Noise and frequency chirping in external-cavity semiconductor lasers

Rongqing Hui and Yizun Wu

Optical Communication Laboratory, P.O. Box 190, Beijing University of Posts and Telecommunications, Beijing 100088, China

Received October 13, 1988; accepted March 26, 1989

The noise spectra and frequency chirping of semiconductor lasers in the presence of arbitrary amounts of optical feedback are analyzed. Short external cavities with strong optical feedback are found to reduce the noise dramatically in semiconductor lasers, especially in the high-frequency regime. Frequency chirping is shown to be closely related to the nonlinear gain effect.

Coherent light-wave transmission and high-bit-rate single-mode fiber communication systems require stable lasers with narrow linewidths. External-cavity semiconductor lasers are a feasible means of meeting this requirement. The intensity and phase-noise power spectral densities of semiconductor lasers in the presence of weak optical feedback have been studied by Spano *et al.*[1] However, recent experimental and theoretical research indicates that strong external-cavity optical feedback is preferred.[2–4] Tkach and Chraplyvy experimentally distinguished the various operating reigmes according to the level of feedback in long external-cavity lasers.[4] In the strong-feedback regime (regime V in Ref. 4) the laser operates stably with narrower linewidths for all phases of the feedback. The linewidth dependence for and dynamic stability of semiconductor lasers with weak and strong optical feedback are known.[5,6] However, a unified study of the intensity and phase-noise spectra and frequency chirping in lasers with arbitrary amounts of optical feedback has not been developed. This Letter presents a theoretical derivation of simple formulas that describe these effects. This analysis also incorporates the effect of nonlinear gain. We show that short external cavities with strong optical feedback dramatically reduce the intensity and phase noises, especially in the high-frequency regime, and that nonlinear gain has a significant impact on frequency chirping but not on the Lorentzian linewidth.

The external-cavity semiconductor laser investigated here is a three-mirror-cavity model (Fig. 1). By taking into account multiple random reflections in the external cavity, the rate equations of semiconductor lasers in the presence of arbitrary amounts of optical feedback are[3]

$$\left(1+\frac{\tau}{\tau_i}H_2\right)\dot{I}(t)=I(t)\left[\Delta G+\frac{2}{\tau_i}H_1-\frac{2\tau}{\tau_i}P_2\dot{\phi}_n(t)\right]+R_s+F_I(t), \quad (1)$$

$$\left(1+\frac{\tau}{\tau_i}H_2\right)\dot{\phi}_n(t)=(\Omega-\omega_0)+\frac{\alpha}{2}\Delta G-\frac{1}{\tau_i}P_1+\frac{\tau}{2\tau_i}\frac{\dot{I}(t)}{I(t)}P_2+F_\phi(t), \quad (2)$$

$$\dot{N}(t)=-GI(t)-\frac{N(t)}{\tau_e}+\frac{M(t)}{e}+F_N(t), \quad (3)$$

where $I(t)$ is the number of photons, $N(t)$ is the number of minority carriers, G is the gain coefficient, ΔG is the net gain coefficient, $\phi_n(t)$ is the optical phase noise, Ω and ω_0 are the optical frequency with and without feedback, respectively, α is the linewidth enhancement factor, and $F_x(t)$ $(x=I,\phi,N)$ represent the Langevin noises. $\tau=n_0L/c$ and $\tau_i=nl/c$ are the round-trip time delays in the passive and active cavities, respectively, R_s is the rate of spontaneous emission, τ_e is the electron lifetime, $M(t)$ is the injection current, and

$$H_1=\frac{1}{2}\ln\left[\frac{1+2(R_3/r_2)\cos\phi+(R_3/r_2)^2}{1+2r_2R_3\cos\phi+(r_2R_3)^2}\right],$$

$$P_1=-\tan^{-1}\left[\frac{R_3(1-r_2{}^2)\sin\phi}{r_2(1+R_3{}^2)+R_3(1+r_2{}^2)\cos\phi}\right],$$

$$H_2=-\frac{1}{\tau}\frac{\mathrm{d}P_1}{\mathrm{d}\omega_0},\qquad P_2=-\frac{1}{\tau}\frac{\mathrm{d}H_1}{\mathrm{d}\omega_0},\qquad \phi=\omega_0\tau.$$

Here r_2 is the amplitude reflectivity of the diode facet facing the external cavity and R_3 is the effective amplitude reflectivity, including coupling loss, of the external mirror.

Equations (1)–(3) are now linearized, and thus limited to the case of relatively short external-cavity lasers. Linearized versions of $I(t)$, $N(t)$, and $M(t)$ in

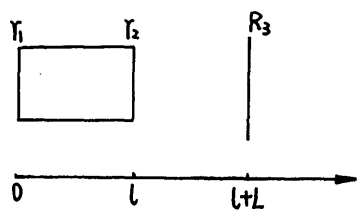

Fig. 1. Three-mirror-cavity model for external-cavity semiconductor lasers.

0146-9592/89/130668-03$2.00/0

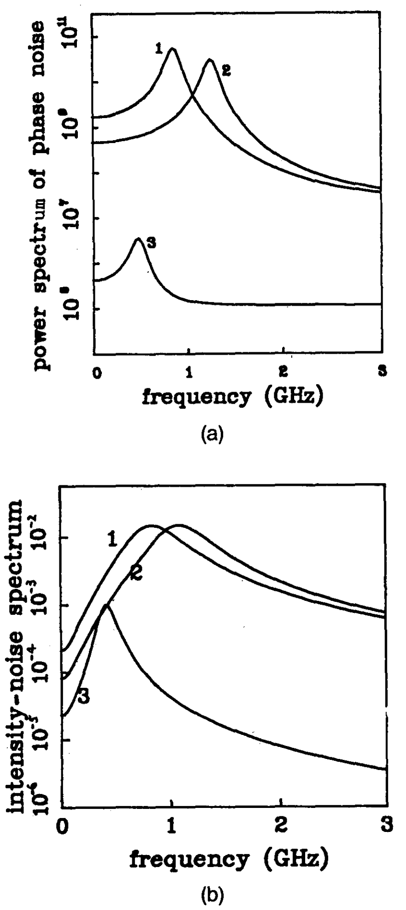

Fig. 2. Noise power spectrum of the external-cavity semiconductor laser for various feedback levels, with $nl = 0.9$ mm, $n_0L = 9$ mm, and $\phi = 240°$. (a) Phase-noise spectrum, (b) intensity-noise spectrum. Curves 1, no feedback; curves 2, weak feedback with $R_3 = 0.01$ and $r_2 = 0.565$; curves 3, strong feedback with $R_3 = 0.4$ and $r_2 = 0.2$.

terms of small deviations around their equilibrium values are written as

$$I(t) = I_0 + \delta I(t), \qquad N(t) = N_0 + \delta N(t),$$
$$M(t) = M_0 + m(t). \tag{4}$$

G is now expressed as

$$G = \Gamma A(N - N_e)(1 - aI), \tag{5}$$

where Γ is the optical confinement factor, A is the differential gain, and a is the gain saturation factor. In single-frequency semiconductor lasers a specifies the spectral hole-burning effect. After Fourier transformation, we have

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} \delta I(iw) \\ \phi_n(iw) \\ \delta N(iw) \end{bmatrix} = \begin{bmatrix} F_I(iw) \\ F_\phi(iw) \\ F_N(iw) + m(iw)/e \end{bmatrix}, \tag{6}$$

where $A_{11} = iw\,[1 + (\tau/\tau_i)H_2] - aG_0I_0 + R_s/I_0$, $A_{12} = iw2I_0\tau P_2/\tau_i$, $A_{13} = -\Gamma AI_0$, $A_{21} = iw[-(\tau P_2/2\tau_i I_0)]$, $A_{22} = iw[1 + (\tau/\tau_i)H_2]$, $A_{23} = -(\alpha/2)\Gamma A$, $A_{31} = \Gamma A[(N_0 - N_e)(1 - aI_0) - a(N_0 - N_e)]$, $A_{32} = 0$, $A_{33} = iw + \gamma$, $G_0 = \Gamma A(N_0 - N_e)(1 - aI_0)$, and $\gamma^{-1} = [\Gamma A(1 - aI)I_0 + 1/\tau_e]^{-1}$ is the damping time associated with relaxation oscillation. $\delta I(iw)$, $\phi_n(iw)$, $\delta N(iw)$, $m(iw)$, $F_I(iw)$, $F_\phi(iw)$, and $F_N(iw)$ are the Fourier transforms of truncated functions corresponding to $\delta I(t)$, $\phi_n(t)$, $\delta N(t)$, $m(t)$, $F_I(t)$, $F_\phi(t)$, and $F_N(t)$, respectively. Following standard treatment (see Ref. 1), the power spectral densities for the intensity and phase noises $S_I(iw)$ and $S_\phi(iw)$, respectively, can be directly obtained:

$$\begin{aligned} S_I(iw) = \frac{R_s}{|Y|^2}\{(2I_0 + 1)|A_{22}A_{33}|^2 \\ + |A_{12}A_{33}|^2/2I_0 + |A_{12}A_{23} - A_{13}A_{22}|^2 \\ - 2\,\mathrm{Re}[A_{22}A_{33}(A_{12}A_{23} - A_{13}A_{22})^*]\}, \end{aligned} \tag{7}$$

$$\begin{aligned} S_\phi(iw) = \frac{R_s\omega^2}{|Y|^2}\{(2I_0 + 1)|A_{23}A_{31} \\ - A_{21}A_{33}|^2 + |A_{11}A_{33} - A_{13}A_{31}|^2/2I_0 \\ + |A_{13}A_{21} - A_{11}A_{23}| - 2\,\mathrm{Re}[(A_{23}A_{31} \\ - A_{21}A_{33})(A_{13}A_{21} - A_{11}A_{23})^*]\}, \end{aligned} \tag{8}$$

where $Y = A_{11}A_{22}A_{33} - A_{12}(A_{21}A_{33} - A_{23}A_{31}) - A_{13}A_{22}A_{31}$. For weak feedback and a negligible nonlinear gain effect, Eqs. (7) and (8) reduce to the results

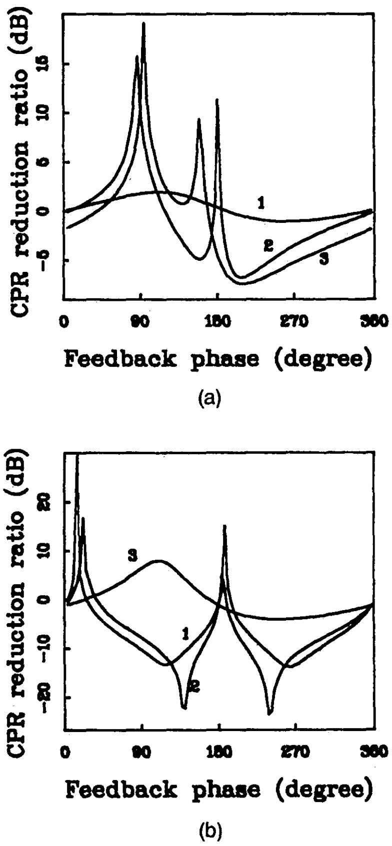

Fig. 3. CPR versus the feedback phase for various feedback levels. (a) $nl = 0.9$ mm and $n_0L = 9$ mm; curve 1, $R_3 = 0.1$ and $r_2 = 0.565$; curve 2, $R_3 = 0.3$ and $r_2 = 0.565$; curve 3, strong feedback with $R_3 = 0.5$ and $r_2 = 0.1$. (b) $nl = 0.9$ mm and $n_0L = 10$ cm; curve 1, $R_3 = 0.01$ and $r_2 = 0.565$; curve 2, $R_3 = 0.3$ and $r_2 = 0.565$; curve 3, strong feedback with $R_3 = 0.5$ and $r_2 = 0.1$.

in Ref. 1 [Eq. (8)]. However, a short external cavity with strong optical feedback can dramatically reduce the phase and intensity noises, as shown in Fig. 2 (curves 3). We define the strong-feedback regime as regime V in Ref. 4. This regime corresponds to the condition that $[1 + (\tau/\tau_i)(H_2 - \alpha P_2)] > 0$ for all values of phase feedback. We have stated[3,7] that the theoretical result quantitatively agrees with the experiment in Ref. 4. The Lorentzian linewidth of the external-cavity semiconductor laser can be expressed as

$$S_\phi(0) = \frac{R_s}{4I_0} \frac{\alpha^2[2I_0 + 1 + (\epsilon\Gamma A)^2] + 2I_0(\epsilon\gamma + 1)^2}{\left\{\epsilon\gamma\left(1 + \frac{\tau}{\tau_i}H_2\right) + \left[1 + \frac{\tau}{\tau_i}(H_2 - \alpha P_2)\right]\right\}^2}, \tag{9}$$

where $\epsilon = [a + R_s/(G_0 I_0^2)]/\Gamma A$. Equation (9) is identical to the result in Refs. 3 and 5, if the nonlinear gain effect is ignored. Quantitatively, ϵ is so small that it can often be omitted.

The chirp-reduction characteristic of the external-cavity semiconductor laser can also be studied by using Eq. (6). The chirp-to-modulated power ratio (CPR) is defined as

$$\mathrm{CPR} = \delta f(iw)/\delta I(iw),$$

where $\delta f(iw) = iw\phi_n(iw)/2\pi$ is the frequency deviation. Neglecting the Langevin noise terms in Eq. (6), we get

$$\mathrm{CPR} = \frac{iw\left[\alpha\left(1 + \frac{\tau}{\tau_i}H_2\right) + \frac{\tau}{\tau_i}P_2\right] + \alpha(aG_0I_0 + R_s/I_0)}{4\pi I_0\left[1 + \frac{\tau}{\tau_i}(H_2 - \alpha P_2)\right]}. \tag{10}$$

The nonlinear gain effect cannot be omitted in Eq. (10). The CPR reduction ratio due to external optical feedback is shown in Fig. 3 for various feedback strengths and external-cavity lengths. The parameters in Figs. (2) and (3) are optical gain $G_0 = 5.8 \times 10^{11}$, linewidth enhancement factor $\alpha = 5.3$, photon number $I_0 = 3.1 \times 10^4$, gain saturation factor $a = 1.58 \times 10^{-8}$, and spontaneous emission rate $R_s = 1.5 \times 10^{12}$. In Fig. 3 the modulation frequency $f = 1$ GHz.

In conclusion, we have investigated noise and frequency chirping effects in semiconductor lasers in the presence of arbitrary amounts of external optical feedback. The most striking feature of this analysis is that the short external cavities with strong optical feedback can dramatically reduce phase and intensity noise, especially in the high-frequency regime. The nonlinear gain effect was shown to play an important role in frequency chirping, but it does not significantly affect the Lorentzian linewidth.

References

1. P. Spano, S. Piazzolla, and M. Tamburrini, IEEE J. Quantum Electron. **QE-20,** 350 (1984).
2. A. R. Chraplyvy *et al.*, Electron. Lett. **22,** 88 (1986).
3. R. Hui and S. Tao, in *Proceedings of IEEE International Semiconductor Laser Conference* (Institute of Electrical and Electronics Engineers, New York, 1988), paper K-5.
4. R. W. Tkach and A. R. Chraplyvy, IEEE J. Lightwave Technol. **LT-4,** 1655 (1986).
5. D. A. Hjelme and A. Mickelson, IEEE J. Quantum Electron. **QE-23,** 1000 (1987).
6. C. H. Henry and R. F. Kazarinov, IEEE J. Quantum Electron. **QE-22,** 294 (1986).
7. R. Hui and S. Tao, "Improved rate equations for external cavity semiconductor lasers," IEEE J. Quantum Electron. (to be published).