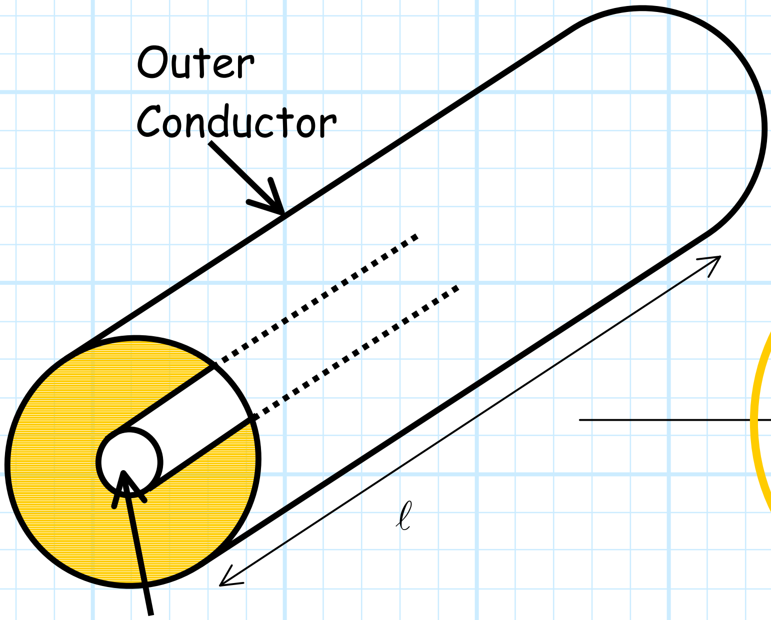
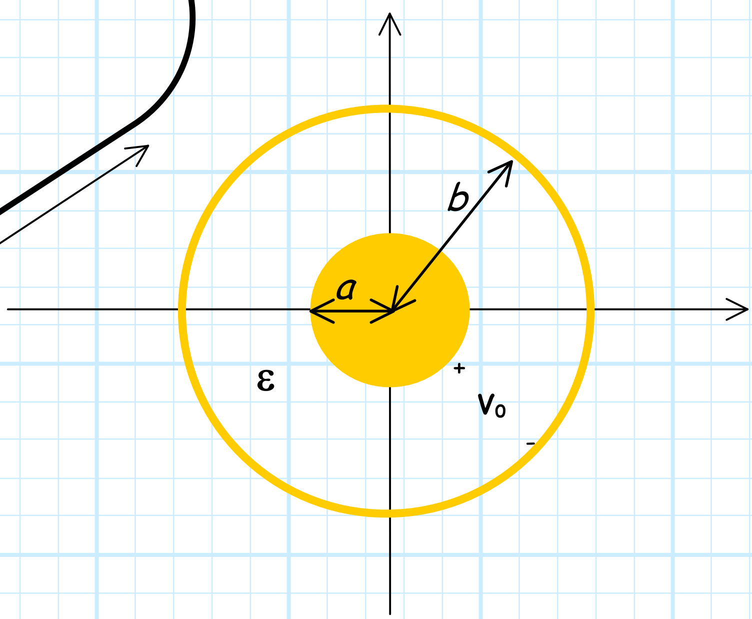
Capacitance of a Coaxial Transmission Line

Recall the geometry of a **coaxial transmission line**:





Coax Cross-Section

We earlier determined that if a **potential difference** of V_0 volts is placed across the conductors, the **surface charge density** on the **inner** conductor is:

$$\rho_{sa}(\bar{r}) = \frac{\varepsilon V_0}{\ln[b/a]} \frac{1}{a} \qquad (\rho = a)$$

The **total charge** Q on the **inner** conductor of a coax of length ℓ is determined by **integrating** the surface charge density across the **conductor surface**:

$$
\begin{aligned}
Q &= \oiint_{S_+} \rho_{sa}(\bar{r})\, ds \\
&= \int_0^{\ell}\int_0^{2\pi} \frac{\varepsilon V_0}{\ln[b/a]}\frac{1}{a}\ \rho\, d\phi\, dz \\
&= \frac{\varepsilon V_0}{\ln[b/a]}\frac{1}{a}\int_0^{\ell}\int_0^{2\pi} a\, d\phi\, dz \\
&= \frac{\varepsilon V_0}{\ln[b/a]}\frac{1}{a}\ \ell(2\pi a) \\
&= V_0 \frac{2\pi\varepsilon}{\ln[b/a]}\ell
\end{aligned}
$$

Note since $\rho_{sa}(\bar{r}) = \mathbf{D}(\bar{r}) \cdot \hat{a}_n$, we would have arrived at the **same** result by using:

$$
Q = \oiint_{S_+} \varepsilon \mathbf{E}(\bar{r}) \cdot \overline{ds}
$$

We can now determine the **capacitance** of this coaxial line!

Since $C = Q/V$, and since the **potential difference** between the conductors is $V = V_0$, we find:

$$
\begin{aligned}
C = \frac{Q}{V} &= \left(V_0 \frac{2\pi\varepsilon}{\ln[b/a]}\ell\right)\left(\frac{1}{V_0}\right) \\
&= \frac{2\pi\varepsilon}{\ln[b/a]}\ell
\end{aligned}
$$

This value represents the capacitance of a coaxial line of length ℓ. A more useful expression is the capacitance of a coaxial line **per unit length** (e.g. farads/meter). We find this simply by **dividing** by length ℓ:

$$\frac{C}{\ell} = \frac{2\pi\,\varepsilon}{\ln\left[b/a\right]} \qquad \left[\frac{\text{farads}}{\text{meter}}\right]$$

Note the **longer** the transmission line, the **greater** the capacitance!

This can cause **great difficulty** if the voltage across the transmission line conductors is **time varying** (as it almost certainly will be!).

For **long** transmission lines, engineers cannot consider a transmission line simply as a "**wire**" conductor that connects circuit elements together. Instead, capacitance (and inductance) make the transmission line **itself** a **circuit element**!

In this case, engineers must use **transmission line theory** to analyze circuits !