An Infinite Charge Plane

Say that we have a very large charge disk. So large, in fact, that its radius $a$ approaches infinity!

**Q:** What electric field is created by this infinite plane?

**A:** We already know! Just evaluate the charge disk solution for the case where the disk radius $a$ is infinity.

In other words:

$$\lim_{a \to \infty} E(x = 0, y = 0, z) = \begin{cases} 
\hat{a}_z \frac{\rho_s}{2\varepsilon_0} \left[1 - \frac{z}{\sqrt{z^2 + a^2}}\right] & \text{if } z > 0 \\
\hat{a}_z \frac{\rho_s}{2\varepsilon_0} \left[-1 - \frac{z}{\sqrt{z^2 + a^2}}\right] & \text{if } z < 0 \\
\frac{\rho_s}{2\varepsilon_0} \hat{a}_z & \text{if } z > 0 \\
-\frac{\rho_s}{2\varepsilon_0} \hat{a}_z & \text{if } z < 0
\end{cases}$$

Therefore, the electric field produced by an infinite charge plane, with surface charge density $\rho_s$, is:
Think about what this says!

* First, we note that the electric field points away from the plane if $\rho_s$ is positive, and toward the plane if $\rho_s$ is negative.

* Second, we notice that the magnitude of the electric field is a constant—the magnitude is independent of the distance from the infinite plane!
The reason for this result is, that no matter how far you are (i.e., $|z|$) from the infinite charge plane, you remain infinitely close to plane, when compared to its radius $a$.

We will find these results are useful when we study the behavior of a parallel plate capacitor.