Spectral Linewidth and Frequency Chirp of Four-Wave Mixing Components in Optical Fibers

J. Zhou, R. Hui, and N. Caponio

***Abstract*— The spectral characteristics of four-wave mixing components in optical fibers have been investigated both theoretically and experimentally, for the first time. The theoretical deductions show different contributions of the spectral linewidth and the frequency chirp of the signal waves to the spectrum of four-wave mixing components. Accurate spectral measurements, relying on a high sensitive heterodyne detection system, fully confirmed the theoretical evaluations. The spectral broadening of the four-wave mixing components, due to the phase noise and the frequency chirp, may degrade the performance of unequally spaced channel HD-WDM and fiber four-wave mixing application systems.**

I. Introduction

The four-wave mixing (FWM) in optical fibers has attracted considerable attention in recent years. The performance of high density wavelength division multiplexed (HD-WDM) optical transmission systems has been found to be degraded by FWM [1], [2]. In order to reduce this degradation, unequally spaced channel HD-WDM has been proposed [3], [4]. According to this technique, each channel should not be overlapped by the FWM components produced by other channels. This implies that the spectral broadening, due to the phase noise and the frequency chirp, of the FWM components is an important consideration when allocating unequally spaced channels. More recently, with optical amplifiers, FWM in optical fibers can be used to perform high speed all-optical demultiplexing [5] and midsystem spectral inversion for compensation of fiber chromatic dispersion [6]. In these applications, both the spectral linewidth and the frequency chirp are also important parameters.

The spectral broadening due to fiber FWM has been investigated by K. O. Hill *et al.* [7]; nevertheless, as the used lasers operated on multiple longitudinal modes, the spectral characteristics of the FWM components were not well described. Generally, it was considered that the relationship between the linewidth of the FWM components and that of the signal waves was the same as for the frequency chirp [8]; however, the Gaussian phase noise distribution may result in an enhanced linewidth broadening of the FWM components, which has been found in semiconductor laser amplifiers [9]. The purpose of this letter is to investigate in detail the spectral linewidth and the frequency chirp of the fiber FWM components, in order to obtain their correct dependence on those of the signal waves.

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II. Theory

Through the FWM process, in the most simple case, two waves of angular frequency ω_1 and ω_2 generate the new spectral components at $\omega_3 = \omega_1 - \delta\omega$ and $\omega_4 = \omega_2 + \delta\omega$ with $\delta\omega = \omega_2 - \omega_1$. The generation of FWM components results from the nonlinear polarization. The lowest-order nonlinear polarization $\mathbf{P}_{\rm NL}$ is proportional to the third power of the optical field $\mathbf{E}$,

$$\mathbf{P}_{\rm NL} = \chi_0 \vdots \mathbf{EEE} \tag{1}$$

where the constant of proportionality χ_0 is the third-order electric susceptibility. It is supposed that the light waves are polarized along the same direction and the optical field can be expressed by

$$E = \frac{1}{2}E_1 \exp(j\omega_1 t) + \frac{1}{2}E_2 \exp(j\omega_2 t) + \text{c.c.} \tag{2}$$

Substituting Eq. (2) into (1), the terms related to the FWM components can be easily determined and the electric fields at the FWM frequencies are [9]

$$E_{3,4} = A_{3,4}E_{1,2}^2 E_{2,1}^* \exp j(2\omega_{1,2} - \omega_{2,1})t \tag{3}$$

with

$$A_{3,4} = j\frac{\pi\omega_{3,4}}{n_{3,4}c}(D\chi_{111})\exp\left(-\frac{1}{2}\alpha L\right) \cdot \left[\frac{\exp(-\alpha L + j\Delta k_{3,4}L) - 1}{j\Delta k_{3,4} - \alpha}\right] \tag{4}$$

where $\Delta k_{3,4} = k_{3,4} + k_{2,1} - 2k_{1,2}$, with the propagation constant $k_i = n_i\omega_i/c$ $(i = 1, \ldots, 4)$; the quantities D, χ_{111}, n_i, α, L and c are, respectively, the degeneracy factor, the electric susceptibility, the fiber-core refractive index at ω_i, the fiber attenuation coefficient, the fiber length and the velocity of light in vacuum. For the sake of simplicity, we neglected the complex conjugate terms. Strictly speaking, the fiber chromatic dispersion and its slope result in the phase-mismatch dependence of the FWM wave-generation efficiency $A_{3,4}$ on the channel spacing and also on the spectral characteristics of the signal waves [2]; this dependence has effect on the spectrum of the FWM components. However, if the channel spacing and the spectral bandwidth of the signal waves are, as usual, sufficiently small, we may take $\Delta k_{3,4}L = 0$ and obtain

$$A_{3,4} = j\frac{2\pi^2}{n\lambda}(D\chi_{111})\exp\left(-\frac{1}{2}\alpha L\right)L_{\text{eff}} \tag{5}$$

where $L_{\text{eff}} = [1 - \exp(-\alpha L)]/\alpha$, and λ, n, k are, respectively, the optical wavelength, the refractive index and the propagation constant, which are assumed to be constant in the working frequency range. On this condition, A_3 and A_4 are proportionality constants.

The effect of the frequency chirp can be taken into account by substituting E_i with $E_i \exp(j2\pi\Delta f_i(t)t)$ into Eqs. (3), where Δf_i represents the frequency chirp of E_i, with $i = 1, \ldots, 4$. Then we have

$$\Delta f_{3,4}(t) = 2\Delta f_{1,2}(t) - \Delta f_{2,1}(t) \tag{6}$$

Generally, as $\Delta f_1(t)$ and $\Delta f_2(t)$ are determined functions, $\Delta f_3(t)$ and $\Delta f_4(t)$ can be evaluated from the Eq (6). If, for example, E_1 is frequency modulated with a modulation index of m_1 and E_2 is not modulated, the frequency modulation index of E_3 is $2m_1$ while that of E_4 is m_1. A similar situation happens when only E_2 is modulated.

In a similar way, the effect of the signal phase noise $\phi(t)$ can be taken into account by substituting $E_{1,2}$ with $E_{1,2}\exp[j\phi_{1,2}(t)]$. However, the above simple deduction for the frequency chirp does not hold in the case of the linewidth because of Gaussian random process of phase noise [9], [10]. With the phase noise, the field autocorrelations of the FWM components are

$$\begin{aligned}\langle E_{3,4}(t+\tau)E^*_{3,4}(t)\rangle = {} & |A_{3,4}|^2|E_{1,2}|^4|E_{2,1}|^2 \\ & \cdot \langle \exp\{2j[\phi_{1,2}(t+\tau) - \phi_{1,2}(t)]\} \\ & \cdot \exp\{-j[\phi_{2,1}(t+\tau) - \phi_{2,1}(t)]\}\rangle\end{aligned} \tag{7}$$

The relationship between the phase noises and the spectral linewidth in a Gaussian process can be written as

$$\langle \exp\{jq[\phi(t+\tau) - \phi(t)]\}\rangle = \exp[-q^2 2\pi\tau\delta\nu] \tag{8}$$

where q is a constant and $\delta\nu$ is the spectral linewidth. Since the phase noises of the signal waves have no correlation, from Eqs. (7) and (8), the linewidth of the FWM components E_3 and E_4 can be obtained respectively as

$$\delta\nu_{3,4} = 4\delta\nu_{1,2} + \delta\nu_{2,1} \tag{9}$$

where $\delta\nu_1$ and $\delta\nu_2$ are the linewidths of the signal waves E_1 and E_2. This simple relationship reveals an enhancement of the linewidths of the FWM components.

This analysis method can be easily extended to the case of FWM with three or more signal frequencies. If three independent signals at angular frequencies ω_i, ω_j and ω_k propagate in a single-mode fiber, through the nonlinear interaction among the three signals, one of the four-wave mixing components will be generated at the angular frequency $\omega_{ijk} = \omega_i + \omega_j - \omega_k$. The relation between the frequency chirp of the FWM component and that of the three signal waves is

$$\Delta f_{ijk}(t) = \Delta f_i(t) + \Delta f_j(t) - \Delta f_k(t) \tag{10}$$

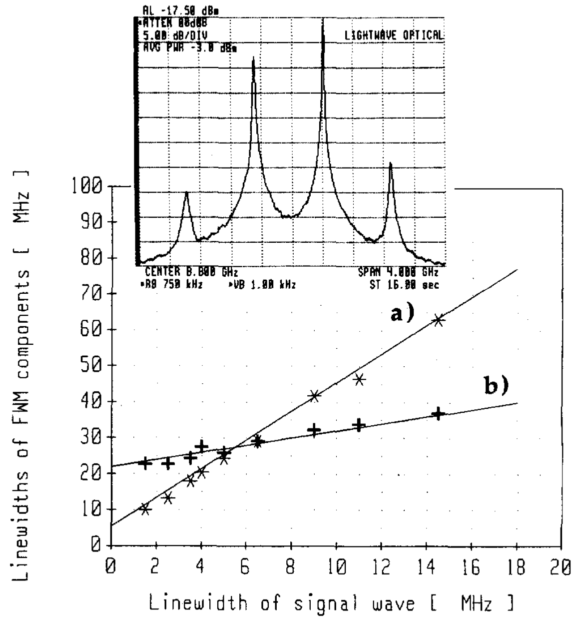

Fig. 1. Linewidths of FWM components versus linewidth of signal wave E_1. a) E_3: theoretical (solid line), measured (stars). b) E_4: theoretical (solid line), measured (crosses). Inset is spectra of FWM components and signal waves with no modulation. From left to right: E_3, E_1, E_2 and E_4.

where $\Delta f_{ijk}(t)$, $\Delta f_i(t)$, $\Delta f_j(t)$ and $\Delta f_k(t)$ are respectively the frequency chirps of the FWM component, and of the signal waves; while, for the linewidth, we have

$$\delta\nu_{ijk} = \delta\nu_i + \delta\nu_j + \delta\nu_k \tag{11}$$

where $\delta\nu_{ijk}$, $\delta\nu_i$, $\delta\nu_j$ and $\delta\nu_k$ are respectively the linewidths of the FWM component, and of the signal waves. It is worthy noting that the linewidth of the FWM components generated by three signals is simply the sum of those of the three signals without the linewidth enhanced coefficient, similarly to the case of the frequency chirp, as shown in (10) and (11).

III. Experiment

In order to investigate the spectral characteristics of FWM components, a demonstrative experiment was arranged as follows. Two commercial DFB lasers, operating at 1.536 μm, were used to generate optical signal waves which were combined by a 3 dB polarization-maintaining fiber directional coupler, to ensure the same state of polarization of the two signals. The output from one branch of the directional coupler was amplified up to +10 dBm by an EDFA and then launched into a 20 km long 1.55 μm dispersion-shifted fiber, where FWM components were generated due to nonlinear interaction. The FWM spectra were measured by means of high sensitive heterodyne detection, with a local oscillator linewidth of approximately 1.0 MHz and an optical detector bandwidth of 22 GHz, followed by a spectrum analyser.

The inset of Fig. 1 reports the spectra of the FWM components and the signal waves, and a enhanced linewidth broadening of the FWM components, with respect to the linewidths of the signal waves, was observed. The linewidth

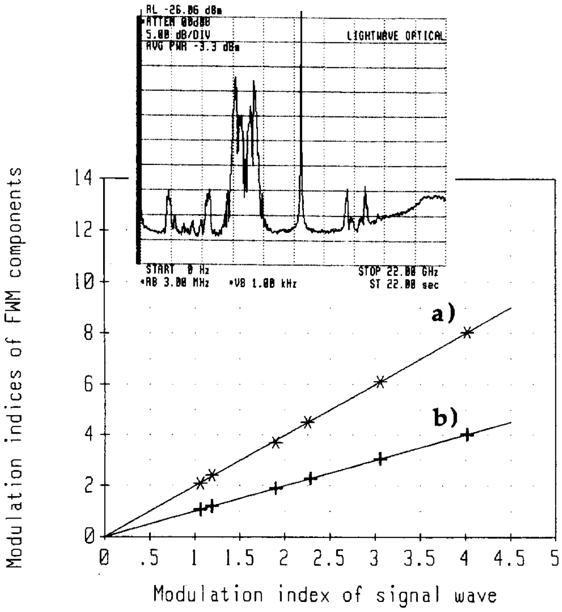

Fig. 2. FSK modulation indices of FWM components versus modulation index of signal wave E_1. a) E_3: theoretical (solid line), measured (stars). b) E_4: theoretical (solid line), measured (crosses). Inset is spectra of FWM components and signal waves with FSK modulation of signal wave E_1. From left to right: E_3, E_1, E_2 and E_4.

of the FWM components was measured at different linewidths of the signal waves, as shown in Fig. 1, and comparison with the theoretical curves calculated using (9) is also presented in the same figure, showing a good agreement.

The effect of the frequency chirp of the signal waves on the spectral broadening of the FWM components was measured by applying direct modulation on the signal laser currents. When one of the signal waves (E_1) was FSK modulated at 622 Mbit/s, the signal wave and the FWM spectra were plotted (the inset of Fig. 2). This figure clearly shows that the FWM component E_3 is also FSK modulated with the modulation index equal to two times that of the signal wave E_1, while the modulation index of E_4 is the same as that of the signal wave E_1. Fig. 2 gives a systematic measurement of the modulation indices of the FWM components versus the modulation index of the signal wave E_1. The measured results well agree with the theoretical predictions given by (6).

IV. Summary

Spectral characteristics of the components generated in the FWM process in optical fibers are reported, for the first time. The results, both theoretical and experimental, indicate the different contributions of the spectral linewidth and the frequency chirp of the signal waves to the spectral broadening of the FWM components. Simple equations to determine the spectral linewidth and the frequency chirp of the FWM components were deduced and were fully confirmed by a demonstrative experiment. This is important for the design of unequally spaced channel HD-WDM, all-optical demultiplexing and midsystem spectral inversion for compensation of fiber chromatic dispersion.

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