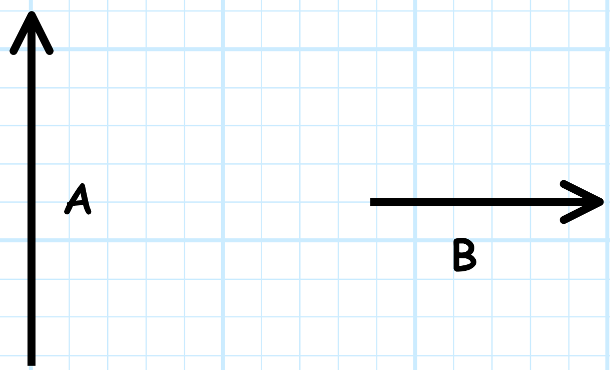
Arithmetic Operations of Vectors

Vector Addition

Consider two vectors, denoted **A** and **B**.

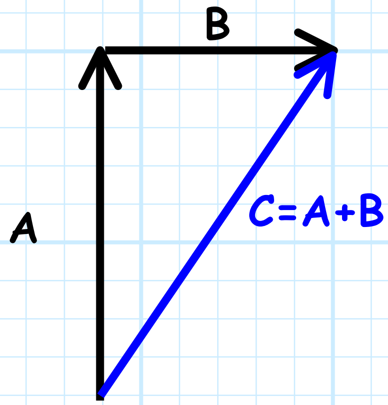


Q: *Say we **add** these two vectors together; what is the **result**?*

A: The addition of two vectors results in **another vector**, which we will denote as **C**. Therefore, we can say:

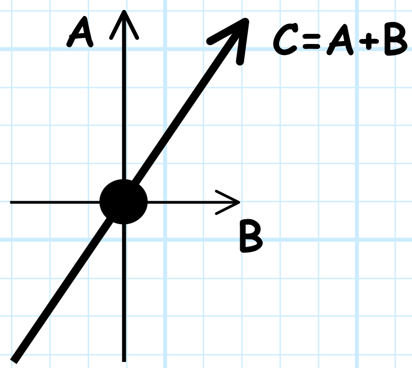
$$\mathbf{A} + \mathbf{B} = \mathbf{C}$$

The **magnitude** and **direction** of **C** is determined by the **head-to-tail rule**.



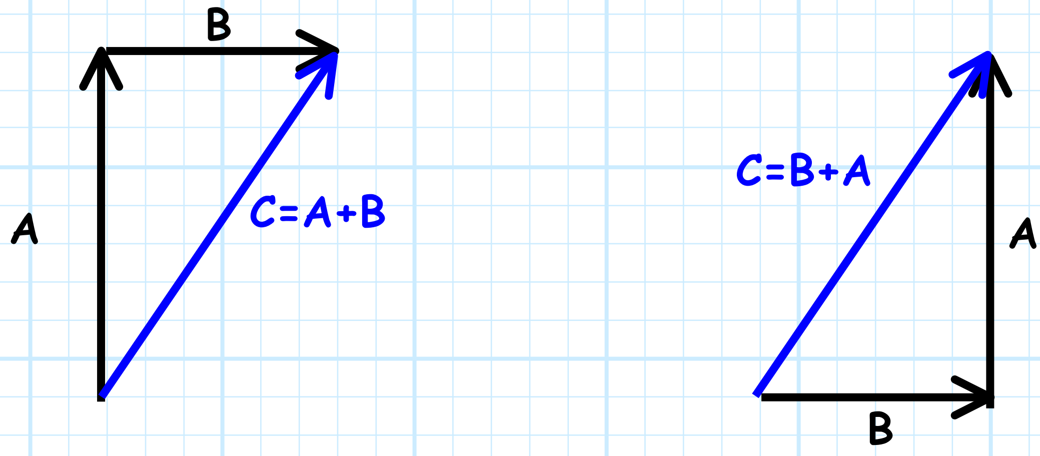
This is not a **provable** result, rather the head-to-tail rule is the **definition** of vector addition. This definition is used because it has many **applications** in physics.

For **example**, if vectors **A** and **B** represent two **forces** acting an object, then vector **C** represents the **resultant force** when **A** and **B** are simultaneously applied.

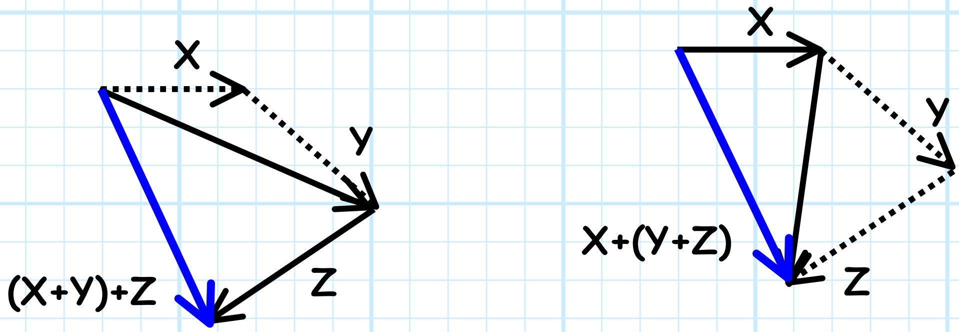


Some important properties of vector addition:

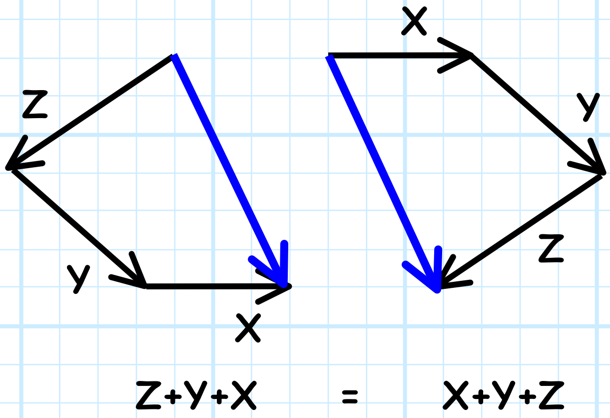
1. Vector addition is **commutative** -> **A+B = B+A**



2. Vector addition is **associative** -> **(X+Y) + Z = X + (Y+Z)**



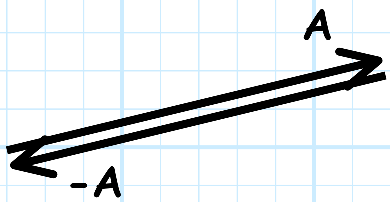
From these two properties, we can conclude that the addition of **several** vectors can be executed in **any order**:



Vector Subtraction

First, we define the **negative** of a vector to be a vector with **equal magnitude** but **opposite direction**.

Note that **A + (-A)** = 0



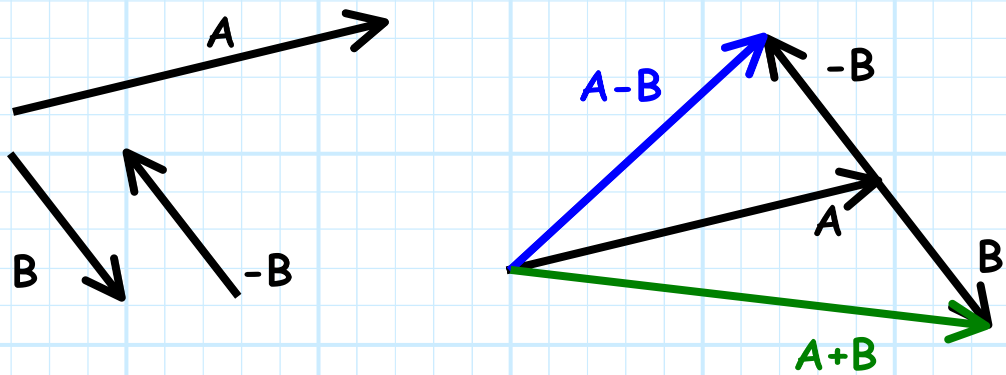
We can therefore consider the addition of a negative vector as a **subtraction**:

A - A = 0

More generally, we can write:

A + (-B) = A - B

E.G.,



Q: *Is A-B = B-A ?*

A: What do **you** think ?