Science of Communication Networks
The University of Kansas EECS 784
Internet Topology and Graph Spectra

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Internet Topology and Graph Spectra

Outline

ST.1 Multilevel graphs
ST.2 Internet topology
ST.3 Graph spectra
• Modelling the Internet
  – Internet is *really* complex system (more later)
    • scale: number of nodes (graph theoretic order)
    • hierarchy: backbones, regional access, subscribers
    • differing structural (graph theoretic properties) of each
Internet Topology
Multilevel

• Modelling the Internet
  – Internet is *really* complex system (more later)
    • scale: number of nodes (graph theoretic order)
    • hierarchy: backbones, regional access, subscribers
    • differing structural (graph theoretic properties) of each
  – Internet is *multilevel graph*
    • physical infrastructure (fibre, wireless links, L2 muxes, switches)
    • engineering underlay (MPLS)
    • IP (layer 3) router topology
    • AS (service provider) graph
    • end-to-end (layer 4) transport flows
    • social interaction graph
Internet Topology
Tomography

- No Internet map
  - service provider secret
- Impedes research on
  - physical structure
  - traffic patterns
  - application use
- *Internet tomography*
  - probe to infer structure
  - logical maps *only*, e.g.
    - IP address structure
    - AS relationships

[Britt wikipedia Image:Internet_map_1024.jpg]
Internet Topology
Approximating Router Topology

• IP and routing tomography
  – infer router interconnectivity of a particular service provider
  – provides an approximate map of L3 connectivity
  – does not provide L2 and physical topology
  – does not provide peering information

• Rocketfuel ISP topology mapping engine
  [http://www.cs.washington.edu/research/networking/rocketfuel]
Multilevel Network Topology

Example: Sprint L3 IP PoP Topology
Multilevel Network Topology

Example: Sprint L3 overlay on L2.5
Multilevel Network Topology
Example: Sprint L2.5 MPLS PoP Topology
Multilevel Network Topology

Example: Sprint L2.5 overlay on L2/1
Multilevel Network Topology

Example: Sprint L1 Physical Fiber Topology
Multilevel Network Topology

Example: Sprint L1–3 Topology
Multilevel Network Analysis
Abstraction of Internet Topology
Multilevel Network Analysis
Multilevel Graph Model

- Multilevel model for unweighted & undirected graphs
- Two requirements for multilevel graph model:
  - nodes at the above level are subset of lower level
  - nodes that are disconnected below are disconnected above
Topography Generation
Flexible and Realistic Topography Generation

- **KU-LoCGen**
  - location and cost constrained
    [TR2009, TSJ2011]
- **Level 1**: backbone realms
  - nodes distributed based on location constraints
  - links generated using various models under cost constraints
- **Level 2**: access network realms
  - distributed around backbone nodes
  - access network connectivity: ring, star, mesh
- **Level 3**: subscribers
  - distributed around access node with preferential attachment
KU-LoCGen
2-level example using Sprint PoP Locations
KU-LoCGen REAL POPS
Population Based Node Generation Example

actual US PoPs:
- Sprint + AT&T + Level3
- population-based

[CCNet-GLOBECOM 2010]
KU-LoCGen
Population Based Node Generation Accuracy

CCDF

offset distance [km]

Sprint
Level 3
AT&T
Combined USA

Population Based Node Generation Accuracy
KU-LoCGen
Node Generation with Technology Penetration

- actual city location
- generated PoP no $\gamma$
- generated PoP with $\gamma$

Cities:
- Delhi
- Patna
- Mumbai
- Kolkata
- Hyderabad
- Bangalore
- Chennai
- Patna
- actual city location
- generated PoP no $\gamma$
- generated PoP with $\gamma$
Internet Topology and Graph Spectra

Internet Topology

ST.1 Multilevel graphs
ST.2 Internet topology
ST.3 Graph spectra
Internet Prehistory
Early Packet Switching Research

  - 1961: Kleinrock – queueing theory shows effectiveness of packet-switching
  - 1964: Baran – packetised voice switching in military nets
  - 1965: Davies proposes UK packet switched network
    - coins term *packet*
  - 1967: NPL begins construction of UK experimental network
  - 1967: ARPANET conceived by US DOD ARPA
  - 1972: CYCLADES project planning in France
  - 1973: first CYCLADES packet switch demonstration
Internet Prehistory

Early Packet Switching: NPL Topology
Internet Prehistory
Early Packet Switching: CYCLADES 1978

[CIGALE packet switch]

[adapted from Pouzin-1982]
Internet History

Early Packet Switching: ARPANET

- 1967-1972: ARPANET emergence
  - 1967: ARPANET conceived by US DoD ARPA
  - 1969:
    - BBN awarded contract to build IMPs (interface msg processors)
    - first ARPANET nodes operational at UCLA, SRI, UCSB, Utah
  - 1972:
    - ARPANET demonstrated publicly
    - NCP (Network Control Protocol) first host-host protocol
    - first e-mail program
    - ARPANET has 15 nodes
  - 1979: ARPANET has 200 nodes
    - but access limited to research institutions with ARPA contracts
Internet History
ARPANET Design Principles

- Minimalism, autonomy
  - no internal changes required to interconnect networks
- Best effort service model
- Robust to failures (or attack)
  - stateless gateways (routers)
  - decentralized control
  - most functionality in end systems
    - note: *very different* from end-to-end arguments!
Internet History
ARPANET 1969

ARPANET 1969

SRI
UCLA
UCSB
Utah
Internet History
ARPANET 1970
Internet History

ARPANET 1971
Internet History

CSNET

  – 1979: first link meeting at Wisc. [Landweber]
  – 1981: initial funding from NSF
  – network for researchers without ARPANET access
    • ARPANET restricted to certain DOD research contracts
  – TCP/IP and other services from ARPANET over:
    • X.25 public networks (initially Telenet)
    • Phonenet (MMDF over leased lines)
    • ARPANET for institutions with ARPA contracts
  – 1989: merged with CSNET into CREN
    • Corporation for Research and Educational Networking

[Denning, Hearn, Kern, ACM SIGCOMM 1983]
Internet History
CSNET
Internet History
Gatewayed Networks to ARPANET/CSNET

- Simplified logical structure
  - some nets use links of others (e.g. PSTN dialup/leased lines)
- Gateways interconnect
  - no seamless addressing
  - mixed formats through gateways (e.g. %-hack)
Internet History

ARPANET → NSFNET

- 1986: NSFNET begun
  - NREN (national research and engineering network)
    - HPCC (high perf. computing & communication) act “Gore bill”
    - funded by National Science Foundation
    - limited to academic institutions and a few govt. contractors

- Progression of
  - 1986: 56kb/s switched by LSI-11 Fuzzball routers
  - 1989: T1 links switched by TR interconnected IBM PC-RTs
  - 1992: T3 links switched by FDDI interconnected IBM RS/6Ks

- Late 1980’s: Gigabit testbeds
  - research testbeds to increase network performance

- Early 1990’s: ARPANET decommissioned
Internet History
Role of Al Gore

• Al Gore was critical in funding the Internet (NSFNET)
  – he never claimed to have invented the Internet, rather:
    “During my service in the United States Congress, I took the initiative in creating the Internet. I took the initiative in moving forward a whole range of initiatives that have proven to be important to our country's economic growth and environmental protection, improvements in our educational system.”
    – Al Gore, 9 March 1999 on CNN
  – his role & statements have been defended by those who did
    “Bob [Kahn] and I believe that the vice president deserves significant credit for his early recognition of the importance of what has become the Internet. ... Gore was talking about and promoting the Internet long before most people were listening.” – Bob Kahn and Vint Cerf, 28 Sep. 2000
Internet History
NSFNET Emerges

- Simplified logical structure
  - some nets use links of others (e.g. PSTN dialup/leased lines)
- Gateways interconnect
  - no seamless addressing
  - mixed formats through gateways (e.g. %-hack)
  - UUNET becomes main UUCPNET–Internet gateway
Internet History
56 kb/s NSFNET Backbone

1986 – 1988

San Diego
Boulder
Urbana-Champaign
Pittsburgh
Princeton
Ithaca

56 kb/s leased PSTN lines
Internet History

448 kb/s over T1 NSFNET Backbone

1988 – 1989

448 kb/s link multiplexed on T1 lines
Internet History

1.5 Mb/s T1 NSFNET Backbone

1989 – 1990

15 Mb/s T1 lines
Internet History
1.5 Mb/s T1 NSFNET Backbone

1990 – 1992

Router
IBM PC-RT
token ring

Seattle
Salt Lake City
San Diego
Palo Alto
Boulder
Lincoln
Urbana-Champaign
Houston
Pittsburgh
Washington
Atlanta
Boston
Princeton
Ann Arbor
Ithaca
Palo Alto
Salt Lake City
San Diego
Palo Alto

15 Mb/s T1 lines

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Internet History

45 Mb/s T3 NSFNET Backbone

1992 – 1994

core    edge
Router
IBM RS/6000
FDDI

45 Mb/s T3 core network links
45 Mb/s T3 edge network links
Internet History

NSFNET Hierarchical Structure

- NSFNET Backbone
  - evolved from 56kb/s leased lines to 45Mb/s DS3
- Regional networks
  - MIDnet, NEARnet, NYSErnet, SURAnet, etc.
- Campus networks
  - KU, WashUStL, UMass, etc.
Internet History

NSFNET Hierarchical Structure

- Strict hierarchal structure
  - advantages?
Internet History

NSFNET Hierarchical Structure

- Strict hierarchical structure
  - simple to understand, measure, and analyse
  - traffic locality exploited
  - each level engineered for
    - local traffic characteristics
    - transit traffic between lower levels
Internet History

NSFNET Regional Network Examples

MIDnet: Midwest Network

NEARnet: New England Academic and Research Network
Internet History

NSFNET and MILNET become the US Internet

- Simplified logical structure
  - some nets use links of others (e.g. PSTN dialup/leased lines)
- Global Internet with NSFNET as backbone
  - seamless addressing using user@DNS and IP addresses
  - corporate networks convert to TCP/IP
  - MILNET is separate but interconnected US military backbone
Internet History
Privatisation

• 1991: NSF lifts access restrictions on NSFNET
  – use of email and netnews explodes
• 1993: NSF solicitation for privatisation plans
• 1995: NSF awards for
  – NAPs: network access point for ISP traffic exchange
  – RA: routing arbiter
  – vBNS: very high performance backbone network service
• 1995 April: NSFNET backbone decommissioned
• 1995 Sept.: NSF ends DNS registration subsidies
• 1998: NAPs and RA transferred to private sector
Internet History
Research Networks

• Internet precursors were research networks
  – privatisation constrained use by researchers
  – drove need for research infrastructure
    • separate from but attached to public Internet

• Research networks
  – US: Internet2 and NLR
  – Canada: CANARIE (Canadian Advanced Network and Research for Industry and Education)
  – Europe: GÉANT
  – UK: JANET
Research Networks

Internet2 Overview

- 1996: Internet2 forms among 34 universities
  - *not* an alternate or replacement for the Internet
    - this is a common misconception by the public
  - consortium of Universities and other research institutions
  - operates network infrastructure with fee to access
    - 2006: Internet2 network with $10 \times 10$Gb/s WDM links
Research Networks
Abilene Network

1999 – 2003

2.4 Gb/s OC-48 links

Internet2
GigaPoP

Los Angeles
Houston
Atlanta
Washington
Indianapolis
Chicago
New York
Denver
Kansas City
Palo Alto
Seattle

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Research Networks
Abilene Network

Seattle
Palo Alto
Los Angeles

Denver
Kansas City

Chicago
Indianapolis
New York
Washington
Atlanta
Houston

10 Gb/s OC-192 links

2004 – 2006

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SCN-ST-50
Research Networks
Internet2 Network

10x10 Gb/s links

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Research Networks
Internet2 Network with L2

10×10 Gb/s links
1991: KanREN (KS Research and Education Network)
- formed to expand connectivity in KS beyond MIDnet
- initial connectivity in 1993 funded by NSF
- GPN (1997) formed as successor to MIDnet
- KanREN attaches to Internet2 through GPN
- 2003 partnership with Kan-Ed for K-12 network
Internet Architecture
Current Structure: Tier 1

- Loose hierarchy but with many interconnections
  - no longer a map of the Internet; ISP internals proprietary

- Tier-1 backbone providers
  - largest with national or international high-speed backbones
  - interconnect with one-another at IXPs
    - Internet exchange points formerly NAP (network access points)
    - *peering* without charging one-another for traffic
  - by bilateral agreement, e.g. Sprint, AT&T
    - or sometimes not: Level3–Cogent peering war in 2005
  - do not purchase transit service from anyone else
Internet Architecture
Current Structure: Tier-1 Providers

- Tier 1 backbone providers
  - at&t
  - Centurylink (Qwest + SAVVIS)
  - Cogent
  - Deutsche Telekom
  - Global Telecom (Tinet/Tiscali Italy)
  - Level 3 with Global Crossing (GX)
  - NTT Communications (formerly Verio)
  - Orange
  - Sprint
  - Tata (India)
  - Telecom Italia
  - Telefonica (Spain)
  - TeliaSonera (Sweden – Finland)
  - Verizon Business (formerly UUNET)
  - XO
  - Zayo (formerly AboveNet)

[Wikipedia Feb 2016]

<table>
<thead>
<tr>
<th>Provider</th>
<th>AS</th>
<th>Value</th>
</tr>
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<tr>
<td>at&amp;t</td>
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<td>2403</td>
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<tr>
<td>Centurylink</td>
<td>AS0209</td>
<td>1580</td>
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<td>Cogent</td>
<td>AS0174</td>
<td>4212</td>
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<tr>
<td>Deutsche Telekom</td>
<td>AS3320</td>
<td>557</td>
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<tr>
<td>Global Telecom</td>
<td>AS3257</td>
<td>1432</td>
</tr>
<tr>
<td>Level 3</td>
<td>AS1/3356/3549</td>
<td>4260</td>
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<tr>
<td>NTT Communications</td>
<td>AS2914</td>
<td>1279</td>
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<td>Orange</td>
<td>AS5551</td>
<td>143</td>
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<td>Sprint</td>
<td>AS1239</td>
<td></td>
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<td>Tata (India)</td>
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<td>653</td>
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<td>Telefonica (Spain)</td>
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<tr>
<td>TeliaSonera (Sweden – Finland)</td>
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<tr>
<td>Verizon Business</td>
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<td>XO</td>
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<td>Zayo (formerly AboveNet)</td>
<td>AS6461</td>
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</tbody>
</table>
Internet Architecture
Current Structure Tiers 2 and 3

- Loose hierarchy but with many interconnections
  - no longer a map of the Internet; ISP internals proprietary
- Tier-1 backbone providers
- Tier-2 ISPs
  - purchases some transit traffic from tier-1 providers
  - peer directly with some networks (tier-1 and tier-2)
  - e.g. BT (AS5400) purchases from GX and Sprint
- Tier-3 local ISPs buys *all* transit service
  - purchases all transit traffic (from tier-2 and perhaps tier-1)
  - provide local Internet access
  - e.g. Sunflower from Sprint & Level3 [cidr-report.org]
Internet Architecture
Current Structure

tier-2 providers

tier-1 providers

NAP

peering points

local ISPs

access lines

NAP

providers
Internet Topology and Graph Spectra

Graph Spectra

ST.1 Multilevel graphs
ST.2 Internet topology
ST.3 Graph spectra
Representation of Graphs

Matrix Types

- **Adjacency matrix** $A(G)$
  - if nodes $i$ and $j$ are connected, $a_{ij}=1$; else 0

- **Laplacian matrix**
  - $L(G) = D(G) - A(G)$
  - $D(G)$ is diagonal matrix of node degrees
  - $L^+(G) = Q(G)$ is signless Laplacian

- **Normalised Laplacian matrix**
  
  \[
  \mathcal{L}(G) = \begin{cases} 
  1, & \text{if } i = j \text{ and } d_i \neq 0 \\
  -1 / \sqrt{d_i d_j}, & \text{if } v_i \text{ and } v_j \text{ are adjacent} \\
  0, & \text{otherwise}
  \end{cases}
  \]

  - *allow us to compare structure of graphs of different sizes*
Matrix Types

Examples

\[ A(G) \]

\[
\begin{bmatrix}
  v_0 & v_1 & v_2 & v_3 \\
 0 & 0 & 1 & 1 \\
 0 & 0 & 0 & 1 \\
 1 & 0 & 0 & 1 \\
 1 & 1 & 1 & 0 \\
\end{bmatrix}
\]

\[ D(G) \]

\[
\begin{bmatrix}
  v_0 & v_1 & v_2 & v_3 \\
 2 & 0 & 0 & 0 \\
 0 & 1 & 0 & 0 \\
 0 & 0 & 2 & 0 \\
 0 & 0 & 0 & 3 \\
\end{bmatrix}
\]

\[ L(G) \]

\[
\begin{bmatrix}
  v_0 & v_1 & v_2 & v_3 \\
 2 & 0 & -1 & -1 \\
 0 & 1 & 0 & -1 \\
 -1 & 0 & 2 & -1 \\
 -1 & -1 & -1 & 3 \\
\end{bmatrix}
\]

\[ Q(G) \]

\[
\begin{bmatrix}
  v_0 & v_1 & v_2 & v_3 \\
 1 & 0 & -0.5 & -0.4 \\
 0 & 1 & 0 & -0.6 \\
 -0.5 & 0 & 1 & -0.4 \\
 -0.4 & -0.6 & -0.4 & 1 \\
\end{bmatrix}
\]

\[ L(G) \]

\[
\begin{bmatrix}
  v_0 & v_1 & v_2 & v_3 \\
 1 & 0 & -0.5 & -0.4 \\
 0 & 1 & 0 & -0.6 \\
 -0.5 & 0 & 1 & -0.4 \\
 -0.4 & -0.6 & -0.4 & 1 \\
\end{bmatrix}
\]
Graph Spectra
Eigenvalues and Eigenvectors

- Given a matrix $M$, eigenvalues $\lambda$, and eigenvectors $x$
- Eigenvalues and eigenvectors satisfy,
  - $Mx = \lambda x$
- Eigenvalues are roots of characteristic polynomial
  - $\det(M - \lambda I)$ for $x \neq 0$
- Spectrum of $M$ is its eigenvalues and multiplicities
  - multiplicity is the number of occurrences of an eigenvalue
Graph Spectra

Important Characteristics of NLS

• Eigenvalues of normalised Laplacian spectra (nls)
  – \( \{0 = \lambda_1 \leq \lambda_2 \leq ... \leq \lambda_n \leq 2\} \)
  – number of 0s represent number of connected components

• \textit{Quasi-symmetric} about 1

• Spectral radius \( \rho(L) \): largest eigenvalue
  – if \( \rho(L) = 2 \), then the graph is bipartite
  – closer to 2 means nearly bipartite

• \( \lambda_2(L) \leq 1 \) for non-complete (non-full-mesh) graphs
  – \( \lambda_2(L) \geq 1/(2eD) \geq 0 \)
  • where \( e \) is the graph size (# of links) and \( D \) is graph diameter
Topological Dataset
US Interstate Highways

- Added 5 highways, 6 interchange nodes, 2 pendants [AASHTO]
Topological Dataset
PoP-Level Topologies – AT&T and Sprint

- Included only 48 US contiguous states [Rocketfuel]
Topological Dataset
Fibre-Optic Routes – AT&T and Sprint

- Included only 48 US contiguous states [KMI]
Topography Analysis of Graphs

Metrics

• Baseline networks *Lecture SCN-RN*
  – star, linear, tree, ring, grid, toroid, mesh
  – order (number of nodes) of the baseline graphs: 10 and 100

• Average degree is same for star, linear, tree
  – *do these graphs share same structural properties?*

• Clustering coefficient is too coarse for baseline graphs
  – except for mesh which the value is 1, rest is 0

• Algebraic connectivity $a(G)$:
  – second smallest eigenvalue of the Laplacian matrix
  – $a(G) = 0.1$ for $n=10$ (linear) and $n=100$ (grid)
  – not very useful for comparing different order graphs
## Topological Analysis via Metrics

### Baseline Networks with n=10

<table>
<thead>
<tr>
<th>Topology</th>
<th>Star</th>
<th>Linear</th>
<th>Tree</th>
<th>Ring</th>
<th>Grid</th>
<th>Toroid</th>
<th>Mesh</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of nodes</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Number of links</td>
<td>9</td>
<td>9</td>
<td>9</td>
<td>10</td>
<td>13</td>
<td>15</td>
<td>45</td>
</tr>
<tr>
<td>Maximum degree</td>
<td>9</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>Average degree</td>
<td>1.8</td>
<td>1.8</td>
<td>1.8</td>
<td>2</td>
<td>2.6</td>
<td>3</td>
<td>9</td>
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<tr>
<td>Degree assortativity</td>
<td>-1</td>
<td>-0.13</td>
<td>-0.53</td>
<td>1</td>
<td>0.28</td>
<td>1</td>
<td>1</td>
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<tr>
<td>Node closeness</td>
<td>0.58</td>
<td>0.29</td>
<td>0.37</td>
<td>0.36</td>
<td>0.44</td>
<td>0.53</td>
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<td>Clustering coefficient</td>
<td>0</td>
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<td>0</td>
<td>0</td>
<td>0</td>
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<td>Algebraic connectivity</td>
<td>1</td>
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<td>0.18</td>
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<td>Network diameter</td>
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<td>5</td>
<td>5</td>
<td>5</td>
<td>3</td>
<td>1</td>
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<tr>
<td>Network radius</td>
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<td>5</td>
<td>3</td>
<td>5</td>
<td>3</td>
<td>3</td>
<td>1</td>
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<tr>
<td>Average hop count</td>
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<td>3.67</td>
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<td>25</td>
<td>24</td>
<td>13</td>
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<td>6</td>
<td>1</td>
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# Topological Analysis via Metrics

## Baseline Networks with n=100

<table>
<thead>
<tr>
<th>Topology</th>
<th>Star</th>
<th>Linear</th>
<th>Tree</th>
<th>Ring</th>
<th>Grid</th>
<th>Toroid</th>
<th>Mesh</th>
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<tbody>
<tr>
<td>Number of nodes</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
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<tr>
<td>Number of links</td>
<td>99</td>
<td>99</td>
<td>99</td>
<td>100</td>
<td>180</td>
<td>200</td>
<td>4950</td>
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<td>Maximum degree</td>
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<td>2</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>4</td>
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<tr>
<td>Average degree</td>
<td>1.98</td>
<td>1.98</td>
<td>1.98</td>
<td>2</td>
<td>3.6</td>
<td>4</td>
<td>99</td>
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<tr>
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<td>0.13</td>
<td>0.04</td>
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<td>10</td>
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Topography Analysis of Graphs
Spectra

- Normalised Laplacian spectrum
  - since normalised, eigenvalues range [0,2]
- Calculate:
  - RF: relative frequency [BJ2009]
  - RCF: relative cumulative frequency [VHE2002]
- Eigenvalue of 2 indicates how bipartite a graph is
- Eigenvalue 1 multiplicity indicates node duplications
  - nodes having similar neighbours
- Spectrum is symmetric around 1
Graph Spectrum
RF for Baseline Networks with n=100

- RF of eigenvalue multiplicities is noisy
  - mesh and star graphs look similar (except $\lambda = 2$ eigenvalue)
Graph Spectrum

RCF for Baseline Networks with n=100
Topology Analysis of Real Networks
Metrics and Spectra

- Communication and transportation networks
- Metrics indicate:
  - physical topologies are closer to transportation network
  - difference between physical and logical topologies
    - higher number of nodes in physical topologies
    - rich connectivity in logical topologies
## Topological Analysis via Metrics

### Real Networks

<table>
<thead>
<tr>
<th>Topology</th>
<th>Sprint Physical</th>
<th>Sprint Logical</th>
<th>AT&amp;T Physical</th>
<th>AT&amp;T Logical</th>
<th>US Highways</th>
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<tbody>
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<td>28</td>
<td>361</td>
<td>107</td>
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<tr>
<td>Number of links</td>
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</table>
• Spectrum of physical topologies resemble motorways
Network Topology
Fiber Relation to Other Infrastructure

L1
Class 1 rail mainlines
Interstate freeways
## Spectral Properties of Real Networks

**cTGD vs. Algebraic Conn. & Spectral Radius**

<table>
<thead>
<tr>
<th>Topology</th>
<th>cTGD</th>
<th>cTGD Rank</th>
<th>a(G)</th>
<th>a(G) Rank</th>
<th>(\rho(L))</th>
<th>(\rho(L)) Rank</th>
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Graph Spectrum
RCF for Communication Networks

- Spectrum of physical topologies resemble motorways
  - Level 3 is richly connected and have small spectral radius
Analysis of Internet Infrastructure

Summary

• Physical topologies resemble motorways
  – known: but not rigorously studied
  – grid-like structures

• The normalised Laplacian spectrum is powerful
  – spectral radius indicates bipartiteness
  – $\lambda = 1$ multiplicity indicates duplicates (i.e. star-like structures)

• Future work:
  – study other physical critical infrastructures
    • railways, power grid, pipelines
  – investigate metrics and relationship to resiliency/connectivity
Analysis of Internet Infrastructure

Review of Graph Spectra

• All but structural graphs have same nodes
Graph Theory
References and Further Reading


• [KMI] KMI Corporation, “North American Fiberoptic Long-haul Routes Planned and in Place”, 1999

• [KU-TopView] https://www.ittc.ku.edu/resilinets/maps
Graph Spectra

References and Further Reading

Graph Spectra

References and Further Reading

Graph Spectra

References and Further Reading

- [CS2011] Dragoš Cvetković and Slobodan Simić, “Graph spectra in Computer Science”, *Linear Algebra and its Applications*, vol. 434, no. 6, March 2011, pp. 1545 – 1562


End of Foils