Tussle and Game Theory

Outline

TG.1 Motivation and tussle
TG.2 Rational choice
TG.3 Games
TG.4 Nash equilibrium

Primary references:
[O]=[O2004]
Tussle and Game Theory

Motivation and Tussle

TG.1  Motivation and tussle
TG.2  Rational choice
TG.3  Games
TG.4  Nash equilibrium
Tussle
Introduction and Motivation

- **Tussle** [CWSB2002]
  - Internet composed of multiple stakeholders
  - have competing interests
  - vie to maximise their own interest

*Network Examples?*
**Tussle**

Introduction and Motivation

- **Tussle** [CWSB2002]
  - Internet composed of multiple stakeholders
  - have competing interests
  - vie to maximise their own interest

- **Examples**
  - subscribers vs. service providers
  - subscribers vs. content providers
  - content providers vs. service providers
  - service providers vs. one another

- Modelled by *game theory*
Tussle and Game Theory
Rational Choice

TG.1 Motivation and tussle
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TG.4 Nash equilibrium
Rational Choice

Definition

- **Rational choice** is an economic definition
  - may not correspond with common use of “rational”

- Individual decision maker chooses actions
  - among available action set $A$
    - subject to constrained subset (e.g. sufficient income)
  - according to preferences
    - defined by a payoff function $u(a_i)$
    - ordinal relation (no sense of relative intensity of preference)
    - transitive relation: $u(a_i) > u(a_j) \land u(a_j) > u(a_k) \Rightarrow u(a_i) > u(a_k)$
Tussle and Game Theory

Games

TG.1  Motivation and tussle
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Games

Introduction

- **Strategic game**
  - set of players \( P \)
  - set of actions \( A \)
  - ordinal preferences over set of action profiles
    - rational choice
Prisoners’ Dilemma

Introduction

• Let’s play a game!
Prisoners’ Dilemma

Scenario

- Let’s play a game!
  - split into two groups (2 players)

- Scenario
  - each group (player) is a suspect of felony (bank robbery)
    - 3 or 4 years in prison
    - only if other player finks you out
  - enough evidence to convict you of misdemeanor (speeding)
    - 1 year in prison
Prisoner’s Dilemma

Actions

• Each player has two possible actions
  – quiet: say nothing about the other player
  – fink: agree to testify that other player is guilty of felony
Prisoner’s Dilemma

Consequences

- you will go free (plea bargain)
  - if you fink out the other player but they don’t fink you out
- you will go to prison for 1 year for misdemeanor unless
  - you fink out the other player but they don’t fink out you
- you will go to prison for only 3 years
  - if you fink them out and they fink you out
- you will go to prison for 4 years
  - if the other player finks you out but you remain quiet
Prisoner’s Dilemma
Preferences

- Preferences
  - you have five minutes to determine your preference
  - return with the choice among \{quiet, fink\} written on paper
Prisoner’s Dilemma

Preferences

• Preferences
  – you have five minutes to determine your preference
  – return with the choice among \{quiet, fink\} written on paper
  – desired payoff function order \( u_1(\text{you, other player}) \)
    • \( u_1(F, Q) > u_1(Q, Q) > u_1(F, F) > u_1(Q, F) \)

possible function?
Prisoner’s Dilemma
Preferences

- Preferences
  - you have five minutes to determine your preference
  - return with the choice among \{quiet, fink\} written on paper
  - desired payoff function order for player 1: \( u_1(a_1, a_2) \)
    - \( u_1(F, Q) > u_1(Q, Q) > u_1(F, F) > u_1(Q, F) \)
  - payoff functions with arbitrary ordinal values:
    - \( u_1(F, Q) = 3 \)
    - \( u_1(Q, Q) = 2 \)
    - \( u_1(F, F) = 1 \)
    - \( u_1(Q, F) = 0 \)
Prisoner’s Dilemma

Preferences

- Preferences
  - desired payoff function order for player 1: \( u_1(a_1, a_2) \)
    - \( u_1(F,Q) > u_1(Q,Q) > u_1(F,F) > u_1(Q,F) \)
  - payoff functions with arbitrary ordinal values:
    - \( u_1(F,Q) = 3, u_1(Q,Q) = 2, u_1(F,F) = 1, u_1(Q,F) = 0 \)
  - maximum payoff function is \( Q,Q = 2+2 = 4 \)

<table>
<thead>
<tr>
<th>Suspect 1</th>
<th>Quiet</th>
<th>Fink</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quiet</td>
<td>2,2</td>
<td>0,3</td>
</tr>
<tr>
<td>Fink</td>
<td>3,0</td>
<td>1,1</td>
</tr>
</tbody>
</table>

- Nash Equilibrium at \( F,F \)
Games
Types

- **Single** vs. **Iterated** game
  - whether game is repeated by same players
- **Sequential** vs. **Simultaneous** game
  - whether player make moves sequentially or in parallel
- **Perfect information**
  - whether players have full information
- **Zero-sum** game
  - loss/gain of one player balanced by gain/loss of others
- **Pareto-optimum**
  - not possible for any player to gain without another losing
Games

Dollar Auction

- Shall we play a game?
  - tic tac toe
  - chess
  - dollar auction
    - sequential game with perfect information
  - global thermonuclear war
Dollar Action

Scenario

• Let’s play a game!
  – entire class participates in the room (perfect information)

• Scenario
  – I have a crisp new dollar for someone to win
  – bidding starts at 10¢
  – high bidder wins
Games
Another Game

• Shall we play a game?
  – tic tac toe
  – chess
  – dollar auction
  – global thermonuclear war
Dollar Action

Scenario

- Let’s play a game!
  - entire class participates in the room (perfect information)
- Scenario
  - I have a crisp new dollar for someone to win
  - bidding starts at 10¢
  - high bidder wins
Dollar Action

Scenario

- Let’s play a game!
  - entire class participates in the room (perfect information)
- Scenario
  - I have a crisp new dollar for someone to win
  - bidding starts at 10¢
  - high bidder wins
    - but wait
  - I keep all bids, including from the loser
Dollar Action

Scenario

• Let’s play a game!
  – entire class participates in the room (perfect information)

• Scenario
  – I have a crisp new dollar for someone to win
  – bidding starts at 10¢
  – high bidder wins
    
    but wait
  – I keep all bids, including from the loser
  – even as the bids go above $1.00
Dollar Action
Scenario

• Let’s play a game!
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• Scenario
  – I have a crisp new dollar for someone to win
  – bidding starts at 10¢
  – high bidder wins
  
  *but wait*
  – I keep all bids, including from the loser
  – even as the bids go above $1.00

• *The only winning move is not to play*
Games
Mutually Assured Destruction

- Shall we play a game?
  - tic tac toe
  - chess
  - dollar auction
  - global thermonuclear war
    - War Games clip  [http://www.youtube.com/watch?v=NHWjlCaIrQo]
Mutually Assured Destruction

Scenario

- Scenario
  - the east and the west both have lots of nuclear weapons
    - enough to obliterate the opponent
  - one side gets angry at the other and launches first strike

*what happens next?*
Mutually Assured Destruction

Scenario

- Scenario
  - the east and the west both have lots of nuclear weapons
    - enough to obliterate the opponent
  - one side gets angry at the other and launches first strike
  - the other side retaliates
    - with all its weapons so they are not destroyed on the ground
  - the first striker launches all its remaining nukes

- *The only winning move is not to play*
Tussle and Game Theory

Games

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Nash Equilibrium

**Introduction**

- Nash Equilibrium
  - player cannot do better by unilaterally changing strategy
Regular Networks

References and Further Reading


- Thanks to Anne-Marie Hoskinson suggestions on game-playing
End of Foils