than 30 dB and the measured linewidth is less than 12 MHz, which is limited by the resolution of the interferometer. The strong optical feedback forces the laser to operate in the stable region as shown in Fig. 3 (by a dot). In more than 12 h continuous observation, no mode jumping occurred. The result was obtained without any external active frequency stabilization. This agrees with the theory of stability in the paper. It seems that this kind of strong feedback external cavity laser may be a practical version.

Recently, Tkach *et al.* [15] have made an elaborate experiment to define the different regimes of optical feedback to semiconductor lasers where the transition from the unstable to the stable regime of strong feedback can be achieved by AR coating the diode facet to $r_2^2 = 5$ percent and increasing the external optical power feedback to about -8 dB. The data, indicated with a triangle in Fig. 3, are just located at the boundary of the stable region. Therefore, the occurrence of the transition is because of the feedback finally dominating the field in the laser. The two squares in Fig. 3 indicate the operating points in [16] where the lasers were stably operated in strong external optical feedback.

VI. Conclusion

A set of improved Langevin rate equations for the external cavity semiconductor laser is derived analytically, which can be used to solve transient as well as steady-state problems. The undulation amplitude of the phase condition curve of the steady state decreases in strong feedback; this facilitates the single-mode oscillation. F defined in (10) is an important factor in ECSL, the chirp reduction is F, the line narrowing ratio is F^2, and the dynamic stable condition is $F > 0$. We also show that the strong feedback induced stable operation region is dependent on the feedback R_3 as well as the diode facet reflectivity r_2. The theoretical result of the stability is verified in the experiment on a strong feedback external cavity semiconductor laser. In 12 h continuous observation, no mode jumping occurred without any external active stabilization. It seems that this kind of strong feedback external cavity semiconductor laser may be a practical version.

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