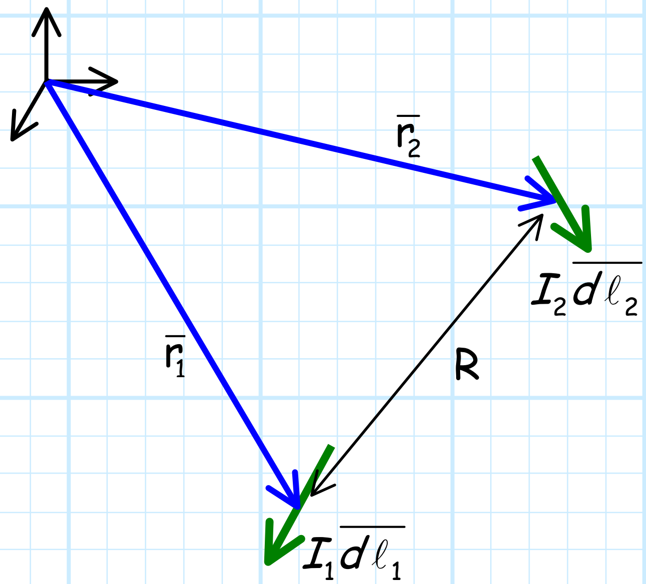
Ampere's Law of Force

Consider the case of two **current filaments** located in space.

One filament has current I_1 flowing along differential displacement distance $\overline{d\ell_1}$, while the **other** has current I_2 flowing along $\overline{d\ell_2}$.



We find that each current filament exerts **force** $d\mathbf{F}$ on the other!

The force depends on the **magnitude** and **direction** of **each** filament vector (I $\overline{d\ell}$), as well as on the **distance** R between the two currents.

Andre Ampere determined this relationship in the 18th century, and we call his result **Ampere's Law of Force**:

$$d\mathbf{F}_1 = \frac{\mu_0}{4\pi} \frac{I_1\,\overline{d\ell_1} \times \left(I_2\,\overline{d\ell_2} \times \hat{a}_{21}\right)}{R^2}$$

Q: *Yikes! What the heck does this mean ?*

A: Well:

* The **unit vector** $\hat{a}_{21}$ is the unit vector directed from filament 2 to filament 1 (just like Coulomb's Law).

* The constant μ_0 is the **permeability of free space**, given as:

$$\mu_0 = 4\pi \times 10^{-7} \quad \left[N / A^2 = \frac{Henry}{meter} \right]$$

* The force $d\mathbf{F}_1$ is the force exerted **on** filament 1 **by** filament 2.

Q: *O.K., but what about:*

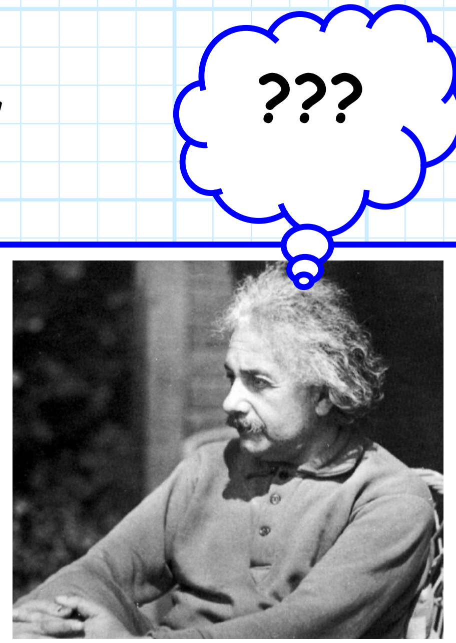
$$I_1 \overline{d\ell_1} \times \left(I_2 \overline{d\ell_2} \times \hat{a}_{21} \right) \quad ?!?$$

A: Using equation B.2 of your book (p. 639), we can rewrite this in terms of the **dot product**!

$$I_1 \overline{d\ell_1} \times \left(I_2 \overline{d\ell_2} \times \hat{a}_{21} \right) = \left(\overline{d\ell_1} \cdot \hat{a}_{21} \right) \overline{d\ell_2} - \left(\overline{d\ell_1} \cdot \overline{d\ell_2} \right) \hat{a}_{21}$$

Therefore, we can **also** write Ampere's Law of Force as:

$$d\mathbf{F}_1 = \frac{\mu_0 \, I_1 \, I_2}{4\pi} \frac{\left(\overline{d\ell_1} \cdot \hat{a}_{21} \right) \overline{d\ell_2} - \left(\overline{d\ell_1} \cdot \overline{d\ell_2} \right) \hat{a}_{21}}{R^2}$$



*See! Didn't **that** help?*

Perhaps **not**. To interpret the result above, we need to look at several **examples**.

But first, let's examine one **very important** property of Ampere's Law of Force. Consider the force **on** filament 2 **by** filament 1—exactly the **opposite** case considered earlier.

We find from Ampere's Law of force:

$$d\mathbf{F}_2 = \frac{\mu_0 I_2 I_1}{4\pi} \frac{\left(\overline{d\ell_2} \cdot \hat{a}_{12} \right) \overline{d\ell_1} - \left(\overline{d\ell_2} \cdot \overline{d\ell_1} \right) \hat{a}_{12}}{R^2}$$

Note in the **numerator** there are **two** vector terms. Let's **compare** them.

We find that the **second** terms in each force expression have **equal** magnitude but **opposite** direction, because $\hat{a}_{12} = -\hat{a}_{21}$.

$$\left(\overline{d\ell_1}\cdot\overline{d\ell_2}\right)\hat{a}_{21} = -\left(\overline{d\ell_2}\cdot\overline{d\ell_1}\right)\hat{a}_{12}$$

However, the **first** vector terms in each expression are related in **neither** magnitude **nor** direction !

$$\left(\overline{d\ell_1}\cdot\hat{a}_{21}\right)\overline{d\ell_2} \neq \left(\overline{d\ell_2}\cdot\hat{a}_{12}\right)\overline{d\ell_1}$$

Therefore, we discover that, in general, the force $d\mathbf{F}_1$ on filament 1, and the force $d\mathbf{F}_2$ on filament 2 are **not** related in **either** magnitude or in direction:

$$d\mathbf{F}_1 \neq d\mathbf{F}_2$$

In fact, we can have situations where the force on one element is **zero**, while the force on the other element is **not!**

This, of course, is much **different** than **Coulomb's Law of Force**, where we found that $\mathbf{F}_1 = -\mathbf{F}_2$ **always**.

André-Marie Ampère **(1775-1836)** was a child prodigy whose early life was marred by tragedy: Ampère's father was beheaded in his presence during the Revolution and, later, his wife died four years after their marriage. As a scientist, Ampère had flashes of inspiration which he would pursue to their conclusion. When he learned of Ørsted's discovery in 1820 that a **magnetic** needle is deflected by a varying nearby **current**, he prepared within a week the first of several papers on the theory of this phenomenon, formulating the law of **electromagnetism** (Ampère's law) that describes mathematically the **magnetic force** between two **circuits**. (*from www.ee.umd.edu/~taylor/frame3.htm*)